

# CYCLIC ELASTO-PLASTIC FRACTURE DIAGRAM FOR A SPECIMEN WITH A CRACK AND ITS ANALYSIS

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## ABSTRACT

Plotting methods of the cyclic elastoplastic fracture diagram for a specimen with a crack (CEPF - diagram) are given. Plotting of two types CEPF - diagram: D - diagram and Q - diagram are proposed. These diagrams are plotted according to the cyclic crack resistance tests of compact specimens made of 45 carbon steel with the thickness of 0.01, 0.02 and 0.04 m; characteristics of the material crack resistance straining and rupture are analysed and presented in this paper. The similarity transformations are proposed and generalized CEPF - diagram for three size-type specimens tested is plotted as a result of.

## KEYWORDS

Crack resistance, stress intensity factor, transverse component of plastic strain, compact specimen, CEPF-diagram, Q - diagram, D - diagram.

## GENERAL DESCRIPTION OF THE CEPF-DIAGRAM

In general case CEPF-diagram is a dependence between the stress intensity factor (SIF)  $K_{I\max}^F$  and transverse component of plastic strain  $\varphi$  taken in a dangerous section of a specimen. (Sosnovskiy, 1990; Sosnovskiy et al., 1990) plotted, for example, according to the results of compact specimens standard cyclic crack resistance tests. The  $\varphi$  value is determined as the residual of  $t_0$  nominal value and  $t_\varphi$  current value of the specimen's thickness, i.e.  $\varphi = t_0 - t_\varphi$  (Fig. 1a).  $K_{I\max}^F$  value is calculated due to the formulas of linear elastic fracture mechanics, but with the extent of ductility correction. For the compact specimen (Fig. 1a) there is

$$K_{I\max}^F = (P_{\max} \sqrt{a} / t_0 B) Y(F_a / F_0),$$

Correction function  $Y(F_a / F_0)$  is calculated taking into consideration actual transverse section  $F_a$  occupied by a crack with length of  $a$ ;  $F_0$  - nominal transverse section. The  $F_a / F_0$  value equals to  $\omega$  and is the value of extent damage, and function  $Y$ ,

thus, takes into consideration not only the loading scheme and geometry of the specimen, but also (integrally) the value of plastic strain in specimen's dangerous section.

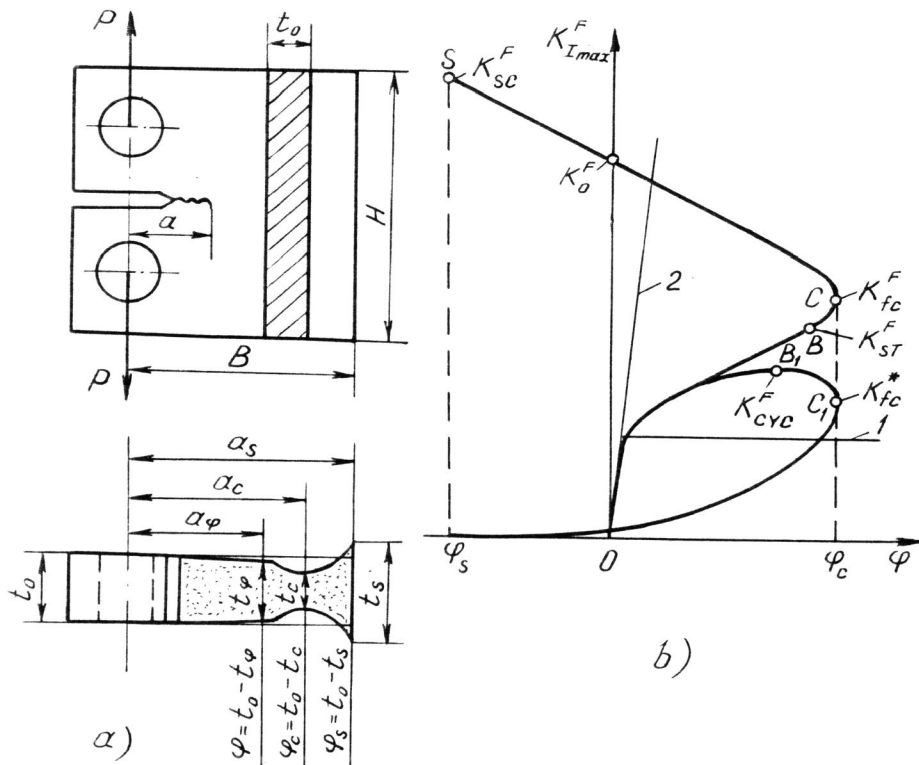


Fig.1. Specimen (a) and CEFF-diagram (b) schemes.

Two types of CEFF-diagram are distinguished (Sosnovskiy and Bogdanovich, 1992). If calculating  $K_{I_{max}}^F$  value by convention maximum rapture load is constant, then OBCD diagram is obtained (Fig.1b). This diagram together with the Y-axis looks as letter D and that is why it is called D-diagram. If calculating  $K_{I_{max}}^F$  the actual value of rapture load is taken into consideration, then  $OB_1 C_1 S_1$  diagram is obtained (see Fig.1b). This diagram resembles letter Q and that is why it is called Q-diagram.

In common case, CEFF-diagram consists of two curves (see Fig. 1,b): cyclic elasto-plastic fracture curve (line segments OBC on D-diagram and  $OB_1 C_1$  on Q-diagram respectively) and quasi-static fracture (rupture) curve line segments CS on D-diagram and  $C_1 S_1$  on Q-diagram respectively). In points C and  $C_1$  the crack reaches its critical length  $a_c$  corresponding to the limit value of narrowing plastic strain and to the SIF limit

value - cyclic fracture toughness ( $K_{fc}^F$  value on D-diagram and  $K_{fc}^*$  on Q-diagram). In points  $S, S_1$  accordingly, the specimen is divided into two parts and simultaneously maximum widening  $\varphi_s$  of its dangerous section takes place. That is the base for determination of another SIF limit value - quasi-static fracture toughness ( $K_{sc}^F$  value on D-diagram, on Q diagram  $K_{I_{max}}^F = 0$  in this point). Crossing point of CS curve and Y-axis gives one more parameter  $K_0^F$  of the crack resistance (see Fig.1,b). Max of Q-diagram on SIF axis (point  $B_1$ ) corresponds with the beginning of the cyclic rapture and characterized by parameter  $K_{cve}^F$ ; parameter  $K_{st}^F$  on Q-diagram corresponds to the beginning of quasi-static rapture and is not a characteristic point of this diagram, but accords with the beginning of sharp lift of the OBC curve (point B on D-diagram). In case of the "ideal plastic fracture", the cyclic elasto-plastic fracture curve transforms into direct line 1. In case of "ideal brittle fracture" ( $\varphi = 0$ ), this curve coincides with the Y-axis. The line 2 divides areas of quasi-brittle and elasto-plastic fracture. Thus, with the help of the CEFF-diagram it is possible to analyze ductile-brittle transition under, for example, the change of the test temperatures or specimen sizes.

THE CEFF-DIAGRAM OF 45 CARBON STEEL

D-diagram. Compact specimens made of 45 carbon steel with the thickness of 0.01, 0.02 and 0.04 m has been tested for cyclic crack resistance with the excentric tension, loading frequency ~ 20 Hz and deassyetry of cycle  $R_\sigma \approx 0,1$  under ambient conditions. D-diagram (Fig.2) for the specimen with the thickness 0.01 m (curve 1), 0.02 m (curve 2) and 0.04 m (curve 3) has been plotted due to the test results of dangerous sections transverse dimensions in consideration of the measurements accuracy to m. The relative strain  $\psi = \varphi/t_0$  is the X-axis here.

Table gives numerical values of crack resistance characteristics for 45 carbon steel.

The treshhold values of SIF  $K_{th}$ ; limit values of SIF  $K_Q^f$  that identisy to the  $K_{fc}$  value only in case of plain strain conditions; parameters C and n of the Paris's equation are determined by usual kinetic diagrams of fatigue failure. The characteristics  $K_{fc}^F, K_0^F, K_{sc}^F$  are determined by D-diagrams accordingly. Thus, from the Fig.2 and table, value  $K_{fc}^F = \text{const}$ , i.e. it doesn't depend on the specimen thickness, that supports experimental data obtained previously. (Sosnovskiy et al., 1990) for carbon steel 20. The values  $K_Q^f < K_{I_{max}}^* = \sqrt{t_0} \sigma_{0,2} / 2,5$ , thus plain strain conditions were not fulfilled under these tests. ( $\sigma_{0,2}$  is yield strength for 45 steel).

Two approaches may be proposed for analitical description of the OBC curve on every D-diagram. The first approach - this curve is described by equation

$$K_{I_{max}}^F = K_w [(\psi - \psi_t) / (\psi_c - \psi)]^m, \quad \psi_t = 0,0006 < \psi < \psi_c,$$

where m - parameter of the strengthening,  $K_w$  - SIF value corres-

ponding to  $\psi = (\psi_c + \psi_t)/2 \approx \psi_c/2$  value.

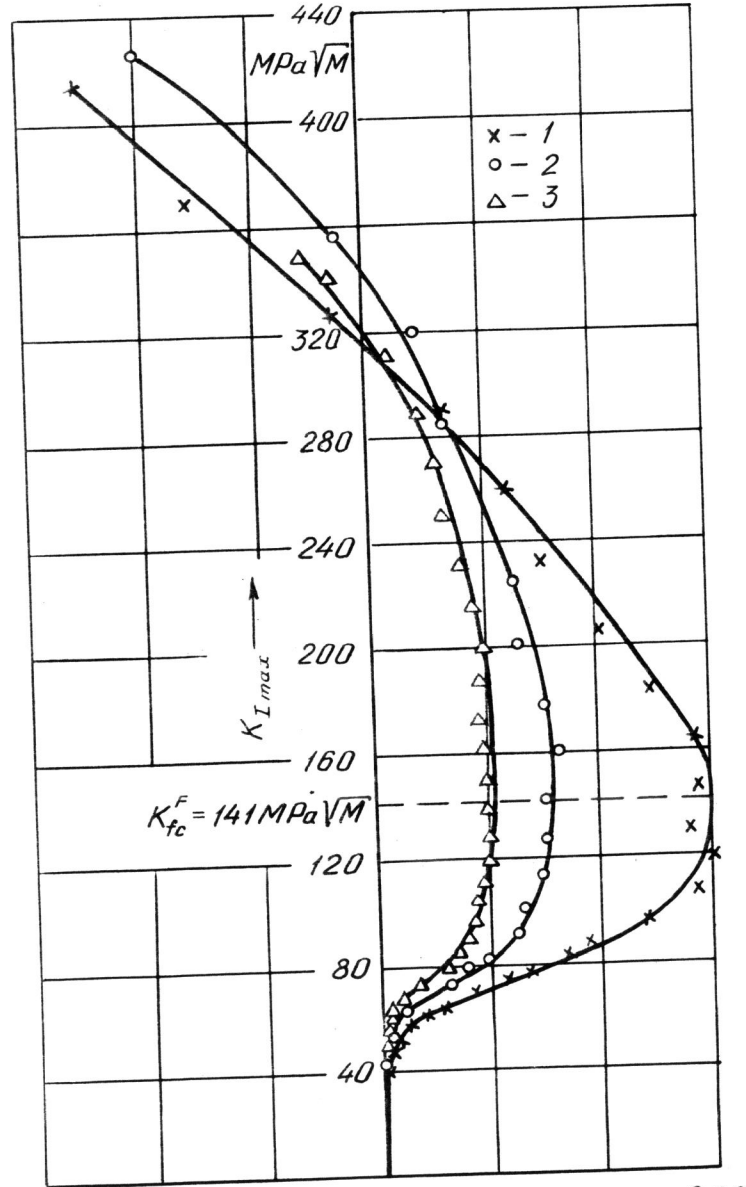


Fig. 2. D-diagrams for specimens of different thickness made of 45 steel.

As a second approach - the right branch of the OBC curve is the direct line in the accordant double logarithm coordinates, hence

$$K_{I,max}^F = K_t (\psi / \psi_t)^{m'}$$

where  $K_t$  is determined as intersection point of the direct line under consideration and the Y-axis. Values  $m$ ,  $K_w$ ,  $m'$ ,  $K_t$  for 45 steel are given in Table.

D-diagrams shown on Fig. 2 are individual for every specimen of given material having determined cracking resistance (ref. Table).

Table. Characteristics of 45 carbon steel cyclic crack resistance

Characteristics	Thickness of the specimen, m		
	0.01	0.02	0.04
$K_{th}$ , MPa√m	1,3	1,9	6,9
$K_{I,max}^*$ , MPa√m	26,6	37,6	53,1
$K_{I,t}^*$ , MPa√m	42,6	45,7	52,5
$K_{I,q}^*$ , MPa√m	38,9	50,2	66,7
$K_{I,w}^*$ , MPa√m	78,0	79,0	81,3
$K_{I,c}^F$ , MPa√m	141,0	142,5	140,0
$K_{I,o}^F$ , MPa√m	317,0	344,0	320,0
$K_{I,c}^F$ , MPa√m	415,0	426,0	349,0
$K_{I,cyc}^F$ , MPa√m	64,0	76,0	72,5
$K_{I,ST}^F$ , MPa√m	63,0	73,0	72,0
$K_{I,c}^{*}$ , MPa√m	33,0	50,0	48,5
$\varphi_c$ , m	$0,58 \cdot 10^{-3}$	$0,62 \cdot 10^{-3}$	$0,80 \cdot 10^{-3}$
$\varphi_s$ , m	$-0,48 \cdot 10^{-3}$	$-0,82 \cdot 10^{-3}$	$-0,45 \cdot 10^{-3}$
$\psi_c$	0,060	0,031	0,020
$\psi_s$	-0,050	-0,041	-0,011
$C$	$4,3 \cdot 10^{-11}$	$5,4 \cdot 10^{-11}$	$1,36 \cdot 10^{-13}$
$n$	3,0	2,72	4,28
$Q_x$	4,27	2,85	2,89
$m'$	0,103	0,120	0,133
$m$	0,133	0,142	0,150

For a given steel by similarity transformations it is possible to plot generalized D-diagram that represents uniform curve for all size- type specimens. For 45 carbon steel such diagram is given on Fig. 3. (1,2,3 - specimens with the thickness

0,01, 0,02 and 0,04 m respectively).

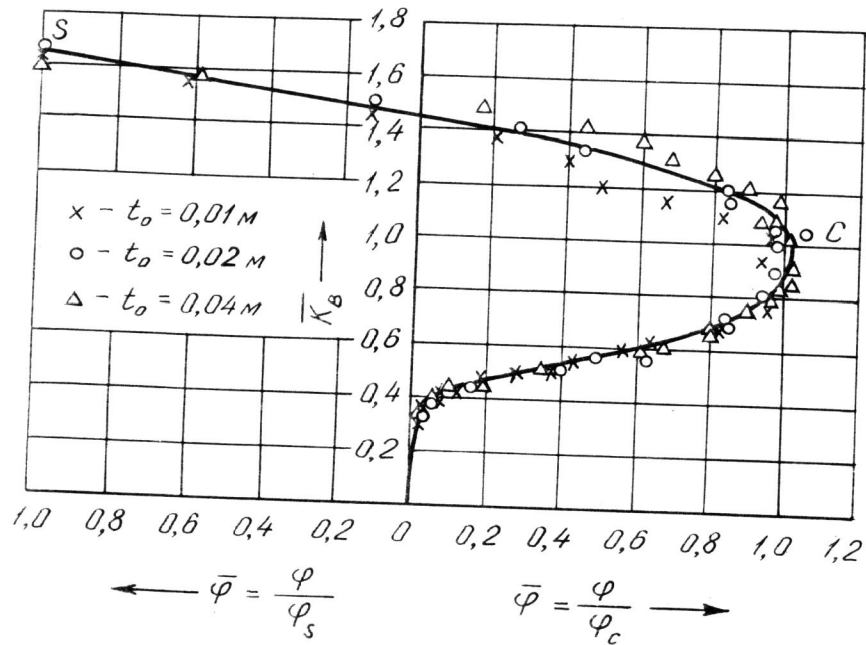


Fig. 3. The generalized D-diagram for 45 steel.

To plot generalized D-diagram it is necessary to perform coordinate transformations  $K_{I_{max}}^F$  and  $\psi$  into  $\bar{K}_B$  and  $\bar{\varphi}$  respectively, where  $\bar{K}_B = K_{I_{max}}^F / K_{fc}^F$  at  $K_{I_{max}}^F \leq K_{fc}^F$  and  $\bar{K}_B = 1 + (K_{I_{max}}^F - K_{fc}^F) / K_{sc}^F$  at  $K_{I_{max}}^F > K_{fc}^F$ , and also  $\bar{\varphi} = \varphi / \varphi_c$  (or  $\varphi / \varphi_s$ ). With the help of generalized D-diagram it is not difficult to plot individual D-diagram for every size-type specimen using four parameters ( $\varphi_c, \varphi_s, K_{fc}^F, K_{sc}^F$ ), and to restore the most important OBC branch of D-diagram only two parameters are sufficient ( $\varphi_c, K_{fc}^F$ ). Then current value  $\varphi$  and SIF are determined as follows:  $K_{I_{max}}^F = \bar{K}_B K_{fc}^F$  for the OBC branch,  $K_{I_{max}}^F = K_{fc}^F + K_{sc}^F (\bar{K}_B - 1)$  for the CS branch (see Fig. 1, b) and  $\varphi = \bar{\varphi} \varphi_c$  (or  $\varphi = \bar{\varphi} \varphi_s$ ).

**Q-diagram.** Q-diagrams has been plotted taking into account the actual value of rupture load of 45 steel specimens with the thickness of 0.01 curve (1), 0.02 (2) and 0.04 m (3) (Fig. 4). The values of crack resistance  $K_{fc}^*$ ,  $K_{cvc}^F$  and  $K_{st}^F$  obtained with the help of these Q-diagrams are given in Table. It is important to note, that cyclic fracture toughness  $K_{fc}^* < K_{fc}^F$

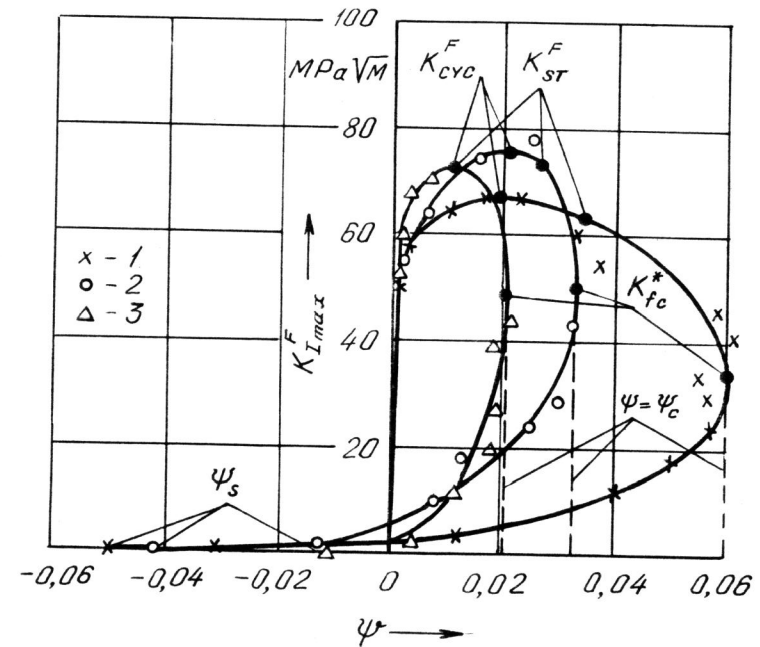


Fig. 4. Q-diagrams for specimens of different thickness made of 45 steel

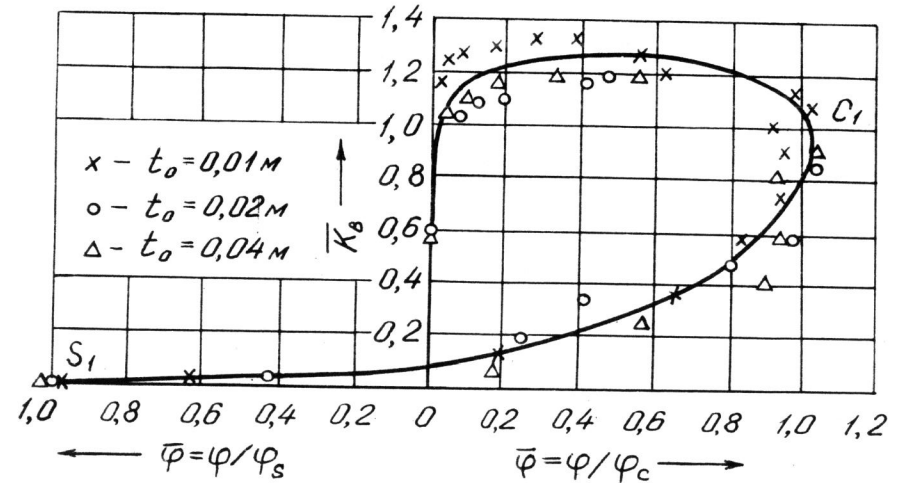


Fig. 5. The generalized Q-diagram for 45 steel.

and relationship  $K_{fc}^F/K_{fc}^* = a_K$  is practically constant for 45 steel at  $t_0 \geq 0.02$  m (see Table).

To plot generalized Q-diagram being uniform curve for all size-type specimens, it is necessary to transit to coordinates  $K_B - \psi$ , where  $K_B = 1 + (K_{I_{max}}^F - K_{fc}^*) / (K_{cyc}^F + K_{fc}^*)$  at  $0 \leq \psi \leq \psi_c$  and  $K_B = K_{I_{max}}^F / K_{fc}^*$  at  $\psi_c \leq \psi \leq \psi_s$ ;  $\bar{\psi} = \psi / \psi_c$  (or  $\psi / \psi_s$ ) (Fig. 5). Individual Q-diagram for a given size-type specimen may be plotted having generalized Q-diagram and value of four parameters ( $\psi_c, \psi_s, K_{fc}^*, K_{cyc}^F$ ) for this specimen. In this case current values  $\psi$  and SIF are determined as:  $K_{I_{max}}^F = K_{fc}^* + (K_B - 1)(K_{cyc}^F + K_{fc}^*)$  for branch  $OB, C_1$ ,  $K_{I_{max}}^F = K_{fc}^*$  for branch  $C_1, S_1$ , and also  $\psi = \bar{\psi} \psi_c$  (or  $\bar{\psi} \psi_s$ ).

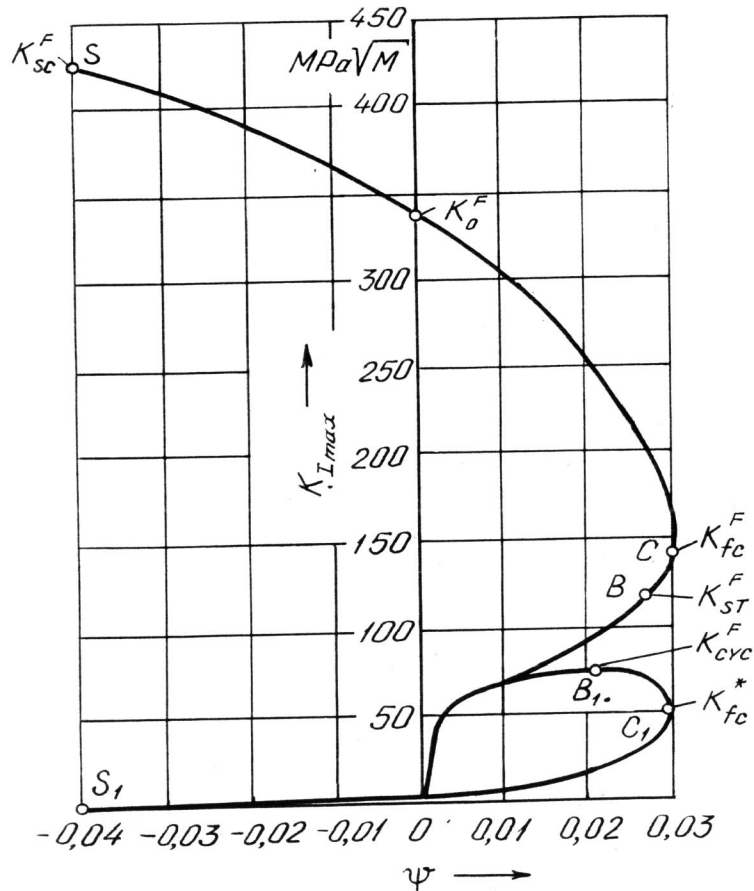


Fig. 6. Aligned Q- and D-diagrams for the specimens of 0.02 m thickness made of 45 steel.

Fig. 6 shows aligned Q- and D-diagrams plotted for the specimens of 0.02 m thickness made of 45 steel. As follows from Fig. 6,  $K_B \ll K_{fc}^F$  value is characteristic point of neither the first nor the second diagram.  $K_{cyc}^F$  value being characteristic point (B) of Q-diagram, for D-diagram is not such. But  $K_{ST}^F$  value being characteristic point ( $B_1$ ) of D-diagram is not Q-diagram characteristic point V.V.

Necessary to note that CEPP-diagram may be plotted in coordinates  $K_{Te} - \psi$  (Sosnovskiy and Bogdanovich, 1992); where  $K_{Te}$  is strain intensity factor (Makhutov, 1981).

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