ELECTROMAGNETIC METHODS OF NONDESTRUCTIVE TESTING IN FRACTURE MECHANICS

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ABSTRACT

The paper presents the basic types of singular integral equations which arise in the theory of electromagnetic nondestructive testing. The quadrature formulae for evaluation of integrals with logarithmic singularity as well as hypersingular (according to Hadamard) integrals are supposed. The direct numerical algorithms of these equations solution are constructed and discussed.

KEYWORDS

Nondestructive testing, diffraction theory, singular integral equations, numerical methods.

INTRODUCTION

Modern calculation of construction elements durability which is founded on the fracture mechanics approaches includes an information on their defects. Such information frequently can be obtained using the electromagnetic methods of nondestructive testing. Diffraction theory forms a basis for these methods. Take into account a wide frequency range of sounding, arbitrary geometry of the defects and their location we can perform computer modelling of the wave interaction with a material inhomogeneities. While solving the scalar problems of diffraction theory one should use the approach supposed by Panasyuk et al. (1984), Nazarchuk (1989).

INTEGRAL EQUATIONS OF THE PROBLEM

Let us designate by L the set of cylindrical surfaces directrices lying in plane xOy and by $W(x, y)$ - the longitudinal (along axis Oz) component of the scattered field. Let it be required to find the finite continuous function $W(x, y)$ satisfying the Helmholtz equation

$$\left(\Delta + \chi^2\right) W = 0, \tag{1}$$

the edge condition of type

$$\lim_{\rho \to 0} \rho \operatorname{grad} W = 0 \tag{2}$$

(ρ is the radius of small circle enclosing irregular point of contour L), the radiation condition of form

$$W \sim e^{i\beta r} O\left(\frac{1}{\sqrt{r}}\right), \quad \frac{\partial W}{\partial r} - i\chi W \sim o\left(\frac{1}{\sqrt{r}}\right) e^{-i\beta r}, \quad \beta = \Im \chi \geq 0, \quad r \to \infty \tag{3}$$

(symbols $\Re$ and $\Im$ designate the real and imaginary value parts) and one of the boundary conditions

$$W(x,y)\Big|_L = -E^*(s), \qquad \frac{\partial W(x,y)}{\partial n}\Big|_L = -\frac{\partial H^*(s)}{\partial n} \tag{4}$$

on contour L (E^* and H^* are the definite functions of arc abscissa s). We consider the solution of the diffraction problem formulated in such a way exists and is a unique one.

Let's agree that symbol z will further designate the affix of point $M(x,y)$ assuming $z = x + iy$. If point $M(x,y)$ with affix z belongs to contour L, it (as well as its affix) will be designated through t. And, finally, complex conjugate values will be designated by a top bar. Let, for definiteness, contour L consists of N open and closed Lyapunov curves L_k, $k = \overline{1,N}$ (contour L_k will be considered the Lyapunov curve if angle ψ_k between the positive normal to L_k and axis O_kx_k as the function of point t_k satisfies the Hölder condition $|\psi_k(t_2) - \psi_k(t_1)| \leq A|t_2 - t_1|^{\mu}$, $0 < \mu \leq 1$). Each of the curves L_k will be considered as a simple one and related to the local Cartesian system $x_kO_ky_k$. In the basic system xOy points O_k are determined by the complex coordinates $z_k^0 = x_k^0 + iy_k^0$ and axes O_kx_k form angles α_k with axis Ox.

Let's introduce the function into consideration

$$W(z,\bar z) = \frac{\pi i}{2}\sum_{k=1}^{N}\int_{L_k}\left[j_k(s)H_0^{(1)}(\chi r_k) - \chi m_k(s)H_1^{(1)}(\chi r_k)\Re\left(\frac{T_k - z}{r_k}e^{-i\Psi_k}\right)\right]ds, \tag{5}$$

where $\Psi_k = \psi_k + \alpha_k$; $r_k = |T_k - z|$; $T_k = t_k\exp(i\alpha_k) + z_k^0$, the densities $j_k(s)$ and $m_k(s)$ are determined by relations

$$j_k(s) = -\frac{1}{2\pi}\left(\frac{\partial W^+}{\partial n} - \frac{\partial W^-}{\partial n}\right), \qquad m_k(s) = \frac{1}{2\pi}\left(W^+ - W^-\right), \tag{6}$$

the superscripts "+" and "−" mark the boundary value when it tends to contour L_k from the left or from the right.

In case of separate location of curves L_k formula (5) gives solution of the boundary value problems (1)-(4) as specified jumps of potential W and its normal derivative $\partial W/\partial n$ on N contours L_k, $k = \overline{1,N}$. In this case to obtain the expression of function W in the local system $x_\nu O_\nu y_\nu$ it is sufficient to put $z = z_\nu^0 + z_\nu\exp(i\alpha_\nu)$ in (5). It can be shown that representation (5) remains valid for the cases when some of the arcs L_k have common end-points. In this case in (6) the boundary values of the Cauchy type integrals at the inner points of curves L_k are taken according to the usual Sokhotskiy-Plemelj formulae. The boundary value at end-points of the arcs L_k are taken as the sum of the boundary values of all the terms included into (5).

Accounting for the above said for the boundary values of potential (5) and its normal derivative we shall obtain the representation

$$W^{\pm}\left(T_\nu^0,\bar T_\nu^0\right) = \pm\pi m_\nu(s_0) + W\left(T_\nu^0,\bar T_\nu^0\right); \quad \frac{\partial W^{\pm}\left(T_\nu^0,\bar T_\nu^0\right)}{\partial n_0} = \mp\pi j_\nu(s_0) + \frac{\partial W\left(T_\nu^0,\bar T_\nu^0\right)}{\partial n_0}, \tag{7}$$

where the direct values of the respective functions are given by the formulae

$$W\left(T_\nu^0,\bar T_\nu^0\right) = \frac{\pi i}{2}\sum_{k=1}^{N}\int_{L_k}\left\{j_k(s)H_0^{(1)}(\chi r_k) - m_k(s)\left[\frac{2}{\pi i}\Re\left(\frac{e^{i\Psi_k}}{T_k - T_\nu^0}\right) + \right.\right.$$

$$\left.\left. + \chi H_1(\chi r_k)\Re\left(\frac{T_k - T_\nu^0}{r_k}e^{-i\Psi_k}\right)\right]\right\}ds, \quad H_1^{(1)}(z) = \frac{2}{\pi i z} + H_1(z), \quad H_1(0) = 0;$$

$$\frac{\partial W\left(T_\nu^0,\bar T_\nu^0\right)}{\partial n_0} = \sum_{k=1}^{N}\int_{L_k}\left\{j_k(s)\left[\Re\left(\frac{e^{i\Psi_\nu^0}}{T_k - T_\nu^0}\right) + \frac{\pi i}{2}\chi H_1(\chi r_k)\Re\left(\frac{T_k - T_\nu^0}{r_k}e^{-i\Psi_\nu^0}\right)\right] + \right.$$

$$+ m_k(s)\left[-\Re\left(\frac{e^{i(\Psi_k + \Psi_\nu^0)}}{(T_k - T_\nu^0)^2}\right) + \frac{\pi i}{4}\chi^2\left(H_0^{(1)}(\chi r_k)\Re(e^{i(\Psi_k - \Psi_\nu^0)}) - \right.\right. \tag{8}$$

$$\left.\left.\left. - H_2(\chi r_k)\Re\left(\frac{\bar T_k - \bar T_\nu^0}{T_k - T_\nu^0}e^{i(\Psi_k + \Psi_\nu^0)}\right)\right)\right]\right\}ds, \quad H_2^{(1)}(z) = H_2(z) + \frac{4}{\pi i z^2}, \quad H_2(0) = \frac{1}{\pi i}.$$

The hypersingular integral figuring in (8) at $k = \nu$ is understood in the sense

$$\int_{L_\nu} m_\nu(s)\Re\left(\frac{e^{i(\psi_\nu + \psi_\nu^0)}}{(t_\nu - t_\nu^0)^2}\right)ds = \int_{L_\nu^*} m_\nu(s)\Re\left(\frac{e^{i(\psi_\nu + \psi_\nu^0)}}{(t_\nu - t_\nu^0)^2}\right)ds + 2m_\nu(s_0)\Im\left(\frac{e^{i\psi_\nu^0}}{\varepsilon_\nu}\right), \tag{9}$$

where $L_\nu^* = L_\nu \backslash Q(\varepsilon_\nu, t_\nu^0)$, $Q(\varepsilon_\nu, t_\nu^0) = (t_\nu^0 - \varepsilon_\nu,\ t_\nu^0 + \varepsilon_\nu)$, the arc abscissa s_0 corresponds to point $t_\nu^0 \in L_\nu$.

With the structure excited by the E-polarized wave the system of integral equations due to (4) will be written in the form

$$\sum_{k=1}^{N}\int_{L_k} j_k(s)\frac{\pi i}{2}H_0^{(1)}(\chi r_k)ds = -E^*(T_\nu^0,\bar T_\nu^0), \quad r_k = |T_k - T_\nu^0|, \quad \nu = \overline{1,N}. \tag{10}$$

With it being solved, the diffracted field at an arbitrary point z of plane xOy will be determined according to the formula

$$E(z,\bar z) = E^*(z,\bar z) + \frac{\pi i}{2}\sum_{k=1}^{N}\int_{L_k} j_k(s)H_0^{(1)}(\chi r_k)ds, \quad r_k = |T_k - z|. \tag{11}$$

When the H-polarized wave falls ($W \equiv H'$) for the diffracted field we shall have the representation

$$H(z,\bar z) = H^*(z,\bar z) - \chi\frac{\pi i}{2}\sum_{k=1}^{N}\int_{L_k} m_k(s)H_1^{(1)}(\chi r_k)\Re\left(\frac{T_k - z}{r_k}e^{-i\Psi_k}\right)ds, \quad r_k = |T_k - z|, \tag{12}$$

and the densities of the transversal currents will be determined from the system of integral equations

$$\sum_{k=1}^{N}\int_{L_k} m_k(s)\frac{\pi i}{4}\chi^2\left[H_0^{(1)}(\chi r_k)\Re(e^{i(\Psi_k - \Psi_\nu^0)}) - H_2^{(1)}(\chi r_k)\Re\left(\frac{\bar T_k - \bar T_\nu^0}{T_k - T_\nu^0}e^{i(\Psi_k + \Psi_\nu^0)}\right)\right]ds =$$

$$= -\frac{\partial H^*(T_\nu^0,\bar T_\nu^0)}{\partial n_0}, \quad r_k = |T_k - T_\nu^0|, \quad \nu = \overline{1,N}, \tag{13}$$

which contains at $k = \nu$ the logarithmic and hypersingular integrals.

From (7) and (8) the generalization of the equations in case of impedance conditions on contours L_k of the form

$$\left(W + \alpha\frac{\partial W}{\partial n}\right)\Big|_L = F(s) \tag{14}$$

is evident. Note that at the end-point c_r (on the arc L_k the arc abscissa s_r corresponds to it), which is not common for several contours, from the condition (2) of the boundary value problem the equality $m_k(s_r) = 0$, $r = \overline{1,M} \le 2N$ follows, which is necessary to be taken into account when building up the algorithm of numerical solution of (13). The case $M = 2N$ corresponds to separate location of open cylindrical surfaces and allows for the transformation of (13) into the system of integrodifferential equations of the form

$$\sum_{k=1}^{N}\Big\{\int_{L_k}\Big[m'_k(s)\Re\Big(\frac{e^{i\varphi_\nu^0}}{i(T_k-T_\nu^0)}\Big)+m_k(s)\frac{\pi i}{4}\chi^2\Big(H_0^{(1)}(\chi r_k)\Re(e^{i(\varphi_k-\varphi_\nu^0)})-$$

$$-H_2(\chi r_k)\Re\Big(\frac{\bar T_k-\bar T_\nu^0}{T_k-T_\nu^0}e^{i(\varphi_k+\varphi_\nu^0)}\Big)\Big)\Big]ds\Big\}=-\frac{\partial H^*(T_\nu^0,\bar T_\nu^0)}{\partial n_0},\quad \nu=\overline{1,N}. \qquad (15)$$

The same equations are obtained from the system (13) also for the problem of diffraction by N perfectly conducting cylinders of an arbitrary profile due to the known formulae for differentiating the integrals of Cauchy type along the closed contour.

Let in the local coordinate system $x_kO_ky_k$ the parametric equation of the L_k arc be known: $t_k = t_k(\tau)$, $-1 \le \tau \le 1$. Then the equations included into the systems (10), (13), (15) have, respectively, the structure

$$-\int_{-1}^{1} j(\tau)\ln|\tau-\xi|d\tau+\int_{-1}^{1} j(\tau)K_e(\tau,\xi)d\tau=F_e(\xi),\quad -1<\xi<1;$$

$$\int_{-1}^{1}\frac{m(\tau)}{(\tau-\xi)^2}d\tau-\int_{-1}^{1}m(\tau)R(\tau,\xi)\ln|\tau-\xi|d\tau-\frac{1}{2}\Big[f_1(\xi)\int_{-1}^{1}\frac{m(\tau)}{(\tau-z)^2}d\tau+$$

$$+f_2(\xi)\int_{-1}^{1}\frac{m(\tau)}{(\tau-\bar z)^2}d\tau\Big]+\int_{-1}^{1}m(\tau)K_h(\tau,\xi)d\tau=F_h(\xi),\quad z\notin[-1,1]; \qquad (16)$$

$$\int_{-1}^{1}\frac{m'(\tau)}{\tau-\xi}d\tau-\int_{-1}^{1}m(\tau)R(\tau,\xi)\ln|\tau-\xi|d\tau+\int_{-1}^{1}m(\tau)K_h(\tau,\xi)d\tau=F_h(\xi),$$

where all the functions occurred are regular in the domain of their arguments variation, the complex variable z may be located near-by the segment $[-1,1]$.

When among the arcs L_k there are closed contours specified by the parametric equations $t_k = t_k(\tau)$, $0 \le \tau < 2\pi$; $t_k(\tau) = t_k(\tau + 2\pi)$, it is sufficient to change the integration limits and the singular kernels in the relation (16) according to

$$\int_{-1}^{1}\to\int_{0}^{2\pi};\qquad \ln|\tau-\xi|\to\ln|\sin\frac{\tau-\xi}{2}|;$$

$$(\tau-\xi)^{-1}\to\frac{1}{2}\cot\big(\frac{\tau-\xi}{2}\big);\qquad (\tau-\xi)^{-2}\to\frac{1}{4\sin^2((\tau-\xi)/2)}. \qquad (17)$$

Thus, solution of the initial diffraction problem was reduced to solving of the type (16) equations (reduction of the relations (11), (12) to the normalized form is obvious). Their direct numerical solution implicates availability of special quadrature formulae for the logarithmic and hyrersingular integrals. Let's consider these cases separatly.

LOGARITHMIC SINGULARITY

The following integrals are meant:

$$J_1(\xi)=\int_{0}^{2\pi} f(\tau)\ln|\sin\frac{\tau-\xi}{2}|d\tau,\qquad f(\tau)=f(\tau+2\pi),\qquad \xi\in[0,2\pi];$$

$$J_2(y)=\int_{-1}^{1} w(x)f(x)\ln|x-y|dx,\qquad y\in[-1,1]. \qquad (18)$$

For approximate calculation of functions $J_{1,2}$ we shall use the interpolation type quadrature formulae built up by means of the proper approximation of density f and further exact calculation of the integrals. Concerning $J_1(\xi)$ such procedure leads to the following well known result. When interpolating the continuous 2π-periodic function $f(\tau)$ in n nodes $\tau_k = 2\pi k/n$, $k = \overline{0,n-1}$ by means of the trigonometric polynomial $f_p(\tau)$ of the order $p = [n/2]$:

$$f_p(\tau)=\frac{1}{n}\sum_{k=0}^{n-1}f(\tau_k)\Big\{1+2\sum_{m=1}^{p}\cos[m(\tau_k-\tau)]-\frac{1+(-1)^n}{2}\cos[p(\tau_k-\tau)]\Big\}, \qquad (19)$$

for the arbitrary $\xi\in[0,2\pi]$ the quadrature formula occurs:

$$J_1(\xi)\approx-\frac{2\pi}{n}\sum_{k=0}^{n-1}f(\tau_k)\Big\{\ln 2+\sum_{m=1}^{p}m^{-1}\cos[m(\tau_k-\xi)]-\frac{1+(-1)^n}{2n}\cos[p(\tau_k-\xi)]\Big\}. \qquad (20)$$

Formula (20) is exact if the density of integral $J_1(\xi)$ is polynomial of order not higher than $[(n-1)/2]$ (symbol [ ] means the integer part of the number). If the weight function in the integral $J_2(y)$ looks like $w(x) = (1-x)^\alpha(1+x)^\beta$, $\Re\alpha > -1$, $\Re\beta > -1$, then the following relations will be the analogues of the formulae (19) and (20):

$$f(x)\approx\sum_{k=1}^{n}A_k^n f(x_k)\sum_{m=0}^{n-1}h_m^{-1}P_m^{(\alpha,\beta)}(x_k)P_m^{(\alpha,\beta)}(x) \qquad (21)$$

(x_k are the roots of the Jacoby polynomial $P_n^{(\alpha,\beta)}(x)$; A_k^n are the Christoffel numbers; h_m are the squares of the norms of polynomials $P_m^{(\alpha,\beta)}(x)$) and

$$J_2(y)\approx\sum_{k=1}^{n}A_k^n f(x_k)S(x_k,y);$$

$$S(x_k,y)=A_0\big(\frac{1-y}{2}\big)-\sum_{m=1}^{n-1}P_m^{(\alpha,\beta)}(x_k)q_{m-1}^{(\alpha+1,\beta+1)}(y)/(mh_m); \qquad (22)$$

$$q_m^{(\alpha,\beta)}(x)=[q_m^{(\alpha,\beta)}(x+i0)+q_m^{(\alpha,\beta)}(x-i0)]/2,\quad q_m^{(\alpha,\beta)}(z)=-\frac{1}{2}\int_{-1}^{1}w(x)\frac{P_m^{(\alpha,\beta)}(x)}{x-z}dx,\quad z\notin[-1,1].$$

As was shown (Nazarchuk, 1989) $A_0(y)$ is expressed in general case through the hypergeometric functions ${}_2F_1$ and ${}_3F_2$.

The above mentioned divergent integrals can be determined in finite part sense according to Hadamard (9) or (which is the same) as a derivative of the principal value of Cauchy type integral. The latter indicates the possibility to obtain the rule for numerical integration of the hypersingular integral

$$I(y) = \int_{-1}^{1} w(x)\frac{f(x)}{(x-y)^2}dx, \qquad y \in (-1,1) \tag{23}$$

by formal differentiation of the quadrature formula for Cauchy type integral

$$I(y) \approx \sum_{k=1}^{n} A_k^n \frac{f(x_k)}{(x_k-y)^2} - 2\Big[f'(y)\frac{q_n^{(\alpha,\beta)}(y)}{P_n^{(\alpha,\beta)}(y)} + f(y)\frac{d}{dy}\Big(\frac{q_n^{(\alpha,\beta)}(y)}{P_n^{(\alpha,\beta)}(y)}\Big)\Big]. \tag{24}$$

Here it is proved (Ioakimidis, 1985) that when f'' belongs to the Hölder class H_μ, the formula (24) uniformly converges in $[-1,1]$ with the rate of $O(n^{-\mu})$. In case the (21) is used for approximation of $f(y)$ the latter formula can be transformed into the form

$$I(y) \approx \sum_{k=1}^{n} \frac{f(x_k)}{x_k - y_r}\Big(\frac{A_k^n}{x_k - y_r} + 2\frac{q_n^{(\alpha,\beta)'}(y_r)}{P_n^{(\alpha,\beta)'}(x_k)}\Big), \qquad q_n^{(\alpha,\beta)}(y_r) = 0. \tag{25}$$

Now we'll consider formulae (16). If the complex variable z does not belong to the integration contour, the integrals it containing are regular and are determined in the usual sense. Hence the usual Gaussian quadrature formula may be used for their calculation. But when the point z is near to segment $[-1,1]$ the convergence rate of such formula essentially decreases. Accounting for this circumstance in case of Cauchy type integral leads to other formula. The appeared additional term in it presents the main contribution of the previously neglected remainder. The above mentioned is also transferred in the case of hypersingular integral. The proper quadrature formula has the form

$$I(z) \approx \sum_{k=1}^{n} \frac{f(x_k)}{x_k - z}\Big[\frac{A_k^n}{x_k - z} + \frac{2}{P_n^{(\alpha,\beta)'}(x_k)}\Big(\frac{q_n^{(\alpha,\beta)}(z)}{x_k - z} + q_n^{(\alpha,\beta)'}(z)\Big)\Big]. \tag{26}$$

In this case it can be shown that with point z removed from the contour of integration the formula (26) coincides with the usual Gaussian approximation.

MECHANICAL QUADRATURES METHOD

The essence of this approach consists in numerical treatment of the integral equations (16) by application of the corresponding quadrature rules (Nazarchuk, 1989). In this case the regular terms are approximated by the Gaussian quadrature formulae. Without describing the details of the calculation procedures for each case, we'll present only some results concerning the grounding of this method. In case of the first kind integral equation with a logarithmical singularity and the periodical desired function the following **theorem** is proved (Gabdulhaev, 1986).

Let the next conditions be fulfilled:

a) functions K_e (for each of the arguments) and $F_e \in W^{r+1} H_\alpha(M) \equiv G$, where $r \geq 0$ is the integer number, $0 < \alpha < 1$, $M = \text{const} > 0$;

b) the equation has a unique solution in L_p for any right-hand part among W_p^1, $1 < p < \infty$;

c) kernels $K_e(\tau,\xi)$ are such that operators $\hat{H}^{-1} : W_p^1 \to L_p$, $1 < p < \infty$ are bounded in the norm in the set.

Then the estimation $\|j - j_n\| \sim n^{-r-\alpha} \ln^{[q]} n$, $q = 1 - 1/p$, $1 \leq p < \infty$ is valid.

When the periodic boundary value problem is solved in case of H-polarization, we assume the solution of integrodifferential equation (16) in class $X = L_2^{(1)}$ of absolutely continuous 2π-periodical functions, the first derivatives of which are square-law summarized with the norm $\|m\|_{L_2^{(1)}} = \|m\|_C + \|m'\|_{L_2}$, exists and is unique one. The space of square-law summarized at $[0,2\pi]$ with a usual norm functions will be designated by $Y = L_2$. Then the following **theorem** is proved (Akhmadiev et al.,1988).

Let be K_h (for both variables), $F_h \in C$ and the equation has unique solution in X with any right-hand part from $F_h \in Y$. Then at n such that

$$\alpha_n = \text{const}\{E_n^\tau(K_h)_C + E_n^\xi(K_h)_C\} < 1 \tag{27}$$

the system of algebraic equations obtained by the mechanical quadratures method has an unique solution and the approximate solutions $m_n(\tau)$ converge to exact $m(\tau)$ with the rate

$$\|m - m_n\|_X = O\{E_n^\tau(K_h)_C + E_n^\xi(K_h)_C + E_n^\xi(F_h)_C\}, \tag{28}$$

where $E_n^\tau(f)_C$ is the best uniform approximation of function $f(\tau)$ for variable τ by the trigonometric polynomials of the order not higher than n.

As a consequence of this theorem we have:

if K_h (for each of the variables uniformly relative to the other of them), $F_h \in H_\alpha^{(r)}$, then the approximate solutions $m_n(\tau)$ converge with the rate

$$\|m - m_n\|_X = O(n^{-r-\alpha}), \qquad \|m - m_n\|_{C^{(1)}} = O(n^{-r-\alpha}\ln n), \qquad r \geq 0, \quad 0 < \alpha \leq 1. \tag{29}$$

To formulate the similar result for integrodifferential equation (16) determined along the open contour, we shall designate $w(\tau)|_{\alpha=\beta=1/2} = \rho(\tau) = \sqrt{1-\tau^2}$ and introduce the spaces: $L_{2\rho}$ - of square-law summarized functions with weight $\rho(\tau)$ and with the usual norm

$$\|\psi\|_{L_{2\rho}} = \Big(\int_{-1}^{1} \rho(\tau)|\psi(\tau)|^2 d\tau\Big)^{1/2} \tag{30}$$

as well as $L_{2\rho}^{(1)}$ - of absolutely continuous functions satisfying the condition of $m_k(s_r) = 0$ and the first derivatives of which are square-law summarized with weight $\rho(\tau)$. Let $X = L_{2\rho}^{(1)}$, $Y = L_{2\rho}$. Then according to Akhmadiev et al. (1988) the next **theorem** is valid:

if F_h and K_h (for both variables) $\in C$ and the integrodifferential equation in (16) has an unique solution in X at any right-hand part from Y, then at n such that

$$\alpha_n = \text{const}\{n^{-3/2} + E_{n-1}^\tau(K_h)_C + E_{n-1}^\xi(K_h)_C\} < 1 \tag{31}$$

the algebraic system of the mechanical quadratures method has an unique solution and the approximate solutions $m_n(\tau)$ converge to exact $m(\tau)$ with the rate

$$\|m - m_n\|_X = O\{n^{-3/2} + E_{n-1}^\tau(K_h)_C + E_{n-1}^\xi(K_h)_C + E_{n-1}^\xi(F_h)_C\} \tag{32}$$

(approximation is implemented by algebraic polynomials of degree not higher than $n-1$). As a consequence of this theorem the solutions convergence in spaces C and H_β, $0 < \beta < 1/4$ is obtained.

REALIZATION OF THE METHOD

The systems (10) and (13) describe the waves scattering by cylindrical bodies of an arbitrary cross-section considered as the superposition of their parts. The respective algorithms are based on consideration of the cross-section as a set of smooth arcs touching by their ends. In this case there arises a question on the behaviour of the integral equation solutions in the contact point. The characteristic part of systems (10) and (13) analysis has shown that in case of the zigzag contour L the Chebyshev weight function extracted in solutions with separate allocation of arcs L_k should be substituted by the Jacoby weight and the limiting values of densities $m_k(s)$ in this point should be equal one to another. The character of the solutions behaviour for equations (10) and (13) at end-points of the L_k arcs for piecewise-smooth profile scatterers is changed due to the terms with stationary singularity in the neighbourhood of zigzag point, extracted in (16). It was found that ignoring of the stationary logarithmic singularity in this case is allowable and does not effect the result obtained. The calculation procedure for solving the equation (13) is stable only on account of relation (26). The developed procedure of solving the problem with an arbitrary singularity of functions $j_k(s)$ and $m_k'(s)$ at the arcs L_k end-points allowed to study the influence of its index deviation from the exact value upon the stability and accuracy of the result obtained. It was proved, for example, that independently of the zigzag profile shape the extraction of Chebyshev weight function in case of the E-polarization is quite allowable. In the H-case ignoring of the finite value of the transversal surface current at the zigzag point leads to error sensed even in the far field diagrams. With account for this value the transition to the Chebyshev weight function practically has not effected the obtained results but led to essential algorithm simplification. Such concept applied sequentially allows, on one hand, to investigate the diffraction properties of arbitrary cylindrical inclusions more efficiently, on the other hand - to progress to a more short-wave range.

CONCLUSION

If we use the including environment Green function in the systems of integral equations describing the diffraction on the above mentioned structures in free space, we shall obtain the respective systems accounting for the availability of the boundaries. Thus,the performed consideration allows to formulate the universal approach to analysis of the boundary value problems for Helmholtz equation in the piecewise-homogeneous area with cuts of arbitrary curvature. This approach promotes the development of rigorous methods in electromagnetic control theory. As a result it will reduce the conservation of testing information and to more correct evaluation of the technical state of the constructive elements.

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