APPROXIMATION FOR J BASED ON THE NEUBER'S RELATION FOR STRESS AND STRAIN CONCENTRATION FACTORS

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ABSTRACT

The problem of evaluation of nonlinear fracture mechanics parameters is compared with methods based on stress and strain estimation in the plastic zone of the round notch by means of Neuber's rule. For revealing of analogies of these approaches, the average strain energy parameter in some small region of the crack tip is introduced. This generalization of Neuber's approach provide some simple assessment for J-integral. This approach is sufficiently precise, as it is shown in comparison with other J-integral assessments, especially for small cracks. It may be useful for the failure assessment diagram method and other similar approaches.

KEYWORDS

Crack tip, stress / strain concentration factor, strain energy density, plastic limit load, J-integral, failure assessment diagram.

INTRODUCTION

In nonlinear fracture mechanics, experimental and analytical investigations have demonstrated that crack initiation and extension are sufficiently described by parameters of J (Cherepanov, 1967; Rice, 1968a, b) and crack opening displacement (COD) (Vitvitskij et al., 1975). These parameters for complex geometries can be evaluated by finite element method, though it requires considerable computation efforts. Therefore, a number of simplified approaches have been developed, such as engineering treatment model (Schwalbe, 1984, 1991), J-based assessment method (Turner, 1979, 1984), reference stress approximation for J (Ainsworth, 1984; Milne et al., 1986), ERPI/GE design scheme (Kumar et al., 1981) and others (Dowling and Townley, 1975; Morozov and Fridman, 1968; Vasiutin, 1988).

From the definition of J- integral (Rice, 1968b), some simple assessments of energy density in the notch tip can be obtained:

$$w_t \sim J / \rho, \tag{1}$$

ρ - notch tip radius. For w_t evaluation before the J method invention several approaches were developed by Neuber (1961) and his successors (Glinka, 1985; Makhutov, 1981; Seeger, et al. 1979, 1980; Topper et al., 1969). In spite of less theoretical justification of these approaches, their advantage is in the direct verification by experimental strain measurements in the notch tip (Makhutov, 1981).

For the generalization of these methods for cracks the parameter of average strain energy density in the small region ahead the crack tip may be concerned.

THE AVERAGE STRAIN ENERGY DENSITY IN SOME SMALL REGION AT THE CRACK TIP

Analysis of stress distribution at the crack tip provided by Hutchinson et al. (1968), Rice and Rosengren (1968) for the power law hardening materials reveals that singularity of product of stress and strain does not depend on the hardening power:

$$\sigma_{ij}\,\epsilon_{ij} \to \frac{f(\theta)}{r}\,; \qquad r \to 0 \tag{2}$$

where σ_{ij}, ϵ_{ij} - are the components of stress and strain tensors, $f(\theta)$ - angle function.

In the paper of Sih (1974), distribution of the strain energy density at the crack tip has been introduced. Let us consider the square element at the crack tip in the polar coordinates $ds = r\,d\theta\,dr$, in which the energy density for this element may be written in the form :

$$\frac{dw(r,\theta)}{ds} = \alpha\,\frac{J}{r}\,f(\theta), \qquad f(0) = 1\,, \tag{3}$$

where α - coefficient that accordingly to Hutchinson (1968) is weakly dependent upon plasticity material characteristics, $f(\theta)$ - angle function.

It may be stressed that the energy density distribution in the case of localized plasticity is practically similar to the elastic material. The same was shown for deep sharp notches by Walker (1974).

Let us introduce the average energy density w_t in ρ-vicinity of the crack tip :

$$w_t(\rho) = \frac{1}{\pi\rho^2}\int_0^{\rho}\int_0^{2\pi}\frac{dw}{ds}(\theta,r)\cdot r\,d\theta\,dr\,. \tag{4}$$

We shall start with the elastic distribution for plain strain:

$$\frac{dw}{ds} = \frac{K_I^{\,2}(1+\nu)}{8\pi E r}\Big[(3-4\nu-\cos\theta)\,(1+\cos\theta)\Big]\,, \tag{5}$$

where K_I - stress intensity factor, ν - Poison's ratio , E - Young's modulus. If one considers ρ as a fixed parameter for a given material, then between J and w_t a simple dependence may be introduced :

$$w_t = b\,J/\rho\,, \tag{6}$$

where for plain strain $b \approx b_e = (5-8\nu)/(8\pi(1-\nu))$. Obviously, relation (6) will make sense if ρ is sufficiently small.

NEUBER'S RULE AND ITS GENERALIZATION

For evaluation of stress and strain in the plastic zone near notch. Neuber (1961) has proposed an approach, which include the case of full plasticity :

$$K_\sigma\,K_\epsilon = K_t^2 \tag{7}$$

where K_σ, K_ϵ - stress and strain concentration factors in plastic zone appropriately, K_t - theoretical elastic stress concentration factor. This relation was obtained for the antiplain shear, but appeared to be very useful to the fatigue problems (Topper et al., 1969).

Rice (1968) has noted that equation (7) is not sufficiently accurate.

When the concept of the plastic limit load is introduced, it results in the following ranges of loading :

1) $P \ll P_y$ - small-scale yielding,
2) $P \to P_y$ - contained yielding,
3) $P > P_y$ - net section yield and gross yield.

Several authors showed (Makhutov, 1981; Glinka, 1985) that in the case of small scale yielding the Neuber's formula gives a slightly magnified assessment for strains and stresses. One of the most successful modifications of such an approach was proposed by Molski and Glinka (1981) :

$$w_t = K_t^2\,w_n\,, \tag{8}$$

where w_t -value of energy density of the notch, $w_n = \sigma_n^2 / 2E'$- nominal value of elastic energy density, σ_n -nominal stress, $E' = E$ for plain stress and $E' = E/(1-\nu^2)$ for plain strain. This formula reflects the Hutchinson statement, that energy density in the plastic zone for small-scale yielding is nearly equal to the appropriate elastic value.

Let introduce the reference stress value :

$$\sigma_{ref} = (P / P_y)\cdot\sigma_y\,, \tag{9}$$

where P - load, P_y - plastic limit load, σ_y - yield stress. We consider a material for which the uniaxial stress-strain curve may be described by :

$$\epsilon/\epsilon_y = \begin{cases} \sigma/\sigma_y\,, & \sigma < \sigma_y\,; \\ (\sigma/\sigma_y)^{1/N}\,, & \sigma > \sigma_y\,, \end{cases} \tag{10}$$

where σ_y, ϵ_y and N are the constants. Thus the expression (8) for $P << P_y$ can be written in another form :

$$w_t = \frac{1}{2} (K_t^*)^2 \sigma_{ref} \epsilon_{ref} , \qquad (11)$$

where $K_t^* = K_t (\sigma_n/\sigma_{ref})$, ϵ_{ref} is, from equation (10), the uniaxial strain corresponding to the stress σ_{ref} Equations (8), (11) are correct for loads $P < P_y$ For the load range $P > P_y$ the Neuber's rule was generalized by Seeger and Heuler (1980) in the following form:

$$w_t = (K_t^*)^2 \frac{\sigma_{ref} \epsilon_{ref}}{N+1} . \qquad (12)$$

These two equations may be unified by the expression, valid for both ranges :

$$w_t = \frac{1}{2} (K_t^*)^2 \sigma_{ref} \epsilon_{ref} F(P/P_y) , \qquad (13)$$

where $F(P/P_y)$ - slightly varying nondimensional function :

$$F(P/P_y) = \begin{cases} 1 , & P/P_y \to 0 \\ 2/(N+1) , & P > P_y \end{cases} . \qquad (14)$$

For entire range of loading we propose the next expression :

$$F(P/P_y) = 1 + \frac{1-N}{1+N} \left[1 - exp\left[- \frac{\epsilon_{ref}^2}{2 \epsilon_y^2} \right] \right] . \qquad (15)$$

For contained yielding $(P < P_y)$ this expression describes the peculiar Irwin correction for J and COD :

$$F(P/P_y) = 1 + \frac{1-N}{2 \cdot (1+N)} (P/P_y)^2 . \qquad (16)$$

Expression (13) reveals correct limit behavior that has experimental approval (Seeger and Heuler, 1981; Glinka, 1985).

APPROXIMATION FOR J

Let express the stress intensity factor in the form:

$$K_I = \sigma_{ref} \sqrt{\pi \cdot a_{eff}} . \qquad (17)$$

Then equation (6) for smal-scale yielding can be written as:

$$w_t = \frac{1}{2} K_t^2 \sigma_{ref} \epsilon_{ref} , \qquad (18)$$

where $K_t^2 = 2\pi \cdot b \cdot a_{eff}/\rho$.

For w_t estimation, as in the case of the notch, we obtain next relation for entire load range, that follows from expression (13) :

$$w_t = \frac{b \cdot \pi \cdot a_{eff}}{\rho} \sigma_{ref} \epsilon_{ref} F(P/P_y) . \qquad (19)$$

Using equation (6) we can rewrite the expression for J estimation:

$$J = J_e \frac{\epsilon_{ref} \cdot E}{\sigma_{ref}} F(P/P_y) , \qquad (20)$$

where $J_e = K_I^2/E'$ elastic value of J. The elastic J value is obtained, when $N=1$.

The comparison of expression (20) with R6 approach (Milne I. et al., 1986) is displayed at Fig. 1. For the same value of J , ratio of load P obtained from R6 approach to load obtained from eq. (20), F_p , is in the range $1.0 < F_p < 1.1$ (for the $P/P_y \to \infty$ limit). From the other hand such comparison of R6 with EPRI approach in terms of F_p , for various crack lengths, gives the range $0.82 < F_p < 1.13$ (Miller et al., 1986). The maximum F_p value is obtained for the case of small cracks. Therefore, we may predicts sufficient precision of the equation (20) for small cracks. The J-design curve is the upper limit for our assessment (20) (Fig. 2).

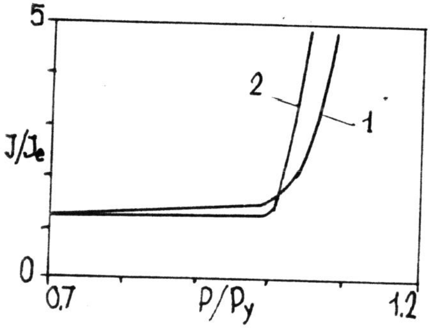

Fig. 1 Estimated values of J/J_e as a function of applied load with $N=0.05$: 1-Ainsworth; 2-formula (20)

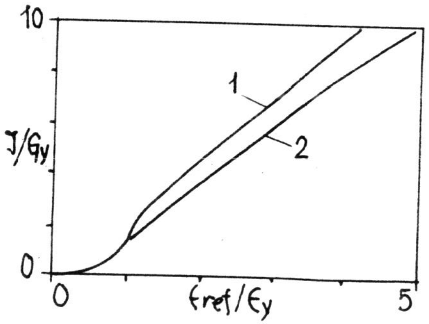

Fig. 2 Relation of J to an effective strain (G_y is value of J_c at $P=P_y$): 1-Turner; 2-formula (20) with $N=0$

ONSET OF THE CRACK GROWTH AND FAILURE ASSESSMENT CURVE

The grounds of the failure assessment curve concept lie in the two-criteria approaches, proposed in the works of Morozov and Fridman (1968), Dowling and Townley (1975).

In the most generalized form it can be defined as a curve in the coordinates $K_r = K_I/K_c$, $S_r = P/P_y$ (Milne, 1986), where K_c is the material toughness and P_y is the limit load, defined for a yield or flow stress. This curve borders the region of safe cracks. For definition of the failure assessment curve, we will start with the condition for onset of the crack growth.

According to the qualitative estimations of the slip line and finite element analysis of stress - strain field in the vicinity of the crack tip, the special process zone is formed. This zone include the large strain region with the depth δ (δ - crack tip opening displacement) and high triaxial stress region near the edge of previous region. Outside this zone the conditions for HRR field are satisfied, and inside the entire energy of deformation depends on J and zone size 2δ. In the moment of onset of the crack growth the size of this zone becomes critical ($\rho_o \sim 2\cdot\delta_c$). Thus, we may concern some zone size parameter ρ_o, which reflects the dimension of this peculiar structure when stable crack growth begin. It is greatly justified to concern the entire energy of this zone without strain and stress field definition. The critical value of the strain energy density in this zone can be chosen as the fracture criteria $w_t = w_c$. For evaluation of the strain energy density in the critical zone we propose the next generalization of expression (6):

$$w_t = b\cdot J/\rho_o + w_n , \qquad (21)$$

where w_n is the nominal value of the strain energy density in the net section. Thus, the criteria for crack growth initiation may be written in the form :

$$J/J_{Ic} + w_n/w_c = 1 . \qquad (22)$$

The value of ρ_o is connected with critical values of J and w:

$$\rho_o = b\cdot J_c/w_c . \qquad (23)$$

When the w_c value is obtained from the fracture experiments with the standard smooth sample, the magnitude of ρ_o lies in the range 10 - 1000 μm, the typical size of grain. Using expressions (20) and (22), the expression for failure assessment curve can be written:

$$K_r = \left[\frac{\sigma_{ref}\,(1 - 0.5\cdot F(\epsilon_{ref})\cdot\sigma_{ref}\,\epsilon_{ref}/w_c)}{E\cdot\epsilon_{ref}\cdot F(\epsilon_{ref})} \right]^{1/2} \qquad (24)$$

where $\sigma_{ref} = S_r\cdot\sigma_y$, $\epsilon_{ref} = f(\sigma_{ref})$ is strain-stress dependance for the smooth sample, $F(\epsilon_{ref})$ - slightly varying function (15). The curve (24), not alike R6 approach, tends to zero at certain value of S_r (Fig. 3).

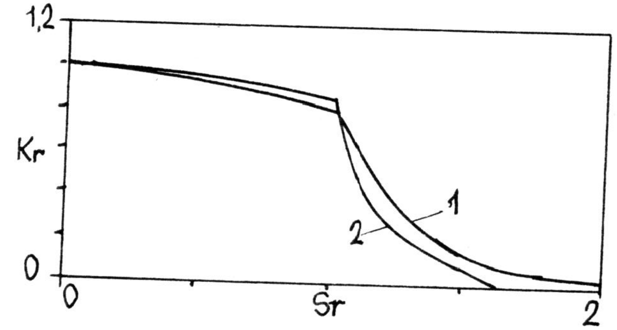

Fig. 3 Failure assessment diagrams :
1- Ainsworth,
2- formula (24) with N=0.1, w_c =120 MPa.

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