CRACKS KINEMATICS FOR BRITTLE FRACTURE OF ROCKS IN BLASTING AND PERCUSSIVE FAILURE

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ABSTRACT

This report presents the authors' investigations on the process of rock fracture in mining. We use methods of fracture mechanics for description of crack kinematics and trajectories. Recently these methods are used in problems of mining.There are the papers of Cherepanov(1974), Ouchterlony (1982) ,Slepian (1985) and others. The first part of this report presents the estimation of fractured zone size in rock blasting.The second part presents investigations on cracks trajectory in blasting near the free surface and in percussive failure.

KEYWORDS

Kinematics, trajectory of cracks, fragmentation, blasting, percussive failure.

A ZONED MODEL OF PROPAGATION OF RADIAL CRACKS IN BRITTLE MEDIA.

The fragmentation process in rock blasting is usually considered in any model taking into account the zone of intensive destruction near the borehole, the intermediate zone of radial cracks and the external zone of elasticity. A difference between such models consists in different criteria of elastic medium fracture on the boundary with the zone of radial cracks. There are used the criterion of tangential stress, the energetical criterion wich take into account expenditure mechanical energy on the growth of cracks and the kinematical criterion.

Sher(1982) suggested to consider the zone of radial cracks and the external zone of elastic medium as one elastic zone with the system of radial cracks. In that case the velocity of cracks is defined from the relation between the velocity of a crack and its stress intensity factor. This relation is given by properties of the elastic material.Such model is realised simply in the case of the cylindrical charge and in terms of static approximation, because there is a large class of investigated problems on equilibrium of the system of radial crack in elastic plate. Analogical problem considered in terms of dinamics is more difficult and approximative solutions are desired. For a large number of cracks the model with the isolated zone of radial cracks may give such approximation.

The radial displacement $u(r,t)$ is determined in the zone of elasticy for $r>l(t)$ by:

$$\frac{1}{a^2}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial t^2} + k\left(\frac{1}{r}\frac{\partial u}{\partial t} - \frac{u}{r^2}\right), \qquad (1)$$

where a is the velocity of elastic waves in the case of cylindrical (k=1) and spherical symmetry (k=2) of the problem.

$$a^2 = \frac{(1-\nu)}{(1+\nu)(1-2\nu)}\frac{E}{\varrho_0},$$

where E, ν, ϱ_0 are the Young's modulus, the Poisson's ratio and density of material respectively.

The radial displacement $u_1(r,t)$ is determined in the zone of radial crack for $r_0(t) < r < l(t)$ by:

$$\frac{1}{c_k^2}\frac{\partial^2 u_1}{\partial t^2} = \frac{\partial^2 u_1}{\partial r^2} + \frac{k}{r}\frac{\partial u_1}{\partial r}. \qquad (2)$$

For cylindrical symmetry: k=1

$$c_1^2 = \frac{1}{1-\nu^2}\frac{E}{\varrho_0}; \quad \sigma_{r1} = \frac{1}{1-\nu^2}\frac{\partial u_1}{\partial r}; \quad \sigma_{\vartheta 1} = 0.$$

For spherical: k=2

$$c_2^2 = \frac{E}{\varrho_0}; \quad \sigma_{r1} = E\frac{\partial u_1}{\partial r}; \quad \sigma_{\vartheta 1} = \sigma_{\varphi 1} = 0.$$

Let us consider the problem with following boundary condition: radial stress is given on the inner boundary of the radial cracks zone:

$$\sigma_r = -p(t); \quad r = r_0(t) \qquad (3)$$

in the front of crack's system

$$u_1 = u \qquad (4)$$

$$\sigma_{r1} + \varrho_0\dot{l}\frac{\partial u_1}{\partial t} = \sigma_r + \varrho_0\dot{l}\frac{\partial u}{\partial t} \qquad (5)$$

$$2\gamma_0 n = V - V_1 + \frac{1}{2}(\sigma_r + \sigma_{r1})\left(\frac{\partial u_1}{\partial r} - \frac{\partial u}{\partial r}\right); \quad \dot{l} \neq 0$$

$$2\gamma_0 n = V - V_1 + \sigma_r\left(\frac{\partial u_1}{\partial r} - \frac{\partial u}{\partial r}\right); \quad \dot{l} = 0. \qquad (6)$$

Where $r=l(t)$; u, u_1-displasements of elastic and cracked zones; V, V_1 - the elastic energy density; $\gamma_0(\dot{l})$ - the surface energy density of cracks, n - the perimetr of cracks per unite of square of crack's front. This equations are followed from the continuite of the mass, impulse and energy flux throw the front of cracks. In the case of cylindrical symmetry $n=N/(2\pi l)$, where N - number of cracks in the zone of radial cracks, γ_0 is given by:

$$2E b^2\gamma_0 R(\dot{l}) = (1+\nu)\dot{l}K_I^2\sqrt{1-\dot{l}^2/a^2}, \qquad (7)$$

$$R(c) = 4\sqrt{1-c^2/a^2}\sqrt{1-c^2/b^2} - (2-c^2/b^2)^2,$$

where a and b are the wave's velocities in the elastic medium, K_I - the stress intensity factor in the end of the cracks. If $K_I(\dot{l})$ is known, equation (7) yields γ_0 and $l(t)$ may be determined from the solution of the problem (1-7). If $l(t)$ is known, $K_I(t)$ may be determined from (6), (7) and from the solution of the problem (1-5).

For example, the case of $p=p_0 t/t$ for $r=v_0 t$ in (3) was investigated for $l=ct$. The factor K_I was obtained for N cracks:

$$K_I = \frac{Q\sqrt{N}}{2\sqrt{\pi ct}}\frac{b\sqrt{(1-\nu)R(c)}\sqrt{1-v_0^2/a^2}\,(1-c^2/(2b^2))}{(1-c^2/a^2)^{3/4}}$$

$$Q = 2\pi v_0 t_0 p_0/N.$$

The function $K_I(\dot{l})$ is determined experimentally for many materials. Important characteristics of this function are values of K_I for $\dot{l}=0$ and the assymptotic value of $K_I(v_m)$, where v_m is the maximal velocity of crack propagation. This function may be approximated by

$$\dot{l} = \begin{cases} v_m\cdot(1-\exp(\alpha(1-K_I/K_{Ic}))); & K_I > K_{Ic} \\ 0 & ; \; K_I \leq K_{Ic} \end{cases} \qquad (8)$$

For P.M.M.A. we have: v_m=650 m/s; α =1; K_{Ic}=1 mPa $\cdot$ $m^{1/2}$. The equation(7,9) complete the problem statement(1-6) and determine the development of crack zone if the number of cracks -N or the density -n in (6) is known. The problem (1-8) was solved numerically for $P(t)=\sin(\omega t)H(t)\cdot H(2\pi/\omega - t)$ in (3).The influence of dynamics was estimated as a result of comparison with the quasi-statics solution.

The problem statement (1-8) was used for estimation of rock fragmentation from explosion of a spherical or cylindrical charge.The pressure p(t) (cf.(3)) in the problem about explosion of isolated chargeis unknown and may be obtained as result of its solution.Such problems were investigated with the help of statement(1-7) in terms of quasi-statics approach (Chernikov (1990)). At the zone of intensive destruction near the borehole the medium was described as the sandy medium with the Qoulomb-Mohr limiting condition.The size of destruction zones was obtained for various densities of the cracks -n. Such data was used to esimate the density ofthe cracks at a distance -r from the charge. The minimumal size of rock fragments at the distance -r is given by

$$x = 91\frac{\gamma_0}{P_0}\left(\frac{r}{r_{00}}\right)^{3,3} \qquad (9)$$

where p_0 is the initial pressure at borehole after explosion, r_{00} is the radius of the charge.Comparing with experimental data in P.M.M.A we conclude that γ_0 in(9) is equal to $\gamma_0(v_m)$.

Dynamics of the blasting process was investigated approximately after taking into account the dinamics of the plastic zone near borehole. The radial cracks zone and the external zone of elasticity was described in terms of the quasi-statics approach. This statement permits to determine the fragmentation as a function of time. The borehole radius -a, the plastic zone radius -b, cracks zone radius -l as a function of time and number of fragments in the zone of radial cracks are shown in Fig.1.

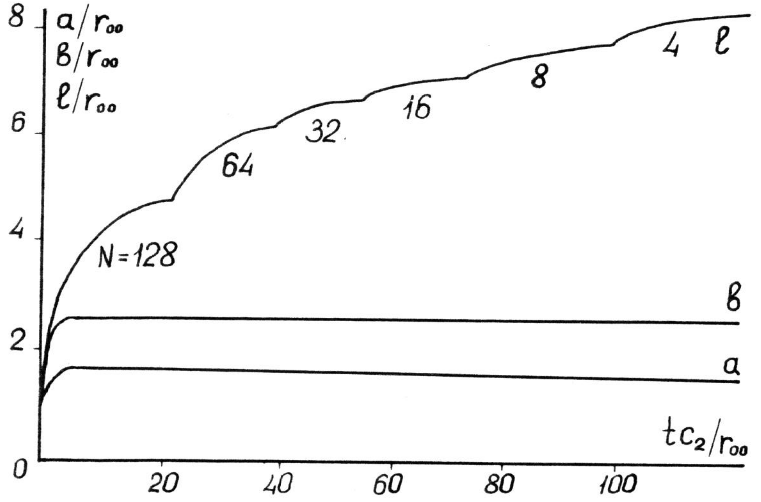

Fig.1.The fractured zone radii as functions of time in the case of explosion of spherical charge in P.M.M.A (r_{00}= 2 mm).

CURVILINEAR DEVELOPMENT OF CRACKS IN THE PROCESS OF DESTRUCTION IN MINING

In the general case cracks trajectories in the process of fragmentation are curvilinear. The deviation of a crack trajectory is caused by the non-uniformity of the stress. This non-uniformity may be caused by other cracks,possible free surfaces and external stresses.

The methods to analyse the problem for curvilinear cracks were attacked in works of Panasyuk(1968) and Savruk et al.(1981,1989a). This method employ σ_θ-criterion for determination of crack trajectory. It is assumed that the direction of development coincide with the direction of the maximal stress for σ_θ . The problem is solved in terms of quasi-static approash. On every transition step the variation of stress field near crack end is taken into account.

Using this approach the following problems were investigated: the confluence of parallel cracks, wich are linear at the first moument, in the external uniform stress field (Kolodko and Martynyuk,1989), the problem of cracks trajectories, when the seam fracture is near the free plane or circular surface (Martynyuk,1989), the propagation of radial cracks caused by the explosion of a cylindrical charge in the two-axis external stress field. The non-symetric stress field in latter problem twists the trajectories of cracks. In solving this problem we estimate the influens of gaz outflow from borehole after explosion. The velocity of

cracks is given by (8).

The brittle fracture occurs in the percussive failure of rocks by some mining machinery.It is an interesting problem to obtaine the volume and shape of the rock fragments , that are formed by percussion of a hard striker on the surface of the rock. For example, Fig.2. shows calculated cracks trajectories resulting from the action of the striker on the rock surface near the vertical free boundary. Depending on the direction of impact we obtain different trajectory.

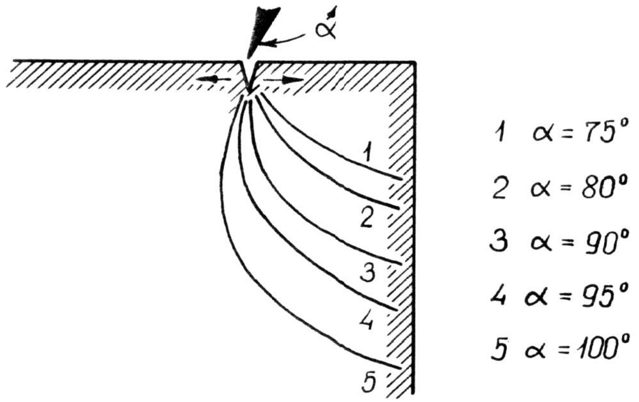

Fig.2. The cracks trajectories in the case of striker impact near vertical free boundary.

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