ON THE SINGULAR CHARACTER OF THERMAL STRESS FOR NON-HOMOGENEOUS BODY WITH SHARP NOTCH

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ABSTRACT

Temperature gradient and stress field in a neighbourhood at the notch tip of bimaterial for thermoelastic problem are considered. Boundary conditions for the temperature fields are $T|_\Gamma = 0$. Initial conditions are arbitrary. Temperature field and temperature gradient are given by expressions of the form $T \sim r^s$, $\nabla T \sim r^{s-1}$ respectively ($s > 0, r$ is polar coordinate centered at the tip of the corner). The least value s equals 1/4. In particular, when a crack locates between dissimilar materials. s equals 1/2. With the help of the results obtained by Kondratev (1967).it is shown that the order of singularity is defined by eigenequation for a homogeneous problem. Temperature field can not increase this order.

KEYWORDS

bimaterial, notch tip, crack, heat transfer problem, thermoelastic problem, singular stress.

INTRODUCTION

In two-dimensional elasticity stresses near sharp notch may exhibit singular behavior. Many authors investigated these equations in the absence of body forces and temperature. Williams (1952), Parton and Perlin (1981) studied singularities near corner with free-free, free-fixed, fixed-fixed boundary conditions. Heat transfer and thermoelastic problems considered here are specific cases of the boundary value problems for elliptic and parabolic equations of general type. The aim of this paper is to study of singularities arising due to a temperature field.

THE TEMPERATURE FIELD

The paper is considering a bimaterial which consists of two plane isotropic domains Ω_1 and Ω_2. The bimaterial is formed by three rays l, l_1, l_2 coming out of point O. l is the interface boundary. The angle of l_1 with l equals ω_1 $(0 < \omega \leq 2\pi)$. In every domain Ω_i temperature field T_i satisfy the following equation

$$b_i^2 \frac{\partial T_i}{\partial t} = \Delta T_i \;,\; t > 0 \tag{1}$$

with the initial and boundary conditions

$$T_i|_{t=0} = \varphi_i(x) \;,\; x \subset \Omega_i \;, \tag{2}$$

$$T_1|_{l_1} = 0 \;,\; T_2|_{l_2} = 0 \tag{3}$$

respectively.

It is asumed that function T_i are continuous in Ω_i $(i = 1, 2)$. The temperature fields T_1 and T_2 on the contact line l satisfy the typical conformity conditions

$$T_1 = T_2 \;,\; k_1 \frac{\partial T_1}{\partial n_1} = k_2 \frac{\partial T_2}{\partial n_2} \;, \tag{4}$$

k is coefficient of the thermal conductivity.

Let r, φ be a polar coordinate system sentered at the point O. Equations $\varphi = 0$, $\varphi = \omega_1$, $\varphi = \omega$ correspond to the rays l_1, l, l_2 respectively. Denote function T that $T = T_1 \; if \; x \subset \Omega_1 \; and \; T = T_2 \; if \; x \subset \Omega_2$. Indicate the asymptotical form of the solution T in a neighbourhood of the point O. The structure of the function T with boundary conditions (3),(4) is studied. From the general theory of parabolic equations (Grisvard, 1992) of the boundary value problem the function T has the form

$$T = C(t) r^s \Phi(\varphi) + o(r^s)$$
$$\frac{\partial T}{\partial x_i} = C_i(t) r^{s-1} \Phi_i(\varphi) + o(r^{s-1}) \tag{5}$$

where $C(t)$ belongs to C^1 , $s > 0$.

Determination of s is as follows: $\Phi(\varphi)$ has the next form

$$\Phi(\varphi) = \begin{cases} C_1 \sin \varphi s & 0 < \varphi < \omega_1 \\ C_2 \sin(\omega - \varphi) s & \omega_1 < \varphi < \omega \end{cases} \tag{6}$$

where C_1, C_2 are constants.

Unknown C_i are determined by the conformity condition (4)

$$C_1 \sin \omega_1 s - C_2 \sin(\omega - \omega_1) s = 0$$
$$k_1 C_1 s \cdot \cos \omega_1 s + k_2 C_2 s \cdot \cos(\omega - \omega_1) s = 0. \tag{7}$$

The system (7) is homogeneous. For non-trivial values of the unknown C_1, C_2 the determinant of this system must vanish:

$$\begin{vmatrix} \sin \omega_1 s & -\sin(\omega - \omega_1) s \\ k_1 \cos \omega_1 s & k_2 \cos(\omega - \omega_1) s \end{vmatrix} = 0, \tag{8}$$

or

$$\tan \omega_1 s + \frac{k_1}{k_2} \tan \omega_2 s = 0. \tag{9}$$

In the capacity of the solution s it is necessary to take the minimal positive root of the equation (9). The least value s equals $1/4$. If $k_1 = k_2$ then $s = \pi/\omega$. In particular, if a crack lokates on the interfase boundary of two dissimilar halfplanes $(\omega_1 = \omega_2 = \pi)$ then s equals $1/2$ for any value k_1/k_2.

SINGULAR SOLUTIONS

First consider boundary value problem of thermoelasticity with the angular point O

$$A\vec{u} \equiv (\lambda + \mu) \; grad \; div\vec{u} + \mu \Delta \vec{u} = (3\lambda + 2\mu) \alpha \; grad \; T \tag{10}$$

and boundary conditions

$$\vec{u}|_\Gamma = \vec{\psi}(x), \tag{11}$$

λ, μ are Lame's coefficients, α is coefficient of expansion.

The temperature field T satisfies (1)-(4). If the right hand side of (10) is absent, the order of singularity is defined by the eigenequations which were given by Williams (1952), Parton and Perlin (1981). So in this case the singular solution with zero displacements must be investigated

$$\vec{u}|_\Gamma = 0. \tag{12}$$

It is proved (Kondratev, 1967) , that for reseach of this solution it is sufficient to explore the solution of the eigenequation

$$A\vec{u} = 0 \tag{13}$$

with boundary conditions (12), when displacements are sought in the form $\vec{u} = r^\lambda \vec{f}(\varphi)$. r, φ is polar coordinate system centered at the point O. λ is complex parameter, which depends on the size of the angle ω and Poisson's ratio. Physically admissible eigenvalues giving singular stresses at the point O must satisfy the condition

$$0 < Re\lambda < 1. \tag{14}$$

Also one can say, that the order of singularity is determined by value λ with minimal real part h. It is known the next statement relative to system (10), (12) coming out of the general theory of the boundary value problem for elliptic equations (Kondratev, 1967; Nazarov and Plamenevski 1991).

Let $|T| \le Cr^s$, $|\nabla T| \le Cr^{s-1}$. *If* $s < h-1$ *then* $|\vec{u}| \le C_1 r^{s+1}$ *and* $|\nabla \vec{u}| \le C_1 r^s$. *If* $s \ge h-1$ *then* $|\vec{u}| \le C_1 r^h$ *and* $|\nabla \vec{u}| \le C_1 r^{h-1}$.

It was shown above, that $s > 0$. So we have. If the singular stresses for homogeneous problem ($T = 0$) are absent ($h \ge 1$), then the considered temperature field does not induce singularities. In presence of the singular stresses of the homogeneous problem the temperature field can not increase this order. All results can be extended to other boundary conditions (free-free, fixed- fixed, free-fixed).

Return to a bimaterial. In this case the singular solution is defined also by the eigenvalue λ of the eigenequation, which includes homogeneous equations and boundary conditions respectively

$$A\vec{u}_1 = 0,\ A\vec{u}_2 = 0,\ a_1\vec{u}_1 = 0,\ a_2\vec{u}_2 = 0, \tag{15}$$

when displacements are sought in the form $\vec{u}_1 = r^\lambda \vec{f}(\varphi)$, $\vec{u}_2 = r^\lambda \vec{g}(\varphi)$.

The conformity conditions on the contact line l are satisfy

$$u_r^1 = u_r^2\ ,\ u_\varphi^1 = u_\varphi^2\ ,\ \sigma_{r\varphi}^1 = \sigma_{r\varphi}^2\ ,\ \sigma_{\varphi\varphi}^1 = \sigma_{\varphi\varphi}^2. \tag{16}$$

The last relationship of (16) in presence of the temperature is non- homogeneous from the difference of coefficients of expansion of materials. But, when the system (15),(16) is solved this term must be discarded. Because by means of the simple change the variable

$$u_\varphi^i = v_\varphi^i\ ,\ u_r^i = v_r^i + Cr^{s+1}\ ,\ (i = 1, 2) \tag{17}$$

arise the situation, which have discussed above.

CONCLUSION

It is possible to prove that in any rational physical processes,temperature gradient in a neighbourhood angular point of bimaterial has the form $\nabla T \sim r^{s-1}$ with $s > 0$. So the main result of this paper holds for other thermoelastic problems. Similar result takes place in the presence of some other body forces.

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