ASYMMETRIC CRACK EXTENSION UNDER STATIC LOADING IN FINITE PLATE

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Utilizing an earlier developed approach based on the energy balance equations in integral form a solution describing an eccentric crack growth process under uniform loading was obtained. Necessity to determine the crack eccentricity change during the crack extension and to consider the factor of an initial crack tip blunting asymmetry was described. A comparison of experimental and calculated data revealed that the proposed model is capable to approximate the test curves.

KEYWORDS

Static loading, crack growth, asymmetric problems, energy balance, crack tip blunting, crack extension trajectory, load intensity.

INTRODUCTION

The case is considered when two parameters, namely the crack length and this crack displacement from the plate centre-line or two coordinates of its tips, are required to describe the crack geometry in order to investigate the static crack extension. This problem gives an opportunity to get some notion about particular features intrinsic to the more general problem-strength of a structure with a system of cracks.

The solution for this problem can be achieved as a next step in development of an approach based on the energy balance equation and the load intensity conception. For the first time the load intensity as a fracture criterion was proposed (Yemelyanov, 1983) to deal with some particular cases when the stress intensity criterion fails to predict the residual strength. The term "LOAD INTENSITY" identifies a quantity which equals to the FORCE TRANSMITTED through a CRACK AREA when there is NO CRACK divided by the CRACK FRONT LENGTH and is denoted by "p". For a through-crack p = S * l, where S - gross stress level and l - half-length of the crack. A substantiation to the proposed parameter was given (Yemelyanov, 1989, 1990) with a help of the energy balance equation in an integral form and also by adopting some ideas proposed by Parton and Morozov (1985). Using this approach some results (Yemelyanov,1989,1990) were obtained for finite and infinite plates under uniform tension.This study is done mainly from the mechanics point of view in order to further substantiate the proposed model in case of asymmetric crack extension.

FORMULATION OF ANALYTICAL MODEL

For this analysis the case of a limited width plate with a crack shifted to one side was chosen as a simple example of the great variety of such problems.
All geometric parameters of the plate and a crack in it are shown in fig 1.The material is assumed to be isotropic and linearly elastic.This plate of a constant thickness is subjected to uniform tension.
To solve the problem it is necessary to establish some relation between the load and the crack length or the crack tip coordinates.
Energy balance equations are written for each crack tip with following assumptions:

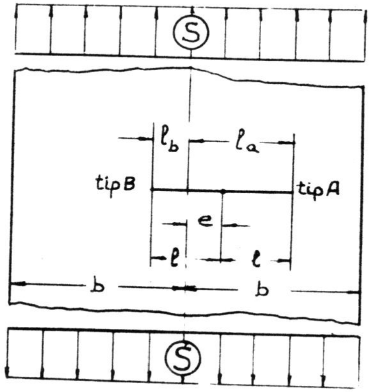


Fig.1

1.Energy release rate at the crack extension is determined by the stress intensity factor:

$$\partial W/\partial l_a = G_a \sim K_a^2/2E = S^2*l*Y_a^2/2E \text{ and}$$
$$\partial W/\partial l_b = G_b \sim K_b^2/2E = S^2*l*Y_b^2/2E, \qquad (1)$$

where Y_a and Y_b are geometrical correction factors for the two crack tips,which depend on the plate geometry,the crack length and also on the crack eccentricity/asymmetry.
2.Energy absorption on a unit crack extension is presented by:

$$\frac{\partial\Gamma}{\partial l_a} = \gamma_0*\left(\frac{l_a-l_{a1}}{l_1}\right)^k \quad \text{and} \quad \frac{\partial\Gamma}{\partial l_b} = \gamma_0*\left(\frac{l_b-l_{b1}}{l_1}\right)^k, \qquad (2)$$

where l_{a1} and l_{b1} - reference crack tip coordinates and $l_1 = (l_{a1} + l_{b1})/2$.
3.Around crack tips only the state of small scale yielding exists and any plastic deformation effects are allowed for by expressions (2).Certainly this can be the case only in some limits of the load and crack length.
4.Beyond those allowed for by expressions (1) and (2) no other energy sources and losses are considered.Then differential energy balance equations for the both crack tips are:

$$\frac{\partial U}{\partial l_{a(b)}} = \frac{\partial\Gamma}{\partial l_{a(b)}} - \frac{\partial W}{\partial l_{a(b)}} = \gamma_0*\left(\frac{l_{a(b)}-l_{a(b)1}}{l_1}\right)^k - \frac{S^2*l*Y_{a(b)}^2}{2*E}, \qquad (3)$$

where U - the system total energy.After integration of the expressions (3),we get:

$$U = \gamma_0*\frac{l_1}{k+1}*\left(\frac{l_a-l_{a1}}{l_1}\right)^{k+1} - \frac{S^2*l^2*Y_{pa}^2}{2*E} + f(S,l_b) \qquad (4)$$

and

$$U = \gamma_0*\frac{l_1}{k+1}*\left(\frac{l_b-l_{b1}}{l_1}\right)^{k+1} - \frac{S^2*l^2*Y_{pb}^2}{2*E} + f(S,l_a) \qquad (5)$$

where

$$Y_{pa}^2 = 2/l^2*\int_{lo}^{l_a} l*Y_a^2*dl_a$$

and

$$Y_{pb}^2 = 2/l^2*\int_{lo}^{l_b} l*Y_b^2*dl_b \qquad (6)$$

The equations (4) and (5) present the same variable and they have to coincide.This is the case if the system total energy can be written as:

$$U = f_1(l_a) + f_2(l_b) + f_3(S) \qquad (7)$$

and if $f_1(la)$ and $f_2(lb)$ are equal to the sums of the first two terms in the equations (4) and (5) correspondingly.
The first terms in $f_1(l_a)$ and $f_2(l_b)$ provide the total energies required to extend the crack by an amount of l_a-l_{a1} or l_b-l_{b1}.
The second terms are the parts of the total elastic deformation energy of the plate released when a crack,with the crack tip coordinates l_a and l_b,is opened under an applied stress S.Their sum gives the total energy released at these conditions.This division of the total energy into two the parts in a way is arbitrary as the lower limit (lo) of the integral in the expressions (6) common for la and lb is arbitrary.
The last term ($f_3(S)$)in the expression (7) depends only on the stress level and can be considered as the elastic energy of the plate having no crack.
5.There is an energy exchange between crack extension process and the plate elastic energy only in the form of elastic energy flow into the crack tips represented by the second terms in the equations (4) and (5);there is no energy exchange in any other form.This assumption permits us to omit the last term from expression (7) or to exclude the elastic energy of the plate without crack in the energy balance equation.
6.Crack extension process is quasi-static.This means that at every load level the plate is in equilibrium.This condition is fulfilled if in every point of a crack extension curve the overall change of the system total energy is zero when varied state is real (variation along the tangent).That is,

$$\delta U = (dU/dl_a)*\delta l_a + (dU/dl_b)*\delta l_b = 0 \qquad (8)$$

According to (8) with arbitrary values of variations δl_a and δl_b it follows:$dU/dl_a = 0$ and $dU/dl_b = 0$.It means that $f_1(l_a) = \text{const1}$ and $f_2(l_b) = \text{const2}$ or:

$$\gamma_0*\frac{l_1}{k+1}*\left(\frac{l_{a(b)}-l_{a(b)1}}{l_1}\right)^{k+1} - \frac{S^2*l^2*Y_{pa(b)}^2}{2*E} = \text{const1(2)}$$

The values of the constants (const1 and const2) are determined as zero as $l_a \to l_{a1}$ and $l_b \to l_{b1}$ gross stress $S \to 0$.
With the following denotations:

$$\sqrt{4*\ *E/((k+1)*l^k)} = A*\alpha^n, \quad (k+1)/2 = n \qquad (9)$$

we get two equations determining asymmetric crack extension under static loading.

$$A*\alpha^n*(l_a - l_{a1})^n - S*l*Y_{pa} = 0$$
$$A*\alpha^n*(l_b - l_{b1})^n - S*l*Y_{pb} = 0 \qquad (10)$$

The value $p = S*l$ (Ymelyanov,1983) identified as the load intensity on the crack front in the case of an infinite plate.It was later (Ymelyanov,1989) found that the non-dimensional values Y_{pa} and Y_{pb} which are functions of non-dimensional crack length and approaching unit when their arguments are close to zero can be considered as correction factors for the load intensity in a finite plate.

Then in the case of a limited plate: $p = S * l * Y_p$. (11)

The denotations (9) were chosen having in view that for a plastic crack extension with a suitable value it can be written from (10):

$$A^{1/n} * \delta_a = p_a^{1/n} , \quad \text{and } A^{1/n} * \delta_b = p_b^{1/n}, \qquad (12)$$

δ_a and δ_b are denoting crack opening displacement at the corresponding crack tip.

The exponent n is indicative of the character of a functional dependence between energy required for a unit crack extension and a finite crack length increase.This function can be ascending (n > 1/2) or descending(n < 1/2) or even be a constant value (n = 1/2,Griffith' case).When n is approaching zero all the crack extension curves which can be obtained with the help of the equations (10) acquire forms that are characteristic of brittle materials.Some experimental data show ability of the load intensity to serve as a fracture criterion in the case of structurally brittle materials.

All the above mentioned assumptions and formulations are applicable to any symmetrical or asymmetrical system.Now it is suitable to begin solution of the problem described in fig.1. For the purpose the following relations in the fig.1 are useful to note:

$$l_a=l + e \text{ and } l_b=l - e \quad \text{or} \quad dl_a=dl+de \text{ and } dl_b=dl-de. \qquad (13)$$

The main difference between the symmetrical and the asymmetrical problems is in the fact that in the second case the crack "trajectory" or the function of the type e = f(l) is not known This function has to be determined on solving the problem. That is why to solve an asymmetrical problem a method based on projection from an infinite plate plane to the finite plate plane which has been utilized in a paper by Yemelyanov (1989) can not be used here.Hence,in the present case the method of step-by-step numeral integration is chosen.From equations (10) and (13) where S and l are common it follows:

$$(e - e_1)/(l - l_1) =(Y_{pa}^{1/n}-Y_{pb}^{1/n})/(Y_{pa}^{1/n}+Y_{pb}^{1/n}) \qquad (14)$$

l_1 and e_1 are l and e values corresponding to zero-stress starting point as was agreed above.Differentiation of the last equation permits to establish following expression:

$$\frac{de}{dl} = \frac{\dfrac{Y_{pa}^{1/n}-Y_{pb}^{1/n}}{Y_{pa}^{1/n}+Y_{pb}^{1/n}} + \dfrac{2(l-l_1)}{l*n}\cdot\dfrac{Y_{pa}^{1/n}*Y_{pb}^{1/n}}{(Y_{pa}^{1/n}-Y_{pb}^{1/n})^2}\left(\dfrac{Y_a^2}{Y_{pa}^2} - \dfrac{Y_b^2}{Y_{pb}^2}\right)}{1 - \dfrac{2(l-l_1)}{l*n}\cdot\dfrac{Y_{pa}^{1/n}*Y_{pb}^{1/n}}{(Y_{pa}^{1/n}-Y_{pb}^{1/n})^2}\,\dfrac{Y_a^2}{Y_{pa}^2} + \dfrac{Y_b^2}{Y_{pb}^2}} \qquad (15)$$

Taking fixed increment dl and using expressions (6) and (13) with the help of any integration procedure values of Y_{pa},Y_{pb} and e can be calculated in function of l.Some remarks concerning details are necessary:

1.Integration should be started from crack length 2l=0 and an arbitrary value e_0.It is necessary to chose some constant de/dl initially so as to arrive at the desired l_1 and e_1 at the same time.Always it is necessary to maintain next condition: $-1.\le de/dl \le 1$.The selection of the starting point is arbitrary but the subsequent crack behaviour depends on it.It can be seen

from (12) that this selection determines a respective bluntness of the crack tips.At this stage expression (15) is not used.

2.At the next stage the calculation is done with the help of expression (15) and the corresponding S,Kr and K_G values (for both tips separately) can be obtained with the aid of the formulae similar to (10) as:

$$S_{a(b)} = A*\alpha^n(l_{a(b)}-l_{a(b)1})^n/l/Y_{pa(b)}$$

$$K_{a(b)r} = S_{a(b)}*\sqrt{l}*Y_{a(b)} , \quad K_{a(b)G} = S_{a(b)max}*\sqrt{l}*Y_{a(b)}. \qquad (16)$$

Extremum conditions for the curves $S=f(l_a)$ and $S=f(l_b)$ are obtained after differentiation of the equations (10) with respect to l keeping in mind that at this point $dS_{a(b)}/dl=0$:

$$A^{1/n}* \alpha n*(Y_{pa}/Y_a)^2 = (Smax*lmax^{1-n}*Y_{pa})^{1/n}$$

$$A^{1/n}* \alpha n*(Y_{pb}/Y_b)^2 = (Smax*lmax^{1-n}*Y_{pb})^{1/n} \qquad (17)$$

Combination of the last two equations (17) permits to show that the extremum of both curves is reached when the denominator of (15) is equal to zero.As $-1.\le de/dl \le 1$.(otherwise dla or dlb becomes negative)previous condition can not be realized at least under uniform tension.Instead of this only one of the two conditions (17) is satisfied at the extremum.Comparison of the stress values according to (16) for two crack tips can serve as an indication of the calculation accuracy.If |de/dl| according to eq.(15) exceeds unit value equal to zero should be ascribed to corresponding dl_a or dl_b and 1. or -1.to de/dl.The gross stress value and other values (16) are determined by the crack tip opposite to the motionless one.To fulfil practically these calculations an analytical approximation of a diagram (Tada et al.,1973;Rooke and Cartwright,1974) of the stress intensity data(Kl_a,Kl_b) relevant to the considered case are calculated specially.It is represented by expressions(18) and illustrated in the fig.2.

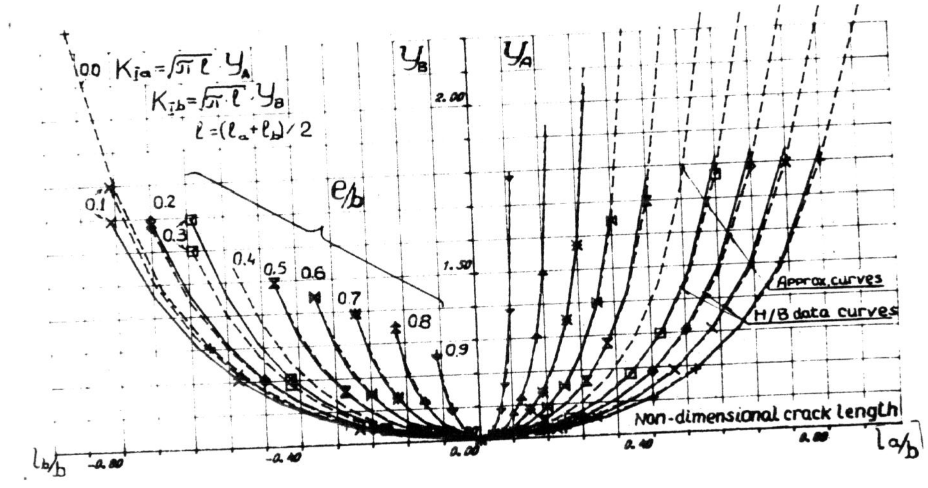


Fig.2 Compares analytical approximation (18) with handbook data (Tada et al.,1973)

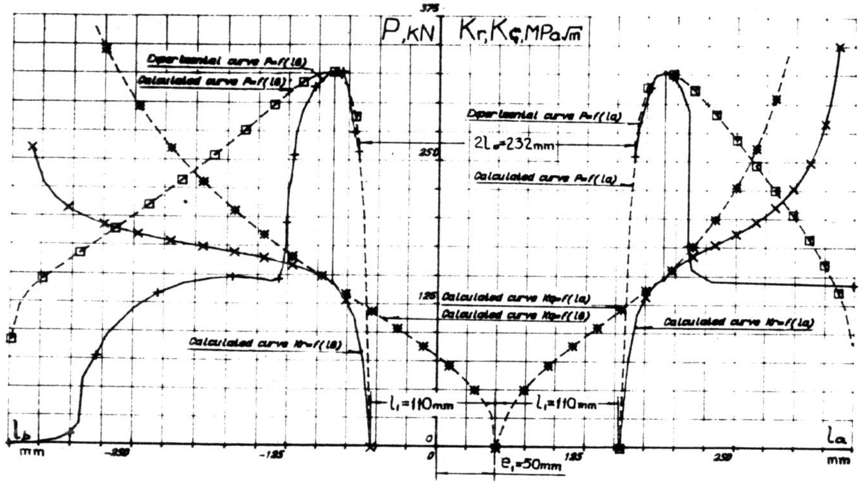
$$Y_{pa}=[1-N*(e/b)^c]*\{[\cos(\pi/2/b/(1-e/b))]^{-1/2}-1\}+1$$

$$Y_{pb}=\cos^{-1/2}\{(\pi/2/b/(1-e/b))*[1-A_1*(e/b)^{n_1}-B_1*(e/b)^{n_2}]\}, \quad (18)$$

where $N = 0.6612, c = 2.4, A_1 = 0.28, B_1 = 0.175, n_1 = 0.4, n_2 = 5.0$

VALIDATION OF THE ANALYTICAL MODEL

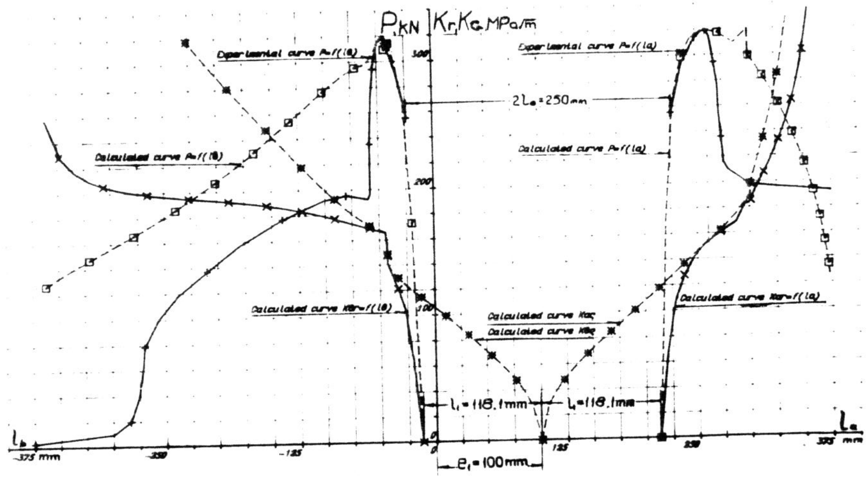
To validate of the proposed model, available experimental data were approximated using this analytic modeling. For this purpose special tests were carried out. Two sheets with eccentrically located cracks (e=50 and 100mm). were tested under uniaxial tension. The sheets with dimensions 3 * 750 * 2120mm were made of the aluminium alloy D16chT(Russian specification similar to 2024-T3). The precracking was performed under constant amplitude loading of Smax=98MPa, R ≈ 0. The crack extension under static loading before fracture and corresponding load were registered using a filming technique and breakable wire indication. The results of this experimental efforts together with the curves calculated using presented theory are given in figs.3 and 4.



Fir.3 Results of the test and analytic efforts are compared for the plate with the crack eccentricity e = 50mm (n = 0.25, b = 349mm, l_1 = 110mm, $(de/dl)_o$ = - 0.03)

For the purpose of obtaining data by calculation, a computer program has been developed based on the above theory. The results of computation include functions relating the plate load and the crack tip positions in the same way as in the case of experimental data. Kc- and Kr-curves also are computed according to the expressions (16). As there exists no statistics of A and α both of them were assumed to be equal to unity. The results of calculations were normalized so that the calculated ultimate load was equal to the experimental one and the corresponding ultimate stress was determined.
As to the value of n according to some previous experience at first it was accepted to be equal to 0.5 (Griffits' case) but it was found that much better approximation can be obtained

for n = 0.25 and 0.31 for the above cases.



Fir.4 Results of the test and analytic efforts are compared for the plate with the crack eccentricity e = 100mm (n = 0.31, b = 375mm, l_1 = 118.1mm, $(de/dl)_o$ = 0.009)

The reference crack length $2l_1$ was taken a little bit smaller than the actual initial crack length to allow for crack extension as the load increases. As precracking was carried out under a constant amplitude loading, it can be expected that crack tip formation conditions were very similar. To this case a value of de/dl approaching zero corresponds. The initial values of de/dl were chosen equal to -0.03 for e=50mm and 0.009 for e=100mm. The computations were terminated when la, or lb, or both exceeded a predetermined value, e.g. 0.999b.

DISCUSSION

The comparison of the computed and experimental curves in figs. 3 and 4 shows a good ability of the proposed model to approximate experimental data within the stage of the stable precritical crack extension. Also it shows the ability to depict truly the crack extension at both crack tips simultaneously.
Since the proposed theory has no allowance for global plastic deformation and dynamic effects it cannot be expected to give a good description of the crack extension process beyond the critical crack length. When comparing the computed and experimental curves in figs.3 and 4 in excess of the critical load point it appears that the curves computed with the constant values of the parameters corresponds to the much grater resistance to the crack extension than the experimental curves. The fact lends a base for a further development so as to include in consideration a postcritical crack extension.
The Kc- and Kr-curves in figs.3 and 4 show the consistency of the data obtained: e.g. the plate load extremum coincides with the mutual tangent point of the Kc- and Kr-curves. Further it

can be seen that beyond the critical point Kr-curves are still ascending reflecting inability of this theory to account for the postcritical effects.

As A, α and n values are combined in equation (10) in one value ($A*\alpha^n$), this value could be used as a fracture criterion for comparison purposes and the dimension of their quantity depends on the exponent n value. That is why this quantity is not usable in the original form. It can be shown that this value can be converted to non-dimensional form by multiplying with b^{n-1}/Su. The corresponding non-dimensional ratio for the two experimental cases including an account of measured residual strength of both test specimens is equal to 0.924. The same value calculated using expression (9) and assuming that material properties of the specimens (Su, E, γ_0) and other factors not included in (9) explicitly are the same is equal to 1.002. These values can be compared to evaluate stability of this parameter.

It is necessary to notice that an initial de/dl value is also a factor of a crack asymmetry: if a crack symmetrical in all other respects has an initial de/dl value not equal to zero, than the behaviour will correspond to an asymmetrical crack.

CONCLUSION

1. The proposed analytical model permits to depict the main characteristic features of asymmetrical crack extension.
2. These features include inability to predict in advance how a crack eccentricity will change as a function of its length and also the necessity to consider the factor of an initial crack tip blunting asymmetry.
3. To compute a crack growth process under static uniform loading for any geometrical configuration any means permitting to calculate stress intensity factor should be available for each crack tip. For a particular case of an eccentric crack in a plate of a limited width an analytical approximation is suggested.

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