A THEORETICAL AND EXPERIMENTAL METHOD FOR FRACTURE PARAMETERS INVESTIGATION OF CRACKED STRUCTURES

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ABSTRACT

A general theoretical and experimental method is presented for fracture mechanics parameters determination. They are elastic and effective stress intensity factors of cracked structural components and stresses on a crack line before its appearing. The crack surface relative displacements measurement provides data for theoretical approach based on the additional problem solution. A crack monitoring system has been developed. The method is considered both for specimen and for structure.

KEYWORDS

Crack, structure, stress intensity factor, gage, relative displacement.

INTRODUCTION

Next parameters are needed for their determination during specimen and structure experimental investigation: elastic and effective stress intensity factors (SIF) of cracked structural components and stresses on a crack line before its appearing. The stresses on a crack line are important for crack initiation analysis. SIF are used to predict crack growth characteristics. Specific feature of the problem is the boundary condition uncertainty in complicated structural components. This circumstance is taken in consideration in theoretical and experimental method (TEM), based on crack surface relative displacements measurements and inverse problem solution (Shkarayev et al.,1988; Shkarayev,1989). The method has been generalized and crack monitoring system application discussed in this paper.

METHOD FOR THE FRACTURE PARAMETERS INVESTIGATION

Consider a structure or a structural component with a crack of length 2ℓ (Fig.1). Cracked component assuming to be planar and elastic with Young's modulus E and Poisson's ratio ν. Plane stress problem is considered.

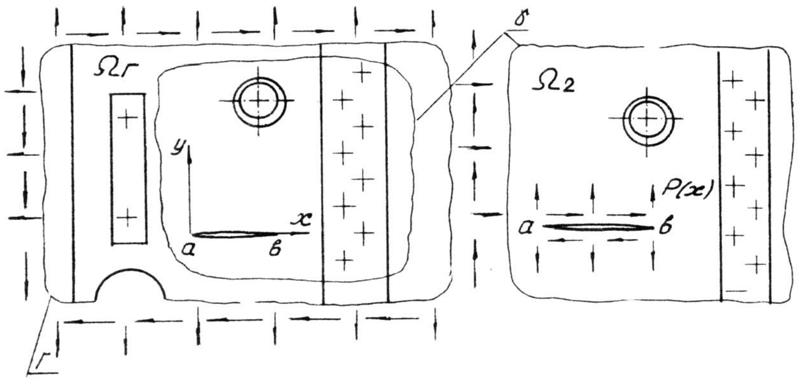

Fig.1. Original Ω_r and additional Ω_2 problems.

There are arbitrary conditions on the boundary Γ of region Ω_r, but not all of them are known. The problem is to determine mixed mode stress intensity factors $K^{a,b}=K_I^{a,b}-iK_{II}^{a,b}$ in a and b crack tips. The second goal is to reconstruct the stresses $N^o(x)=\sigma_y^o-i\tau_{xy}^o$ on the line segment $[a,b]$ in that problem with no crack.

Additional Problem Definition. In the structural component to be analyzed the additional region Ω_γ is formed, which contains the crack, part of the holes and stiffeners. Unknown loads are acting on the boundary γ. They are equal to stresses on γ in the problem Ω_r. The crack is deformed under these loads. The crack surface relative displacements are $\Delta U(x)$ in x axis and $\Delta V(x)$ in y axis direction. Write the function $\Delta g(x)=\Delta U(x)+i\Delta V(x)$. Consider Ω_1 and Ω_2 problems superposition on the Ω_γ region. Acting loads in the Ω_1 problem are the same as in Ω_γ. There is no crack in Ω_1. Internal region of Ω_2 is free from loads (Fig.1), and the crack surface is loaded by self - equilibrium loads $p(x)=p_n(x)-ip_t(x)$. They are equal and opposite to stresses $N^1(x)=\sigma_y^1-i\tau_{xy}^1$ on $[a,b]$ in Ω_1 problem. The performed transformations are identical to SIF determination. Ω_r, Ω_γ and Ω_2 have equal SIF and $\Delta g(x)$. Concerning $N^1(x)$ they will be as close to $N^o(x)$ as less difference in boundary conditions on γ in Ω_γ and in the same region, but with no crack. Thus we have $N^o(x)=N^1(x)$ when $\gamma=\Gamma$. An additional problem solution combines $\Delta g(x)$, $p(x)$ and SIF:

$$\Delta g(x)=D[p(x)]; \tag{1}$$

$$K^{a,b}=G[p(x)], \tag{2}$$

where $D[\ ]$ and $G[\ ]$ - integral operators.

Governing System. Let we have experimentally measured the crack surface relative displacements $\Delta g_1^*, \Delta g_2^*, \dots, \Delta g_r^*$ $(\Delta g_i^*=\Delta U_i^*+i\Delta V_i^*)$ in $x_1, x_2, \dots, x_r$ of the original problem. Using this data the solution of (1) has to be found. Then SIF will be discovered from formulae (2).

In general, crack surface loads will be presented as a linear combination of basic functions $F_{nj}(x)$ and $F_{tj}(x)$:

$$p(x)=\sum_{j=0}^{m_1} A_{yj}F_{nj}(x)-i\sum_{j=0}^{m_2} A_{xj}F_{tj}(x), \tag{3}$$

where A_{yj} and A_{xj} - unknown parameters.
After substitution (3) in (1) one derives

$$\Delta g(x)=\sum_{j=0}^{m_1} A_{yj}[\Delta U_{nj}(x)+i\Delta V_{nj}(x)]+\sum_{j=0}^{m_2} A_{xj}[\Delta U_{tj}(x)+i\Delta V_{tj}(x)], \tag{4}$$

where $\Delta U_{nj}(x)$, $\Delta U_{tj}(x)$ - tangential and $\Delta V_{nj}(x)$, $\Delta V_{tj}(x)$ - normal components of crack surface relative displacements under $F_{nj}(x)$ and $F_{tj}(x)$, respectively. Governing system will be structured in least - square sense

$$\sum_{i=1}^{r}\{[\Delta U_i^*-\Delta U^s(x_i)]^2+[\Delta V_i^*-\Delta V^s(x_i)]^2\} \to min; \tag{5}$$

$$\Delta U^s(x)=\sum_{j=0}^{m_1} A_{yj}\Delta U_{nj}(x)+\sum_{j=0}^{m_2} A_{xj}\Delta U_{tj}(x);$$

$$\Delta V^s(x)=\sum_{j=0}^{m_1} A_{yj}\Delta V_{nj}(x)+\sum_{j=0}^{m_2} A_{xj}\Delta V_{tj}(x).$$

An additional problem solution and basic functions choosing. Analytical solutions of the additional problems are available for the infinite and semi - infinite plate with internal and edge crack (Shkarayev 1989). Algorithm of the additional problem solution based on the finite element method has been created also (Sergeev and Shkarayev 1991).

The formulated problem has been an inverse one. Its correctness is considered in the sense of (Tihonov and Arsenin 1986). Thus, the solution has to be obtained with help of specific approaches, utilizing a priori information. The first way is the choosing of $p(x)$ in the form, involving data of its behavior. The second way is to use probabilistic empirical risk criteria for Chebishev polynomial approximation. Both procedures are used.

CRACK MONITORING SYSTEM

The measuring and processing system has been created based on this method - crack monitoring system. The system consists of: clip gages; interface; computer and software. A computer connection makes it possible to perform the measurements in real test time.

There are gages designed for measuring normal and tangential relative displacements. Normal type gage is shown in Fig.2. Two cantilever beams scheme with the foil sensing elements has been used in this gage. The gage is installed on the needles in the opposite crack surface points symmetrically on 1 mm to both sides. In the range of ± 0.5 mm the maximal error of the displacements measuring is 0.25%. This characteristic is obtained during numerous static calibration.

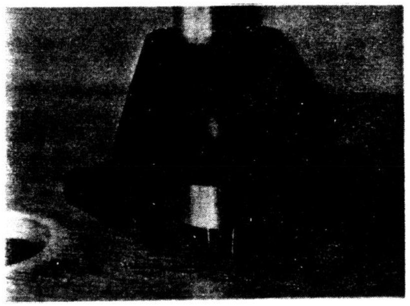

Fig.2. Gage of normal type.

Specimen Geometry and Measurements. This experiment goal was to verify crack surface relative displacements measurement and fracture parameters evaluation accuracy with help of TEM. Geometry of the specimen is shown in Fig.3. Sawcut of length 2ℓ = 20 mm was fabricated under 45 deg. to the symmetry axis from small hole of diameter 2 mm in center of the specimen . There are two holes of diameter 20 mm on the distance 30 mm from center. The material thickness is 3.67 mm, Young's modulus E = 65400 MPa.

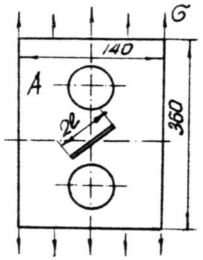

Fig.3. Specimen geometry.

The specimen is subjected to uniform tensile σ = 113 MPa. The normal ΔV_i^* and tangential ΔU_i^* crack surface relative displacement components were measured with help of monitoring system. Results are presented in Table 1.

Table 1. Crack surface displacements in μm.

	x_i, mm	2	4	5	6	7	8
Gages	ΔU^*	17.3	16.5	14.9	14.5	13.2	11.4
	ΔV^*	20.4	19.3	16.5	15.5	14.8	11.8
FEM	$\Delta U(x)$	18.42	17.40	16.56	15.44	13.94	11.88
	$\Delta V(x)$	21.05	19.41	18.14	16.53	14.53	12.02

TEM Analysis. The finite element method solution of the problem serves as the basis for the experimental data analysis. Obtained with help of program (Hilton and Gifford 1982) normal $\Delta V(x)$ and tangential $\Delta U(x)$ displacements are presented in Table 1. Theoretical SIF are: K_I = 5.74 MPa$\sqrt{m}$; K_{II} = 6.305 MPa$\sqrt{m}$.

Consider the application of the theoretical and experimental method in three variants of additional problem. Additional region A is identical to original one (Fig.3). Additional problems B and C are shown in Fig.4. Region B is a finite cracked plate and C is infinite one.

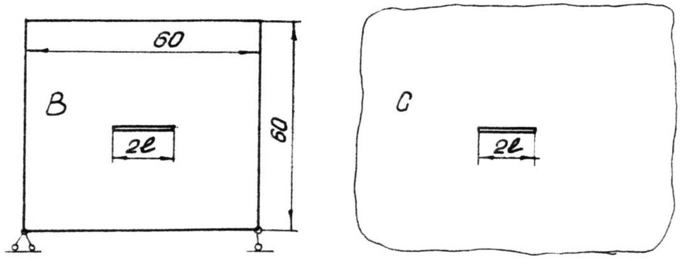

Fig.4. Additional problems B and C.

Chebishev polynomials $F_{nj}(x) = F_j(x) = T_j[\omega(x)]$ are used in crack surface loads approximation (3). Additional problems A and B are solved by procedure (Sergeev and Shkarayev 1991) based on finite element method. For the infinite cracked plate analytical formulas are utilized.

Mixed mode SIF for several values $m_1 = m_2$ are obtained with help of TEM based on the experimental ΔU^* and ΔV^* (see Table 2.). Comparison of the theoretical and experimental SIF is witnessing, that the accuracy of SIF determination depends lesser on form and greater on solution procedure of additional problem (analytical or numerical). SIF K_I^* has higher accuracy for $m_1 = m_2$ = 0; 2, and K_{II}^* - for $m_1 = m_2$ = 2; 4. The best results are obtained for quadratic approximation of loads $p(x)$. This is natural, if to mention the stresses distribution on line [$-\ell$, ℓ] in specimen with no crack. Dotted lines in Fig.5,6 are obtained from FEM solution of uncracked specimen.

Table 2. Experimental SIF.

Problem	$m_1 = m_2$	0	2	4
A	K_I^*	5.71	5.45	6.25
	K_{II}^*	5.29	5.76	6.37
B	K_I^*	5.64	5.53	6.37
	K_{II}^*	5.58	6.02	6.63
C	K_I^*	5.85	5.49	6.55
	K_{II}^*	5.19	5.63	6.35

Normal stresses distribution σ_y^* is shown in Fig.5 reconstructed by TEM with help of additional problem A for several $m_1 = m_2$. Linear approximation is averaging the stresses. For m_1 = 4 polynomial approximation is deviating from theory significantly, especially near the points $\pm\ell$. In the case of m_1 = 2 maximal error is 7.6 %. The error is reducing to $\pm\ell$.

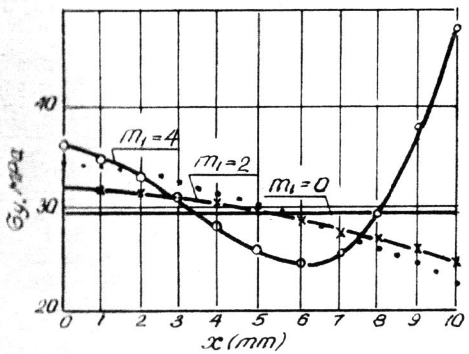

Fig.5. Normal stresses.

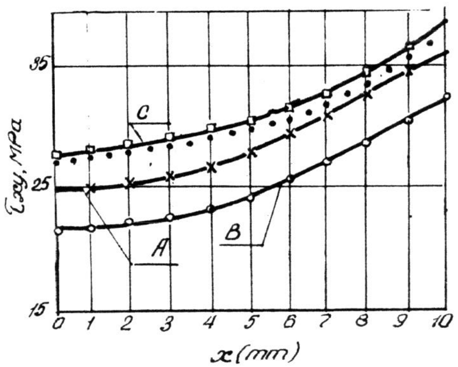

Fig.6. Shear stresses.

Shear stresses τ_{xy}^* are reconstructed by TEM with A - C additional problems. The stresses τ_{xy}^* distribution is shown in Fig.6 for $m_1 = m_2 = 2$. This worths to mention their quality consistency to theoretical results. The curvature of the shear graphs is inverse to normal one. As it was expected the accuracy both σ_y^* and τ_{xy}^* is falling for B additional problem, which form differs from original problem. In spite of this result, SIF are determined with the same accuracy for A and C problems.

Thus, if crack loads approximation can be established a priori, then TEM provides good accuracy in SIF and stresses determination.

Empirical Risk Criteria. For many cases it is difficult to choose m_1 and m_2 without additional information. Empirical risk criteria was proposed (Vapnik et al 1980) to the inverse problem probabilistic solution. Here this criteria is written for 2-D case :

$$I(m_1,m_2)=\sum_{i=1}^{r}[\Delta U_i^* - \Delta U^s(x_i)]^2/\{1-\sqrt{[(m_2+1)(\ln r-\ln(m_2+1)+1)-\ln(1-P)]/r}\}+$$
$$+\sum_{i=1}^{r}[\Delta V_i^* - \Delta V^s(x_i)]^2/\{1-\sqrt{[(m_1+1)(\ln r-\ln(m_1+1)+1)-\ln(1-P)]/r}\}, \quad (6)$$

where P - confident probability of the estimation.

The optimal m_1 and m_2 are forwarding to minimum in (6). The results of (6) calculations for A problem are presented in Table 3. Optimal powers are $m_1 = 0$; $m_2 = 2$. SIF for these values are close to data of Table 2 for $m_1 = m_2 = 2$. This proves the criteria applicability for powers choosing when Chebishev polynomials are using in crack loads approximation.

Table 3. $I(m_1, m_2)$ for P = 0.8.

m_2 \ m_1	0	2	4
0	2.161	3.576	8.363
2	2.127	3.541	8.326
4	3.009	4.422	9.207

EXPERIMENTAL INVESTIGATION OF CRACKED WING PANELS

Panel Geometry. TEM and crack monitoring system was widely applied to investigate cracking of stiffened panels during laboratory tests of the aircrafts. The upper panels of the wing has been fabricated from aluminum alloy and stringer S joins two of them (Fig.7). The wing was tested under cyclic loading. Fatigue crack appeared from sawcut after 7000 cycles of constant amplitude loading. Two crack lengths were investigated: 2ℓ = 32.9 mm ($\Delta\ell$ = 8.1 mm) and 2ℓ = 41.6 mm ($\Delta\ell$ = 12 mm).

The strain measurement on the distance 80 mm from crack plane was performed in this panel. Average normal stresses are σ_u = -121.3 MPa ("up" load) and σ_d = 75 MPa ("down" load).

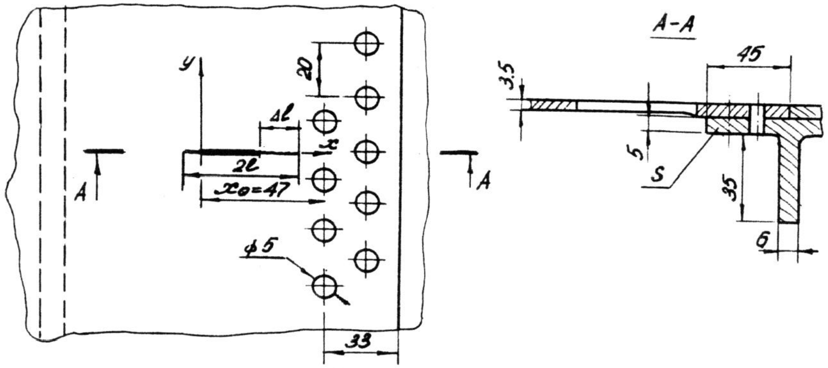

Fig.7. Cracked stiffened panel.

Measurements. Crack monitoring system was used to measure crack surface displacements. Fig.8 shows displacement ΔV^* in the point x = 26 mm for crack length 2ℓ = 41.6 mm as a function of a load, applied to wing. The opening load is 55 KN. It was found, that experimental normalized crack - opening stresses 0.335 agrees with 0.283, obtained from analytical closure model (Newman 1990). For TEM application the values of ΔV^* were calculated from the linear part of diagram load - displacement (Fig.8) as $\Delta V^* = V' - V''$.
The results are presented in Fig.9.

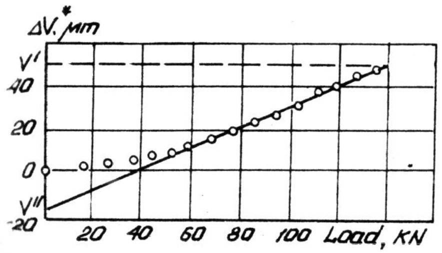

Fig.8. Relation between displacements in x = 26 mm and load.

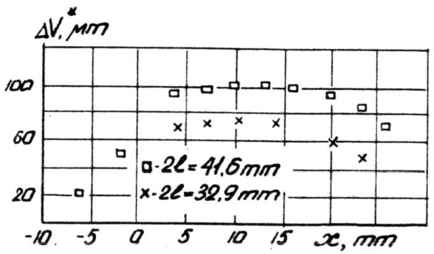

Fig.9. Crack surface relative displacements.

During wing testing crack speed was measured. For two considered crack lengths the crack speed ratio was 0.693. Taken into account, that for panel alloy the exponent in Paris equation is 3.06, SIF ratio equals 0.887.

TEM Analysis. Analytical solution for infinite plate was chosen as an additional problem. The number of computer modeling shows, that for the crack, which tip is near the stiffener line, the load approximation has to be

$$p(x)=-\sigma^*+Q^* y_Q\,[(1-\nu)(x-x_Q)^2+(3+\nu)y_Q^2]/2\pi/[(x-x_Q)^2+y_Q^2]^2 \quad (7)$$

The component of the forces in the rivet points x_Q = 47 mm, y_Q = 10 mm is included in (7) to account stringer influence.

SIF obtained by TEM are next: K_I^* = 18 MPa $\sqrt{m}$ (2ℓ = 32.9 mm); K_I^* = 20.6 MPa$\sqrt{m}$ (2ℓ = 41.6 mm), and SIF ratio is 0.872. This result is in a good agreement with one from crack speed measurement. Reconstructed stresses σ^* = 79.4 .. 85.2 MPa have reasonable accuracy in comparison with strain measuring σ_d .

CONCLUSIONS

The evaluation of the accuracy and applicability of a theoretical and experimental method and crack monitoring system have been demonstrated. The method is based on crack surface relative displacements measurements and inverse problem solution. It makes possible to determine elastic and effective mixed mode SIF in complicated structural components. With the help of this method, crack growth prediction is possible under variable amplitude loading using a closure model. Reconstruction of the stresses in uncracked structure is also performed.

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