DEVELOPMENT OF METHODS AND ALGORITHMS FOR NONDESTRUCTIVE ELECTROMAGNETIC TESTING OF THE LAMELLAR COMPOSITES

B.I. KOLODIY and O.B. LJASHCHUK

Karpenko Physico-Mechanical Institute,
5 Naukova Str., 290601 Lviv, Ukraine

ABSTRACT

A new approach to development of testing methods is proposed. It depends on the solution of inverse multiparameter problems and consists in finding the position of absolute minimum of some function in the domain of tested parameters. This function amounts to the sum of modulus discrepancy between theoretical and measured reflection coefficients values taken over all frequencies of sounding electromagnetic waves. Two mathematical models are considered in order to determine reflection coefficients:integral representation and recurrent formula. The algorithms for finding absolute minimum position are developed for numerical solution of the inverse problems. Two-layers structure with delamination is investigated. The influence of measuring errors and the number of sounding frequencies on testing accuracy has been investigated.

KEYWORDS

Lamellar composit, delamination, thickness, plane electromagnetic wave, reflection coefficient, direct problem, inverse problem, absolute minimum, testing accuracy.

FORMULATION OF PROBLEM

Using polymer composite materials in many branches of engineering requires the determination of their mechanical and geometrical parameters. Dielectric characteristics (the permittivity and dielectric loss tangent) are connected with mechanical parameters. Determination of foregoing parameters permit to provide the nondestructive testing of materials and products. Such testing can be effectively realized on the basis of investigated objects sounding by means of electromagnetic waves in the superhigh frequencies range. Theoretical basis of foregoing testing is solution of electrodynamical direct and inverse problems. Some of these problems for composite materials are solved and methods developed on this basis for electromagnetic diagnosis are represented below.
While solving these problems the reflection or transmission coefficients as frequency functions are considered as an ini-

tials data. In the case of one-side access to the tested objects reflection coefficients V are initial values. Describe the geometric and electromagnetic characteristics of the tested objects by some vector $\vec{p}$. Then we have the equation

$$V = A [\vec{p}], \qquad (1)$$

where A is a wholly continuous operator with nonbounded inverse one ($\vec{p} \in P$, P is metric space).

The direct problem on electromagnetic waves interaction with the tested object consists in solution of equation (1), that is in determining reflection coefficients V on the basis of known characteristics $\vec{p}$. Inverse problem of interaction of electromagnetic field with the object, as applied to the nondestructive testing, consists in determination of $\vec{p}$ on the basis of known $\vec{V}$. The solutions of direct problems are significant for nondestructive testing. Analysis of reflection coefficients and, in general, field vectors give significant information for the understanding of physical processes in the tested objects and for the possibility of creation of new approaches for determination of simple structure parameters (Kolodiy et al, 1988). Moreover, in the numerical solution of inverse problem we use for many times the direct problem solutions. Therefore, the development of effective algorithms for direct problems solutions is an important task.

THE EXPRESSIONS FOR REFLECTIVE COEFFICIENTS

V as a function of geometrical and electromagnetic parameters is determined by the mathematical models which describe tested objects response to a sounding field. Below we shall give the expression for V in the case of a normal incidence of a plane electromagnetic polarized wave with time-dependence $\exp(i\omega t)$ on the dielectric plate located on semiinfinite subground, where ω is circular frequency. Assume that a body occupies the domain $0 < x < \infty$ and wave is incident on plane $x = 0$. We consider the body with arbitrary distribution of dielectric characteristics and the body with piece-wise homogeneous parameters against thickness. The permeability is taken as for vacuum.

Arbitrary Distribution of Parameters. It is assumed that the material is characterized by wave value k(x). We consider a body as an equivalent source of electromagnetic field. The equation for electric intensity E has the form:

$$\frac{d^2E}{dx^2} + k^2(x)E = 0 \qquad (2)$$

and it may be written as

$$\frac{d^2E}{dx^2} + k_0^2 E = - [k^2(x) - k_0^2] E, \qquad (3)$$

where k_0 is wave value for vacuum. The solution of equation (3) can be represented in the form:

$$E(\omega,x) = \int_0^\infty [k^2(x') - k_0^2] E(\omega,x') G(\omega;x,x')dx' + E(\omega,x'), \qquad (4)$$

where $E_0(x)$ is an electric intensity of an incident wave, $G(\omega; x, x')$ is one-dimensional Green function:

$$G(\omega; x, x') = -\exp[-ik_0(x-x')]/2ik_0. \qquad (5)$$

For the surface x = 0 we have the evident relationship:

$$E(\omega; 0) = [1 + V(\omega)] E_0(\omega; 0). \qquad (6)$$

From Eqs (4) and (6) it follows that

$$V(\omega) = \int_0^\infty [k^2(x') - k_0^2] E(\omega, x') G(\omega; 0, x')dx'. \qquad (7)$$

The formulae (7) may be applied in step-by-step approach methods. The first approximation is received by substitution E(ω,x) for $E_0(\omega,x)$ under the integral. It is Born approximation.

Body with Piece-Wise Homogeneous Parameters. We shall consider the structure which consists of M -1 dielectric layers with thickness $h_m (m = 1,..., M -1)$ and located between two halfspaces $x < 0$ and $x > \sum_{m=1}^{M-1} h_m$ It is assumed that every layer has the flat boundaries and is homogeneous and isotropic. The lamination that would arise between the layers or inside separate layers are also represented as layers with flat boundaries. The investigated structure is characterized by the number of layers, their thicknesses and dielectric parameters. So, this structure may be described by the vector $\vec{p}$ in 3M-dimensional space P:

$$\vec{p} = (M; \varepsilon_1, tg\delta_1, h_1; ...h_{M-1}; \varepsilon_M, tg\delta_M), \qquad (8)$$

where ε_m and $tg\delta_m$ are relative permittivity and dielectric loss tangent of corresponding layer.

In the considered case equation (2) has the form:

$$\frac{d^2E}{dx^2} + k_m^2 E = 0, \qquad (9)$$

$$k_m^2 = \omega^2 \mu_0 \varepsilon_m'; \quad \varepsilon_m' = \varepsilon_0 \varepsilon_m (1 - i tg\delta_m). \qquad (10)$$

Here m = 0,1 ..., M; m = 0 and m = M correspond to upper and lower halfspaces; ε_0 and μ_0 are absolute permittivity and permeability for vacuum. Using the continuity conditions for tangential electric and magnetic components and the radiation condition, one can find the electric intensity E and similar to (Brechovskich, 1973; Wait, 1970) expression for reflection coefficient:

$$V = (N_0 - Y_1)/(N_0 + Y_1),$$

$$Y_m = N_m \frac{Y_{m+1}(\exp z_m + 1) + N_m(\exp z_m - 1)}{N_m(\exp z_m + 1) + Y_{m+1}(\exp z_m - 1)}, \qquad (11)$$

$$z_m = 2i\omega h_m \sqrt{\varepsilon'_m \mu_0}, \quad N_m = \sqrt{\varepsilon'_m/\mu_0}, \quad Y_m = N_m.$$

DIRECT PROBLEM SOLUTION

Further we shall consider dielectric plate located on semiinfinite dielectric subground. Let the plate has the thickness d, there is the delamination in the plate with thickness γ at distance h_1. from the plate surface, at which a sounding wave is incident. The calculations of V have been performed according to relationship (11). The dependence $|V(h_1)|$ for the sounding wave frequency $f = 10^{10}$ Hz, d = 0.12 m and ε = 4, tgδ = 0.02 for plate materials are presented in Figs.1 and 2.

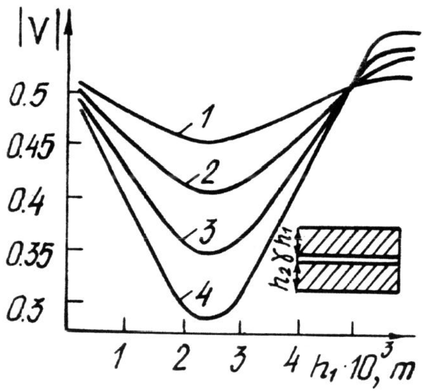

Fig.1.Behaviour of |V| versus h_1 for the plate located in vacuum.

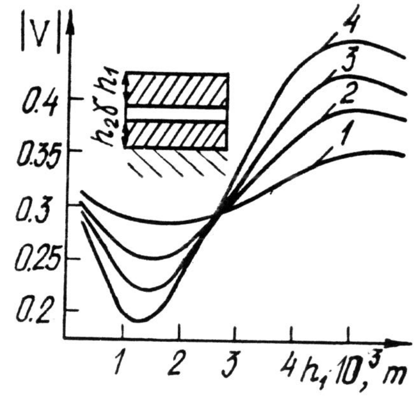

Fig.2.Behaviour of |V| versus h_1 for the plate located on semiinfinite layer.

Information in Figs.1 and 2 concerns the plate located correspondingly in vacuum and on the halfinfinite underlayer with ε = 5 and tgδ = 0.02. Lines 1-4 in these Figs. correspond to delamination thicknesses γ = 0.25; 0.50; 0.75 and 1.0 mm. One can see that dependencies $|V(h_1)|$ are

ambiguous, so for structures with different h_1 and γ the values |V| can be the same.

The relationship between IVI and frequency is represented in Figs.3 and 4 for such cases as for Figs.1 and 2.

In Fig.3, for plate in vacuum, curves 1, 2, 3 correspond to γ = 0; 2.5; 5.0 mm with h_1 = 2 mm. For the plate on subground (Fig.4) curves 1 and 2 correspond to delaminations with h_1 = 18 mm and γ = 2.5; 5.0 mm; curve 3 corresponds to h_1 = 10 mm and γ = 5 mm. Similar dependencies exist for the real and imaginary parts of V. Hence, on the basis of direct problem solutions we can not unambiguously determine the delaminations parameters in the case of their change in broad domain. For this aim it is necessary to solve the inverse problem.

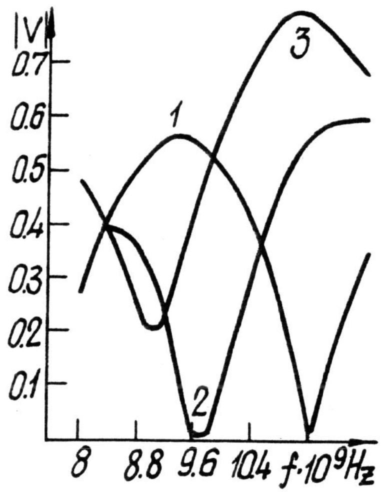

Fig.3.Behaviour of |V| versus f for the plate located in vacuum.

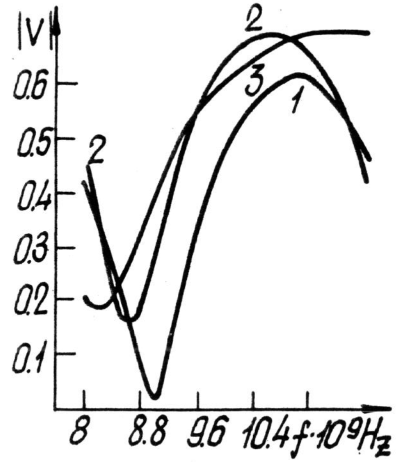

Fig.4.Behaviour of |V| versus f for the plate located on semiinfinite layer.

INVERSE PROBLEM

For numerical solutions of the inverse problems usually two methods, exhausting and regularization, are applied (Tichonov et al, 1979). Exhausting methods allow to reduce the initial inverse problem of electromagnetic testing to a nonlinear programming problem which consists in finding the position of absolute minimum of some function. The minimum coordinates are parameters to be find. According to the "quasisolution" conception (Ivanov, 1963) we have made the statement of inverse problem for lamellar composites parameters determination. Vector $\vec{p}$ represented by relation (8) is an unknown quantity. There are the evident restrictions for layer parameters: $1 \leqslant \varepsilon_m \leqslant \varepsilon_{max}$; $0 \leqslant tg\delta_m \leqslant tg\delta_{max}$; $0 < h_m \leqslant h_{max}$; $M > 1$. In a finitedimensional space R^{3m} these restrictions form compact set $\vec{p} \in P$. Function whose absolute minimum must be found amounts to the sum of modulus discrepancy between theoretical $V_t(f,\vec{p})$ and measures reflection coefficients values $V_e(f)$ taken over all frequencies of sounding waves. Thus the problem on $\vec{p}$ determination can be formulated as

$$\vec{p}: \quad inf \sum_{n=1}^{N} |V_t(f, \vec{p}) - V_e(f)|^2, \quad \vec{p} \in P. \qquad (12)$$

N is the number of sounding frequencies. In (Kolodiy et al, 1987; Kolodiy et al, 1990; LJashchuk, 1990) it was shown that the function in (12) has many local minima. The problem consists in development of an optimal algorithm for finding absolute minima.

Uniqueness and existance of the above formulated problem are based on the following theorems.
Theorem 1. Complex reflection coefficient V(f) uniquely determines the number of layers and the set of thicknesses and dielectric parameters (M; h_1; ε_1, $tg\delta_1$;...,$tg\delta_{M-1}$; ε_m, $tg\delta_m$) for multilayer structure. The proof of this theorem is based on the analysis of additive representation of reflection coefficients (Glasko et al, 1980).

Theorem 2. If P is a compact set in R^{3M}-space and function in (12) is continuous in P, a solution of problem (12) exists. The proof of theorem 2 is based on Weierstrass theorem.

Numerical Solutionof Inverse Problem. As an example we determine the delamination thickness and its depth in dielectric plate located on subground. For solving this task we consider function as in relation (12)

$$F = \sum_{n=1}^{N} |V_t(f_n, h_1, \gamma) - V_e(f_n)|^2. \qquad (13)$$

Solution is carried out on the basis of numerical experiment. Dielectric parameters are such as in foregoing direct problem and h_1 = 10 mm, γ = 5 mm. Frequencies f_n of the sounding electromagnetic field are equal to 9.8, 10.0, 10.2 and 10.4 GHz. In Figs.5 and 6 dependencies $F(h_1)$ for γ = 5 mm and $F(\gamma)$ for h_1 = 10 mm are presented correspondingly. Curves 1 and 2 refer to the plates in vacuum and on semiinfinite subground correspondingly. Absolute minimum for curve 1 corresponds to the delamination depth h_1 = 10 mm is F_m = 3.763×10^{-9} and that for curve 2 corresponds to the delamination thickness γ = 5 mm is $F = 4.472 \times 10^{-9}$.

As it is seen from the obtained results, function F is not unimodal and consequently the standard methods of nonlinear programming (Himmelblau, 1972) can't be applied to

determination of the absolute minimum position F^* and consequently the desired parameters. For unique determination of delaminations thickness and depth the combined algorithm was developed.

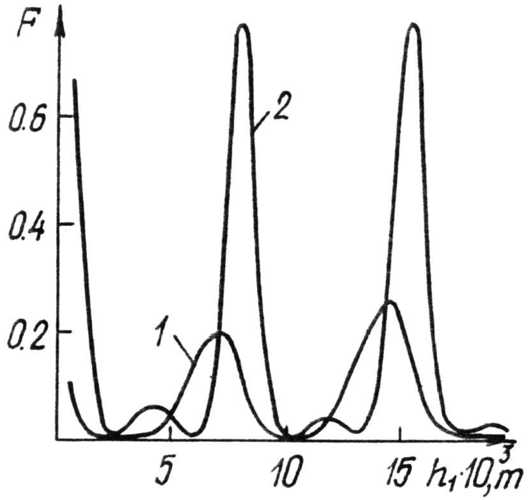

Fig.5. Behaviour of function F versus h_1.

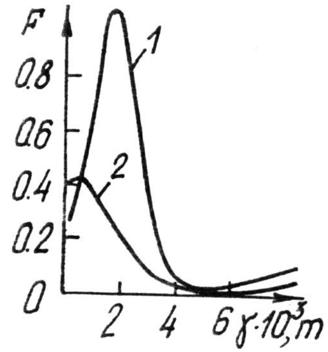

Fig.6. Behaviour of function F versus γ.

The Accuracy of Parameters Determination. In practice it is important to study the influence of the errors of an initial data V_e and the number N of sounding frequencies on the accuracy of desired parameters determination. For numerical simulation some errors covered on the calculated values V_c of the reflection coefficients as follows:

$$V_e = V_c\,(1 + sq), \qquad (14)$$

where s is the experimental data accuracy and q is a random value in the interval (-1, 1). Value q is obtained from the relation

$$q = 2q' - 1,$$

where q is a random value in the interval (0, 1) and is produces by the random number generator (for example, by program RANDU). The numerical results for the plate with delamination located on semiinfinite subground are represented in Table 1. The parameters of the plate and of the subground are the same as for Figs.5 and 6. From the table it is seen that the absolute minimum value is increasing with errors increase, and the position of this minimum is shifting relatively to the true values of parameters.

Table 1. Dependencies of F^*, h and γ on accuracy of $|V|$.

S	10%			20%			30%		
N	$F^* x10^3$	h_1,mm	γ,mm	$F^* x10^3$	h ,mm	γ,mm	$F^* x10^3$	h ,mm	γ,mm
5	3.338	10.0	5.15	12.65	9.9	4.90	40.09	10.20	4.4
10	5.697	10.1	4.85	32.66	10.0	5.00	121.7	10.15	4.6
20	2.813	10.1	4.85	92.58	10.2	4.55	312.9	9.95	5.25

CONCLUSIONS

On the basis of electromagnetic waves reflection coefficients as function of wave frequency it is possible to determine the thickness and depth of delaminations in multi-layers composite material. It can be made by means of inverse problem solutions. An increase of the sounding frequencies number permits to obtain more accurate values of an unknown parameters. The developed methods can be applied for determination of other parameters of multi-layers structures.

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