ON ONE POSSIBLE APPROACH TO ESTIMATING FRACTURE TIME UNDER CREEP CONDITIONS

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ABSTRACT

A model for long-term strength of structural materials within the context of a reliability theory is proposed. For the creep and damage power laws the mean time of specimen fracture under constant load and its standard deviation have been estimated. It is shown that an increase in a fracture time variation coefficient is observed with a decrease of stress in the case of brittle fracture. The range of stresses is obtained in which material hardening during creep does not practically affect this coefficient. The comparison of theoretical calculations with the results of experiments on long-term strength of Type X18H10T corrosion-resistant steel specimens is given.

KEYWORDS

Creep fracture, long-term strength, damage, reliability, mean time before fracture, standard deviation of fracture time.

INTRODUCTION

Delayed fracture of structural elements is the process of the microcrack initiation and development under applied load. For this process description Yu.N.Rabotnov's kinetik theory is apPicable involving the parameter corresponding to damage accumulated as one of the state characteristics. One failed so far to formulate general theory covering main creep fracture effects and describing quantitatively available experimental data with sufficient accuracy.

In developing the methods of long-term strength computation and these methods application to real structure analysis it is necessary to take into account a rather great time spread in test data before fracture (Rabotnov, 1969; Kachanov, 1974). The most natural way of describing the scatter is to develop a physically adequate probability model for fracture and to use statistical physics methods.

This way as applied to the problem under consideration is associated with great difficulties due to the complexity and different-scale inhomogeneity of solids. In general the possibilities of obtaining quantitative long-term strength characteristics for modern engineering materials in this way are rather limited.

From the point of view of application statistical methods based on direct experimental data on spread and the simplest probability models are more effective.

PROBLEM FORMULATION

In the present paper a creep-fracture model based on the positions of the reliability theory is being developed.

Let us assume that in the specimen fracture under constant load a functional dependence exists between the value ε of creep deformation accumulated in time t and specimen non-fracture probability at the given instant P .

In the reliability theory an index is widely used referred to as "failure rate" which is related to no-failure operation probability (non-fracture probability) $P(t)$ as follows

$$P(t) = exp\left[- \int_0^t \lambda(t)dt \right] . \tag{1}$$

In most cases the failure-rate function $\lambda(t)$ varies unmonotonously in time and possesses an U -shaped form (Bolotin, 1990). The typical curve of a creep deformation rate versus time t is of a similar nature. The function $\lambda(t)$, accurate to a constant, can be expected to be expressed in the form

$$\lambda(t) = C\,\dot{\varepsilon}(t) . \tag{2}$$

Substituting (2) into (1) and taking into account that $\varepsilon(0)=0$ at the initial instant of time we have

$$P(t) = exp[-C\ \varepsilon(t)].$$

The constant C is determined from the condition that $P = P_*$ with $\varepsilon = \varepsilon_*$ where P_* is a specimen non-fracture probability if creep deformation reaches the value ε_* ; ε_* is a creep deformation mean value at fracture moment. Finally, having performed non-complicated transformations we have

$$P(t) = exp[-m\ \varepsilon(t)/\varepsilon_*] \tag{3}$$

where $m = -\ln P_*$ is a parameter estimated by the results of material tests on creep before fracture.

Mean time before fracture $\langle t_* \rangle$ and life standard deviation $\langle \sigma_t \rangle$ for a rod stretched by constant force under creep conditions are estimated using the first and the second moments of a distribution

$$\langle t_* \rangle = \int_0^\infty P(t)dt , \tag{4}$$

$$\langle \sigma_t \rangle = \left\{ 2 \int_0^\infty tP(t)dt - \left(\int_0^\infty P(t)dt \right)^2 \right\}^{\frac{1}{2}} . \tag{5}$$

According to the fracture model considered Poshivalov (1989) the equation of creep is expected to be independent of the damage parameter. However, the third period of creep observed not rarely cannot be fully explained only by the decrease in specimen cross-sectional area. In this case the micropore development and cracking are important. For this process description Rabotnov (1969) included the damage ω $(0 \le \omega \le 1)$ as a structural parameter into the equation of creep.

Simplifying the further analysis the equations of creep and long-term strength are taken in the form of an exponential function when specimen necking in deforming is accounted for

$$\dot{\varepsilon}\varepsilon^{\alpha} = a\sigma_0^n\ exp(n\varepsilon)\ (1-\omega)^{-q} , \tag{6}$$

$$\dot{\omega} = c\sigma_0^k\ exp(k\varepsilon)\ (1-\omega)^{-r} . \tag{7}$$

Here, a , c, k, n, r, q, α - material constants at the given temperature; σ_0 is a nominal stress related to the initial cross-sectional area. It is worth noting that in a general case the parameters a , c, k, n, r, q, α are random values the necessary information of which can be given only in the result of statistical processing of a great number of test data on specimens during creep. Subsequently, under a , c, k, n, r, q, α we understand mean values of corresponding quantities. Having divided the equation (6) by (7) we get a differrential equation for ε in a function of ω . Integrating it, when the initial condition $\varepsilon(0) = 0$ is accounted for, we have

$$1-\omega = \left\{ 1 - \frac{c(r-q+1)}{a}\ \sigma_0^{-(n-k)} \int_0^\varepsilon \varepsilon^{\alpha} exp[-(n-k)\varepsilon]d\varepsilon \right\}^{\frac{1}{r-q+1}} \tag{8}$$

Substituting the expression (3) into (4) and (5) and considering (6) and (8) yield finally

$$\langle t_* \rangle = nt_1 \int_0^{\varepsilon_*} \varepsilon^\alpha exp[-(n + m\varepsilon_*^{-1})\varepsilon] f(\varepsilon) d\varepsilon \quad , \tag{9}$$

$$\langle \sigma_t \rangle = nt_1 \left\{ 2 \int_0^{\varepsilon_*} \left[\int_0^{\varepsilon} \varepsilon^\alpha exp(-n\varepsilon) f(\varepsilon) d\varepsilon \right] \times \right.$$

$$\times \varepsilon^\alpha exp[-(n + m\varepsilon_*^{-1})\varepsilon] f(\varepsilon) d\varepsilon - \tag{10}$$

$$\left. - \left(\int_0^{\varepsilon_*} \varepsilon^\alpha exp[-(n + m\varepsilon_*^{-1})\varepsilon] f(\varepsilon) d\varepsilon \right)^2 \right\}^{\frac{1}{2}}$$

where

$$f(\varepsilon) = \left\{ 1 - \frac{n-k}{\upsilon} \int_0^{\varepsilon} \varepsilon^\alpha exp[-(n-k)\varepsilon] d\varepsilon \right\}^{\lambda - 1}, \quad \upsilon = \frac{\lambda}{\mu + 1} \frac{t_2}{t_1},$$

$\lambda = (r+1)/(r-q+1)$, $\mu = k/(n-k)$.

Here, t_1 and t_2 are the respective times of ductile and brittle fracture defined by the relations

$t_1 = 1/(an\sigma_0^n)$, $t_2 = 1/[c(1+r)\sigma_0^k]$.

Two versions for the problem solution are possible.

1.The creep deformation value tends to infinity. The equality (8) implies that a certain finite damage value exists $\omega = \omega_*$ corresponding to breaking moment

$$\omega_* = 1 - \left\{ 1 - \frac{c(r-q+1)}{a} \sigma_0^{-(n-k)} \int_0^{\infty} \varepsilon^\alpha exp[-(n-k)\varepsilon] d\varepsilon \right\}^{\frac{1}{r-q+1}}.$$

If α are integers the integral can be calculated in the following way

$$\omega_* = 1 - \left[1 - \frac{\alpha!}{\upsilon(n-k)^\alpha} \right]^{\frac{1}{r-q+1}}.$$

In this case the relations (9) and (10) take the form

$$\langle t_* \rangle = nt_1 \int_0^{\infty} \varepsilon^\alpha exp(-n\varepsilon) f(\varepsilon) d\varepsilon \ , \qquad \langle \sigma_t \rangle = 0 \ . \tag{11}$$

2.The fracture condition $\omega = 1$, as follows from (8), is reached with a certain finite creep deformation value $\varepsilon = \varepsilon_*$ which is defined by the relation

$$\frac{a\sigma_0^{n-k}}{c(r-q+1)} = \int_0^{\varepsilon_*} \varepsilon^\alpha exp[-(n-k)\varepsilon] d\varepsilon \tag{12}$$

From the equation (12) we find creep deformation at breaking moment ε_* and then from (9) and (10) - a mean time before fracture $\langle t_* \rangle$ and its standard deviation $\langle \sigma_t \rangle$. From the relation (12) with integers α it is not difficult to obtain the condition of brittle fracture

$$\sigma_0 \le \left[\frac{c(r-q+1)\alpha!}{a(n-k)^{\alpha+1}} \right]^{\frac{1}{n-k}} = \sigma_0' \ .$$

Let us consider the case when material hardening does not take place during creep ($\alpha = 0$).

In the fist version with $\sigma_0 > \sigma_0'$ $(\upsilon > 1)$ from (11) we have a determinate solution, obtained by Rabotnov(1969). In the second version, taking into account (12) , we otain the solution from (9) and (10) with $\sigma_0 < \sigma_0'$ $(\upsilon < 1)$ in the form

$$\frac{\langle t_* \rangle}{t_2} = \lambda \int_0^1 (1 - \upsilon + \zeta\upsilon)^{\mu - m \ln^{-1}(1-\upsilon)} \zeta^{\lambda - 1} d\zeta \ , \tag{13}$$

$$\frac{\langle \sigma_t \rangle}{t_2} = \lambda \left\{ 2 \int_0^1 \left[\int_\zeta^1 (1 - \upsilon + \zeta\upsilon)^{\mu} \zeta^{\lambda - 1} d\zeta \right] (1 - \upsilon + \right.$$

$$+ \zeta\upsilon)^{\mu - m \ln^{-1}(1-\upsilon)} \zeta^{\lambda - 1} d\zeta - \left[\int_0^1 (1 - \upsilon + \right. \tag{14}$$

$$\left. \left. + \zeta\upsilon)^{\mu - m \ln^{-1}(1-\upsilon)} \zeta^{\lambda - 1} d\zeta \right]^2 \right\}^{\frac{1}{2}}$$

The integrals in the right-hand sides of equality (13) are expressed through hypergeometric functions.

For small υ 's from the relations (13) and (14) we get approximately

$$\frac{\langle t_* \rangle}{t_2} = 1 - \frac{[\mu - m \ln^{-1}(1-\upsilon)] \ \upsilon}{\lambda + 1} \ , \tag{15}$$

$$\frac{\langle \sigma_t \rangle}{t_2} = \sqrt{-\frac{\upsilon m\, \ln^{-1}(1-\upsilon)}{2\lambda + 1}\left\{1 - \frac{2\lambda+1}{(\lambda+1)^2}\,[\mu - m\,\ln^{-1}(1-\upsilon)]\upsilon\right\}} \,. \quad (16)$$

Rabotnov's (1969) solution follows from (13)-(16) at $m = 0$. If the equation of creep (6) is independent of the damage parameter ω ($q = 0$) then the relations (13) and (14) are integrated in quadratures

$$\frac{\langle t_* \rangle}{t_1} = \frac{\ln x}{\ln x - m}\,(1 - e^{-m}x)\,, \quad (17)$$

$$\frac{\langle \sigma_t \rangle}{t_1} = \left\{ 2\left[\frac{\ln x}{\ln x - m}\,(1 - e^{-m}x) - \frac{\ln x}{2\ln x - m}\,(1 - e^{-m}x^2)\right] - \frac{\ln^2 x}{(\ln x - m)^2}\,(1 - e^{-m}x)^2\right\}^{\frac{1}{2}} \quad (18)$$

where

$$x = [1 - t_2/(t_1\eta)]^{\eta}, \quad \eta = \mu + 1.$$

NUMERICAL RESULTS

The numerical analysis of creep fracture to be used here is based on the solutions (13),(14) and (17),(18).

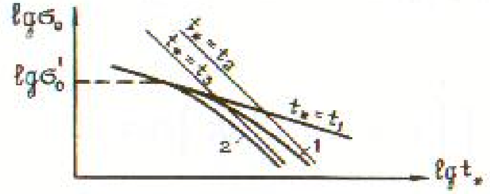

Fig.1.Diagrams of the long-term strength.

Fig.1 shows a long-term strength curve 1 by Kachanov's (1974) solution and curve 2 - by a revised solution (17). The straight lines $t_* = t_1$ and $t_* = t_2$ characterize pure ductile and pure brittle fracture.

Curve 2 lies beneath curve 1 ; transition to pure ductile fracture occuring simultaneously.

Curve 2 in logarithmic coordinates has an inclined asymptote the equation of which is defined in the form

$$t_* = t_3 = \frac{1 - e^{-m}}{m}\, t_2 .$$

Fig.2 shows the dependence of time fracture variation coefficient $\upsilon_t = \langle \sigma_t \rangle / \langle t_* \rangle$ on the parameter $S = [(n-k)^{\alpha+1} a / c(r+1)\alpha!]\sigma_0^{n-k}$

at $m = 0.5$ and $q = 0$ for different values of the constant η. Here, the solid lines correspond to $\alpha = 0$, dashed lines - to $\alpha = 1$, dot-and-dash ones-to $\alpha = 2$.

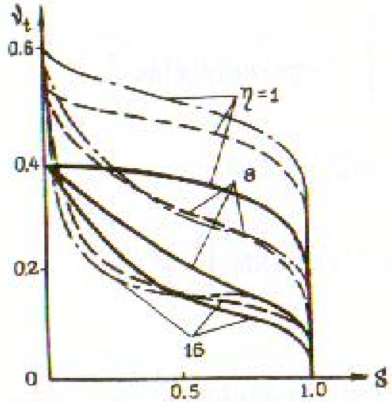

Fig.2.Graphs of time fracture variation coefficients υ_t versus stress σ_0.

It is seen from the picture that there is an increase in spread of experimental data with a decrease in stress in the case of brittle fracture. In addition, judging by the results of calculation accounting for material hardening during creep does not practically influence the value of a fracture time variation coefficient υ_t for $S \geq 0.1$. υ_t is a monotonically decreasing function and in this case has two finite limits

$$\lim_{\sigma_0 \to \sigma_0'} \upsilon_t = 0,$$

$$\lim_{\sigma_0 \to 0} \upsilon_t = \begin{cases} \dfrac{(1 - 2me^{-m} - e^{-2m})^{\frac{1}{2}}}{1 - e^{-m}}, & \alpha = 0 \\[2ex] \dfrac{[5 - e^{-m}(m+2)(m^2+m+2) - e^{-2m}(m+1)^2]^{\frac{1}{2}}}{1 - e^{-m}(m+1)}, & \alpha = 1 \\[2ex] \dfrac{[76 - 2/3e^{-m}(m^5+5m^4+20m^3+54m^2+108m+108) - e^{-2m}(m^2+2m+2)^2]^{\frac{1}{2}}}{2 - e^{-m}(m^2+2m+2)}, & \alpha = 2 \end{cases}$$

A comparison of theoretical calculations with the results of experiments on long-term strength of Type X18H10T corrosion-resistant steel specimens at temperature 1123K (Lokoshchenko at al.,1979) is made. The material constants are evaluated after corresponding experimental results processing on creep and long-term strength

$a = 0.63 \times 10^{-8}\ MPa^{-3.2}/h$; $n = 3.2$; $q = 0.7$; $\alpha = 0$;
$c = 0.58 \times 10^{-7}\ MPa^{-3.12}/h$; $k = 3.12$; $r = 1.56$; $m = 0.4$.

Table 1.Theoretical results and experiment.

N	σ_0,MPa	t_*,h	σ_t,h	$\langle t_* \rangle$,h	$\langle \sigma_t \rangle$,h
10	40	51.3	14.5	52.3	16.1
11	50	21.8	5.1	26.0	8.0
6	60	15.4	5.0	14.7	4.5
2	80	6.0	0	6.0	1.8

The Table 1 presents measured (the third and the forth columns) and calculated (the fifth and the sixth columns) values of specimen fracture time and their standard deviations evaluated from (13) and (14). Here, the number of specimens N tested at different stress levels are also indicated.

The integrals in the right-hand sides of the equlities of (13) and (14) are determined by numerical methods.It is directly seen from the Table that the theoretical results obtained are in a satisfactory agreement with experimental results both by fracture time and its standard deviation.

CONCLUSIONS

•In this paper while establishing relationship between specimen non-fracture probability and creep deformation accumulated in time of load action the assumption was made on the similarity of creep deformation rate curves and failure rate. In a general case a more complex functional relation must exist between these values.Not pretending to complete description of creep fracture statistical character nevertheless assumed approach seems to make it possible to qualitatively and quantitatively describe natural spread in experimental data versus fracture time.

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