A NEW ANALYTICAL METHOD OF THE DETERMINATION OF J_{IC} BY USING SINGLE SPECIMEN

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ABSTRACT

A new analytical method was advanced for the determination of J_{IC} value by using single specimen method, in which the correction item of elastic compliance was eliminated and a new regression method of determining the value of parameter m was introduced. The existence and the uniqueness of this new analytical solution was verified and the error distribution of the solution was also analysed.

Keywords: Single specimen, analytical method, critical J integral

1. INTRODUCTION

The multi-specimen procedure has been considered to be the most reliable method for the determination of J_{IC} value, now widely used as the elastic-plastic fracture criterion. Its application, however, is subjected to quite a lot of restrictions. For instance, it is often not possible to get the requisite number of specimens in the failure analysis of a machine part; besides, one could hardly keep the different specimens tested under similar experimental parameters either in the same environmental condition,or in the case of using specimens with shallow cracks. Hence,a great number of investigations have been done to establish a practical single specimen method[1-3].

The advantages of the analytical determination of J_{IC} value as compared with the other single specimen methods lie in avoiding the using of the precision testing machine, which is usually necessary in the case of compliance unloading technique, or the using of sensitive instruments for the detection of the initiation point of crack extension.

2. THEORETICAL ANALYSIS

An exponential relationship between the plastic displacement of the loading point, (Δp) and the applied load, P, has been proposed[5]

$$(\Delta p) = k \left[\frac{P}{B(W-a)^2}\right]^m \tag{1}$$

where k and m are material parameters, and B, W and a are the thickness, the width and the initial crack length of the specimen respectively. The P-Δ curve is shown in Fig. 1, in which C is the crack initiation point and S the unloading point. The applied load and the plastic displacement associated with C and S are P_c, $(\Delta p)_c$ and P_m, $(\Delta p)_m$ respectively. On loading from C to S the increment of the plastic displacement of the loading point is

$$\delta(\Delta p)_m = (\Delta p)_m - (\Delta p)_c = DS - BC \qquad (2)$$

where $\delta(\Delta p)_m$ is composed of the following two parts:(1).The plastic displacement increment of the loading point, $\delta(\Delta p)_1$, due to the load increment $\Delta P = P_m - P_c$. (2). The plastic displacement increment of the loading point, $\delta(\Delta p)_2$, due to the crack extension Δa. $\delta(\Delta p)_2$ is the summation of the direct and the indirect contributions to the plastic displacement. The former comes from Δa, while the latter from the variation of elastic compliance caused by the crack extension.

2. 1. Theory of the analytical method and the effect of elastic compliance

It is supposed that the material parameters k and m keep constant on loading from C to S. Thus, the total differentiation of equation (1) can be expressed as

$$\delta(\Delta p) = \frac{2m(\Delta p)}{W-a}da + \frac{m(\Delta p)}{P}dP \qquad (3)$$

The first term on the right hand side of equation (3) corresponds to the total increment of the plastic displacement caused by the contribution of the extension of initial crack length including the contribution of elastic compliance. The correct values of k and m can be obtained by the linear regression of the points (P_i, $(\Delta p)_i$) read on the P-Δ curve on the assumption that the compliance be constant.

The differential equation (3) can be rewritten in the form of finite difference

$$(\Delta p)_m - (\Delta p)_c = \frac{2m(\Delta p)_c}{W-a}\Delta a + \frac{m(\Delta p)_c}{P_c}(P_m - P_c) \qquad (4)$$

The expression of stable crack extension, Δa, can be derived from equation(4)

$$\Delta a = \frac{(\Delta p)_m - (\Delta p)_c - m(\Delta p)_c(P_m - P_c)/P_c}{2m(\Delta p)_c/(W-a)} \qquad (5)$$

By solving simultaneously equations (1) and (4) , one can obtain

$$k\left(\frac{2\Delta a m}{W-a} + 1 - m\right)\left[\frac{P_c}{B(W-a)^2}\right]^m + \frac{k_m P_m}{B(W-a)^2}\left[\frac{P_c}{B(W-a)^2}\right]^{m-1} - (\Delta p)_m = 0 \qquad (6)$$

The equation (6) can be expressed in the following form

$$\alpha P_c^m + \beta P_c^{m-1} + \eta = 0 \qquad (7)$$

The J_{IC} value can be calculated by substituting the value P_c obtained from equation (7)and the corresponding displacement of loading point Δc into the formula given by [2]

$$J_{IC} = \left(1 + \beta\frac{J_e}{W-a}\right)J_e \qquad (8)$$

where

$$J_e = \frac{M_J P_c \Delta c}{B(W-a)}$$

$$M_J = 0.2227 + 2.6839(a/W) - 2.3119(a/W)^2$$

$$\beta = \frac{E}{6\pi(1-\nu^2)\sigma_y^2}$$

where E, ν and σ_y are elastic modulus, Poisson's ratio and flow stress respectively.

2.2 Existence, uniqueness and error estimation of the real solution of equation (7)

The equation (7) , which is a high degree equation of non-integer power,can be solved by using the iteration method on computer. For convenience, this equation can be converted into an equivalent equation in another form. Substituting (1) into (6)and after adequate simplification, we obtain

$$X^m = mX + b \qquad (9)$$

where $X=(P_m/P_c)$, $b=2m\Delta a/(W-a)+1-m$. Eqation (9) implies that the unique existing solution of the equation is the abscissa value at the interaction of the the parabola, X^m, and the straight line mX+b, as indicated by the example in Fig.2.The effect of the parameters m, Δa and (W-a)on the precision of the solution is shown in Fig.2.

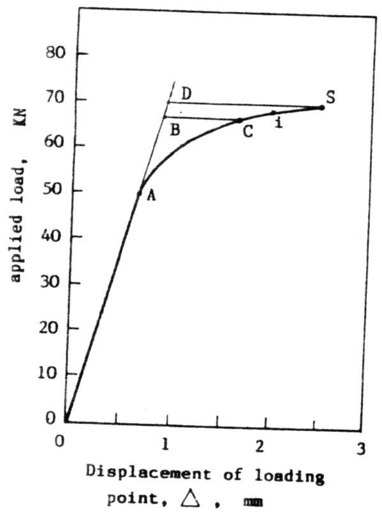
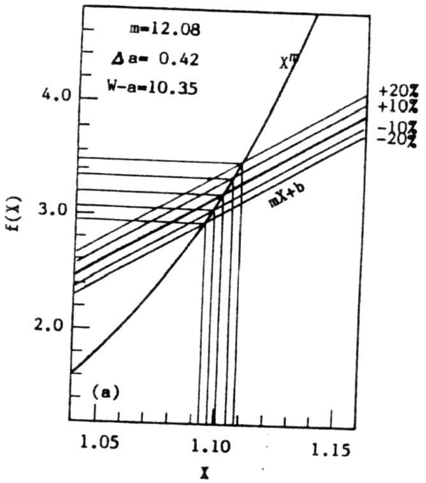
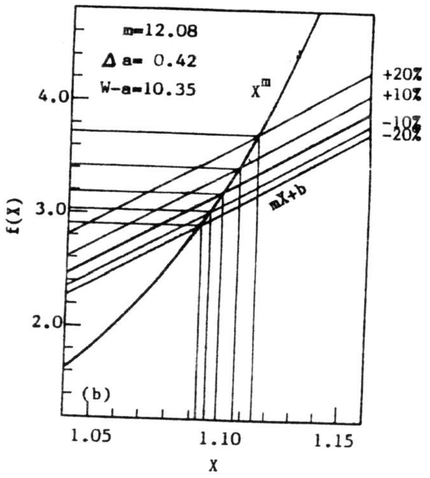


Fig.1 Schemalic P-△ Curve

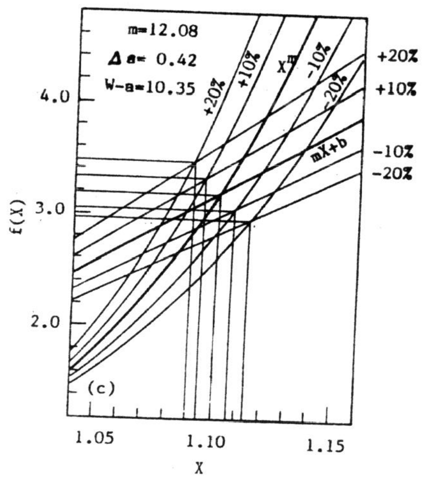


(a). effect of △a

Fig.2 Error estimation on solution of equation (10)



(b). effect of W-a



(c). effect of m

Fig.2 Error estimation on solution of equation (10)

The effect of the measurement error, △a, is shown in Fig.2a. It will not bring about any change in both the position and the shape of the parabola and will only make the straight line move parallel to itself up or down in the case of positive or negtive deviation and therefore will yield a greater or smaller value of P_c. The calculation errors of P_c and $(\triangle p)_c$ are less than 0.4% and 4% respectively, if the measurement error of △a is controlled within ±10%. In this way, the calculation error of J_{IC} is limited to less than 5%.

The effect of the measurement error of W-a on P_c is similar to that of △a as shown in Fig.2b. The calculation errors of P_c and $(\triangle p)_c$ are less than 0.5% and 7% respectively, if the measurement error of W-a is controlled within ±10%. The final calculation error of J_{IC} is still less than 5%.

The effect of m is schematically shown in Fig.2c. The regression error of value m alters not only the position and the curvature of the parabola but also the slope and the intercept of the straight line. If the regression error of m is controlled within ±10%, the calculation errors of P_c and $(\triangle p)_c$ are less than 0.5% and 7% respectively, and the final calculation error of J_{IC} is limited to less than 10%.

2.3 Regression method of value m

The main assumptions on which this new analytical method is based are as follows: (1). The relationship between the plastic displacement of the loading point, $(\triangle p)$, leaving the variation of elastic compliance out of consideration, and the applied load, P, obeys the power law, as indicated by equation(1). (2). On loading from C to S, the parameters of k and m remain unchanged, as it is supposed in order to derive the differential equation (3). (3). On loading from C to S, the increments of both the applied load, △P, and the crack extension, △a, should be small, as indicated by equation (4). Among the above mentioned assumptions, the first one has been proved experimentally[3-5] and the third one can be achieved by the proper control of the experiment. However, the second one has not yet been satisfied.

The general method to determine value m is as follows. As shown schematically in Fig.3, when a straight line is obtained, the second assumption is naturally satisfied. But if a broken line is obtained by regression, the situation will become complicated and a further discussion is required. The first problem is how to determine objectively the position of its inflection point. Secondly, when the inflection point is determined the second assumption may still not be satisfied, if the initiation point appears on segment I of the line. It is noticed that the parameter k is not required in solving P_c, the root of the equation (9), and we are only interested in the value m associated with the segment of the line containing the points C and S. It is obvious that the reliable value m depends mainly on the points close to the unloading point S, and the data near point A will inevitably introduce quite a large error and uncertainty to the results of regression. The following procedures are recommended for determining the value m. From equation (1), we obtain

$$Y = X^m \tag{10}$$

where

$$Y = \frac{(\Delta p)_m}{(\Delta p)_i}, \qquad X = \frac{P_m}{P_i}$$

Find a regression straight line, passing through the original point, that best fits the recorded data close to S with lgY and lgX as the ordinate and the abscissa respectively. The slope of the regression line represents the value m. The analysis shows that all the points involved in the regression analysis fall within the shaded triangle bounded by the lines having the slope values of $\Delta m = \pm 0.03m$, as shown in Fig.4. This means that the deviation of the regression values m is less than 3%. The regression straight line turns out to be a parabola of mth degree in the rectangular coordinate system as given in Fig.2.

The new procedures of J_{IC} determination are recommended as follows. (1). Compute with the least squares method the value m in a double logarithmic coordinate system by using equation (10); (2). Calculate X_c, P_c, $(\Delta p)_c$ and Δc by substituting the value m and measured values of Δa and W-a into the equation (9) and solving it with the iteration method; (3). Calculate the J_{IC} value by using equation (8).

3. EXPERIMENTAL VERIFICATION

To examine the accuracy of this analytical method, the practical determination of J_{IC} value was conducted with TPB specimens of BHW35 and 19Mn6 steels. The specimen size is BxWxL=20x24x110 mm. The experimental method was in accordance with the standard procedure. The material strengths obtained are: $\sigma_{0.2}$ = 537 MPa and σ_u = 670 MPa for BHW35 steel, and $\sigma_{0.2}$=358 MPa and σ_u = 569 MPa for 19Mn6 steel.

3.1 Efficacy of the improved analytical method in judging

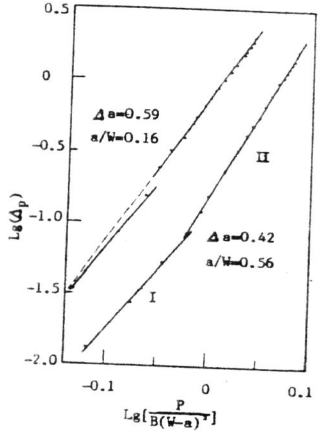


Fig.3 Conventional regression method for m

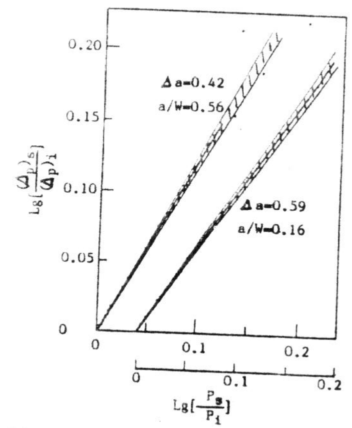


Fig.4 The new regression method for m

Table 1. Comparison between the J_{IC} values obtained from conventional and analytical method

a/W		J_{IC}, KN/m		
Tested	Average	Multi-specimen Method	Analytical Method	
			Calculated	Average
0.158 0.161 0.160 0.155 0.147 0.154	0.156	106.60	132.49 107.48 96.11 123.18 90.71 112.09	110.33
0.550 0.533 0.548 0.551 0.543 0.556	0.54	169.27	168.49 183.98 175.64 164.07 162.40 166.92	170.25

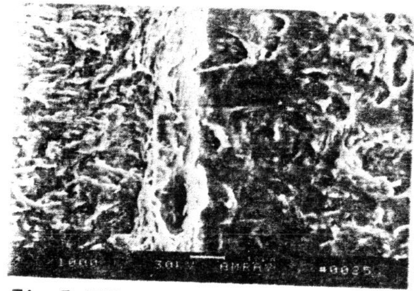


Fig.5 SEM photo showing critical point determined by analytical method, a/W=0.16

Table 2. Comparison between tested and calculated Δa values

No	Crack Extension, mm		Deviation mm	a/W	Correlation Coefficient	Number of Regression Points
	Tested	Calculated				
1	0.196	0.228	-0.032	0.556	0.9987	21
2	0.280	0.236	0.044	0.567	0.9990	25
3	0.420	0.403	0.017	0.563	0.9980	19
4	0.540	0.576	-0.036	0.573	0.9920	29 *
5	0.570	0.502	0.068	0.556	0.9987	25
6	0.890	0.453	0.437	0.580	0.9665	21
7	1.040	0.479	0.561	0.624	0.9830	32
8	1.446	0.861	0.585	0.567	0.9940	24 *
9	1.580	0.893	0.687	0.547	0.9887	26 *
10	0.428	0.493	-0.065	0.152	0.9980	26
11	0.476	0.537	-0.061	0.167	0.9990	23
12	0.585	0.546	0.039	0.161	0.9985	24

* represents the results on 19Mn6 steel.

the crack point

A pair of specimens of almost the same size were used to examine the efficacy of the new method. One of the specimens was tested to acquire the P_c value after being loaded up to the crack extension point. The other one was loaded up to a point just above the P_c value and then unloaded. The second specimen was re-fatigued and then broken. A typical SEM photo of the fracture surface taken from the second specimen is shown in Fig.5. The crack extension, Δa, was measured to be 55μm, including a SZW of 35μm. The calculated crack extension point was proved to be satisfactory.

3.2 Comparison between J_{IC} values determined by this new analytical method and the multi-specimen method

The J_{IC} values of the specimens with different crack lengths were determined by using this analytical method and the multi-specimen method. The results are listed in Table 1. It is seen that J_{IC} values obtained by both methods are in good agreement. The applicable crack length of the specimen for this method is between 0.1 and 0.7 times the depth, W.

3.3 Reasonable range for Δa

Table 2 shows the influence of different crack extensions, Δa, on K_{IC} values of the tested materials. The results indicate that the experimental and calculated results agree well when $\Delta a < 0.60$mm. The difference between them becomes significant only when $\Delta a > 0.8$mm, because here the third assumption is no longer satisfied. The reasonable range of Δa in this work is concluded to be 0.15-0.60mm.

4. CONCLUSION

The new single specimen analytical method is simple and reliable in calculation, and may be applied to the accurate calculation of the critical point load, Pc, and the stable crack extension, Δa. The J_{IC} values obtained by the experiment and the analytical technique are in good agreement in the range of Δa=0.15 - 0.60mm. This new method applies well to specimens with crack length between 0.1 and 0.7 times W and the results obtained agree well with those obtained from the standard multi-specimen method.

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