A NUMERICAL METHOD FOR MODELLING CREEP CRACK PROPAGATION BY DIFFUSIVE VOID GROWTH

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ABSTRACT

This paper describes a numerical method which has been developed to model the growth of a macroscopic crack in a non-linear viscous material. The material surrounding the damage zone which forms ahead of the crack is modelled using an incompressible, non-linear viscous, plane strain finite element formulation. Steady-state crack growth is incorporated into the formulation by assuming that the growth is controlled by boundary-diffusion void growth in the damage zone. Crack velocities are obtained for cracks growing in a C*-integral field. The resulting computations are in agreement with simple analytical models of fully constrained and unconstrained void growth, and provide a means of interpolating between these extreme situations.

KEYWORDS

Creep crack growth; finite elements; diffusive void growth.

INTRODUCTION

The integrity of power-generating plant operating at high temperatures is potentially threatened by excessive creep deformations of components, and by the propagation of dominant, macroscopic cracks. The phenomenon of high temperature crack growth has been extensively studied both experimentally and theoretically, and has been reviewed by Riedel (1987) and Saxena (1991).

Cracks frequently grow by the nucleation, growth and coalescence of grain-boundary microvoids in a damage zone ahead of the crack tip. Cocks and Ashby (1982a) have shown that the growth of these voids may be controlled by boundary diffusion, surface diffusion, power-law creep, or by a combination of these mechanisms.

Theoretical models have been developed by Cocks and Ashby (1982b) and Riedel (1980) which describe the growth of cracks by void growth. Both assume that the growth is accompanied by extensive creep of the component, when the near-tip stress and strain-rate fields are characterized by the C*-integral, Landes and Begley (1976). C* is analogous to the J-integral of post-yield fracture mechanics, Hutchinson (1968) & Rice and Rosengren (1968); and has been found to correlate with creep crack growth rates for extensive creep conditions.

Finite element (FE) analysis has been used to study a number of different aspects. Needleman and Rice (1980) analysed the growth of grain boundary voids, while Tvergaard (1985) also included the effects of grain boundary sliding. Crack-tip stress and strain rate fields have been compared with the corresponding analytical HRR fields by Bassani and McClintock (1981) and Li et al.(1988a). Finally, transient creep crack growth rates have been obtained by FE analysis: for example, Hawk and Bassani (1986), Li et al.(1988b) and Wang et al.(1991).

This paper uses FE analysis to model the steady-state propagation of a crack by boundary-diffusion void growth. Extensive creep is assumed, with the crack and damage zone enclosed in a C* field.

GOVERNING EQUATIONS

Non-Linear Viscous Material

For a non-linear viscous material undergoing small, plane strain deformations the principle of virtual work may be written as (Needleman and Shih, 1978):

$$\int_A \sigma_{ij} dA - \int_\Gamma T_i \dot{u}_i ds = 0 \qquad (1)$$

where σ_{ij} and $\dot{\epsilon}_{ij}$ are the stress and strain rate tensors ,T_i denotes the tractions, and $\dot{u}_i$ are the displacement rates throughout the material in region A, with boundary Γ.

The constitutive relationship we employ for power-law creep is:

$$\dot{\epsilon}_{ij} = (1+\nu)\dot{\epsilon}_0\left(\frac{\sigma_e}{\sigma_0}\right)^{n-1}\left[\frac{s_{ij}}{\sigma_0} + \frac{1-2\nu}{3\nu}\frac{\sigma_{kk}}{\sigma_0}\delta_{ij}\right] \qquad (2)$$

where σ_e is the von Mises effective stress, s_{ij} are the deviatoric stresses, ν is Poisson's ratio, and $\dot{\epsilon}_0$, σ_0 and n are material parameters. When ν=0.5 eqn.(2) reduces to the standard expression for power-law creep of an incompressible material. In the present paper incompressibility has been approximated by using a Poisson's ratio of 0.4999. This allows the displacement finite element method to be used.

Damage Zone

The situation to be modelled is illustrated in Fig.1, where a crack with a damage zone is introduced into a creeping body. The damage zone comprises a length, δ of grain boundary co-linear with the crack. Along the grain boundary a series of spherical microvoids exists, with a radius r, and spacing 2l.

We will consider the extension of the crack by the growth and coalesence of the voids by grain-boundary diffusion. The volumetric growth rate of the voids by boundary diffusion is given by Cocks and Ashby (1982a) as:

$$\dot{v} = \frac{4\pi\Omega D_b\delta_b}{kT\ln(1/f_h)}\sigma_n \qquad (3)$$

where Ω is the atomic volume, D_b the grain-boundary diffusion coefficient, δ_b the grain-boundary thickness, k is Boltzmann's constant, T is temperature, f_h is the area fraction of the voids (=r^2/l^2), and σ_n is the stress normal to the damage zone.

Eqn.(3) can be expressed in terms of the displacement rate normal to the grain boundary as:

$$\dot{u} = D(u)\sigma_n = -\frac{6\Omega D_b\delta_b}{kTl^2\ln F}\sigma_n \qquad (4)$$

where

$$F = 3u/4l - f_i^{3/2}$$

and f_i is the initial value of f_h.

If attention is confined to situations where the crack and damage zone form a line of symmetry in the body, then the virtual work principle in eqn.(1) becomes:

$$\int_A \sigma_{ij}\dot{\epsilon}_{ij} dA - \int_\Gamma T_i\dot{u}_i ds + \frac{1}{2}\int_\gamma \dot{u}\sigma_n ds = 0 \qquad (5)$$

where γ represents the boundary of the damage zone.

Crack Growth

The crack shown in Fig.2 is assumed to propagate with steady state velocity $\dot{a}$. The normal displacement rate throughout the damage zone in the y-direction is given by:

$$u(x) = \frac{1}{\dot{a}}\int_x^\delta \dot{u}(x')dx' \qquad (6)$$

At the crack tip the displacement may be equated to the critical crack tip opening displacement u_c, which we take to be a property of the material:

$$u_c = u(0) = \frac{1}{\dot{a}}\int_0^\delta \dot{u}(x')dx' \qquad (7)$$

Rearranging eqn.(7) gives the crack velocity, which may be substituted into eqn.(6) to yield:

$$u(x) = u_c\int_x^\delta \dot{u}(x')dx' \Big/ \int_0^\delta \dot{u}(x')dx' \qquad (8)$$

FINITE ELEMENT FORMULATION

Incompressibility

Four noded isoparametric, quadrilateral elements have been used to implement eqn.(5) into a finite element scheme. Apart from the numerical problems associated with modelling incompressible material in plane strain, this implementation is a standard application of the displacement FE method, see for example Hughes (1987).

As the incompressible limit, with a Poisson's ratio of 0.5, is approached, elements of the type used become susceptible to mesh locking. That is the mesh becomes over-constrained as a result of the imposed incompressibility conditions. There are a number of recognised solutions to this problem, normally categorized as either mixed or penalty methods, and reviewed by Hughes (1987).

Mixed methods involve the introduction of an additional unknown, the hydrostatic pressure p. The incompressible constraint p plays the role of a Lagrange multiplier in the system of FE equations.

Penalty methods, one of which will be used here, allow for slight compressibility with ν approaching 0.5, Hughes (1987). In order to eliminate mesh locking however, reduced or selective integration procedures are necessary.

In our case we have used selective integration, with four Gauss points for the deviatoric terms, and a single

Gauss point for the volumetric terms. Then eqn.(5) may be written as:

$$\{\dot{u}\}^T \sum_{\alpha=1}^{4} ([\bar{B}]^T [D] [\bar{B}])_\alpha \{\dot{u}\} - \{T\}^T \{\dot{u}\} + \{\dot{u}\}^T [k_D] \{\dot{u}\} = 0 \qquad (9)$$

where [D] is obtained from eqn.(2), α denotes the Gauss points, and $[\bar{B}]$ is a modification of the strain displacement matrix [B] to implement selective integration, Hughes (1980).

The matrix $[k_D]$ is a function of the void size and spacing within the damage zone. For a prescribed distribution of damage the set of non-linear equations represented by eqn.(9) can be solved directly to yield the instantaneous displacement and void growth rates. In the following section we postulate the existence of a steady state. This places restrictions on the size distribution of voids within the damage zone which is determined as part of the solution process.

<u>Crack Growth</u>

In Fig.3 finite elements numbered from 1 to N form the damaged grain boundary along the element sides with nodes numbered from 1 to n. Node 1 is situated at the crack tip. The second integral of eqn.(8) is then given by:

$$\int_0^\delta \dot{u}(x')dx = \lambda(\dot{u}_1/2 + \sum_{i=2}^{n} \dot{u}_i) \qquad (10)$$

where $\dot{u}_i$ are the nodal displacement rates normal to the damage zone, and λ the element lengths.

At the extreme right-hand end of the damage zone $\dot{u}_n = 0$. By a similar treatment of the first integral the displacement at the ith node becomes:

$$u_i = u_c(\dot{u}_i/2 + \sum_{k=i+1}^{n} \dot{u}_k) \,/\, (\dot{u}_1/2 + \sum_{k=2}^{n} \dot{u}_k) \qquad (11)$$

Substitution of u_i into eqn.(4) yields the function D(u) at each of the nodes in the damage zone, and the matrix $[k_D]$ of eqn.(9). These can be treated as nodal stiffnesses to be appended to the existing element stiffness matrices for elements 1 to N. Further details may be found in Witts (1992).

<u>Solution of Equations</u>

On assembly of the overall stiffness matrix, from the element stiffness matrices and the constraint of eqn.(11), the resulting non-linear equations have been solved by the iterative Newton-Raphson method. This follows the example of Shih and Needleman (1984). Additionally, in order to obtain convergent solutions for values of creep exponent n>1, parameter tracking has been used. That is, the solution for n=1 has been used as the initial conditions for n=2, and so forth to larger values of n. The computed displacement rate field can then be used to calculate the crack velocity via eqn.(7), and the distribution of void size within the damage zone through eqn.(6).

FINITE ELEMENT MODELLING

The FE model used to obtain the results in the following section is illustrated in Fig.4. It consists of 136 4-noded elements, with a maximum of 8 elements adjacent to the damage zone. The radius of the semi-circular boundary is approximately 10 times the damage zone length.

Tractions have been specified around the semi-circular boundary in order to impose a C* field on the region analysed. The tractions were computed from the equation given by Riedel (1987):

$$\sigma_{ij} = [C^*/I_n B r]^{1/(n+1)} \tilde{\sigma}_{ij} \qquad (12)$$

where I_n and $\tilde{\sigma}_{ij}$ are given by Riedel (1987), r = radius and B = $\dot{\epsilon}_0/\sigma_0^n$.

On the lower boundary, nodes not in the damage zone, and not on the crack face, were restrained in the y-direction.

MODEL PREDICTIONS & COMPARISON WITH OTHER STUDIES

Initial computations have been performed for a range of C* values, over a wide range of ϕ_0. Cocks and Ashby (1982a) defined the material property ϕ_0 as:

$$\phi_0 = \frac{2D_b\delta_b\Omega}{kTl^3}\frac{\sigma_0}{\dot{\epsilon}_0} \qquad (13)$$

which provides a measure of the ratio of deformation-rate resulting from damage growth to that resulting from power law creep at a suitably chosen reference stress σ_0. When ϕ_0 is small the rate of grain-boundary diffusion is sluggish compared to power-law creep and the last term of eqn.(9) can be neglected when determining the stress field in the body. Conversely when ϕ_0 is large the rate of grain-boundary diffusion is much faster than power-law creep, and deformation within the damage zone is completely constrained by the surrounding power-law creeping material.

Experimental and theoretical studies of crack growth-rate in the limit where C* determines the remote stress and displacement-rate fields suggest a crack growth-rate law of the form:

$$\dot{a} \sim (C^*)^m \qquad (14)$$

where m is a material property. Riedel (1980) assumed that the stress field is unaffected by the presence of the damage and predicted m = 1/(n+1). Cocks and Ashby (1982b), Thouless (1988) & De Voy and Cocks (1992) assumed that the damage zone is fully constrained, and obtained a law with the form of eqn.(14) with m = n/(n+1).

The results of a series of computations over a wide range of ϕ_0 are summarized in Figs.5 and 6, for σ_0 = 25 MPa, $\dot{\epsilon}_0$ = 0.002 /s, n = 3.0, f_i = 0.02, l = 0.01 mm, u_c= 0.005 mm and δ = 0.076 mm. For a given value of ϕ_0 the results correspond with eqn.(14). When ϕ_0 is small m = 1/(n+1), and when it is large m = n/(n+1), in agreement with the simple analytical models described above. The transition between these two extreme conditions occurs over a wide range of ϕ_0, with m increasing from 1/(n+1) to n/(n+1) as ϕ_0 is increased from 10[illegible] to 10^4.

In the unconstrained limit the theoretical models predict the following relationship between crack velocity and damage zone size:

$$\dot{a} \sim \delta^p \qquad (15)$$

where p = n/(n+1). This result is confirmed by the present work, as shown in Fig.6.

The results of this preliminary study serve to validate the numerical procedures described in the paper, while also identifying the range of applicability of the simple analytical models.

CONCLUDING REMARKS

In the case where void growth is controlled by grain-boundary diffusion, and the crack grows in a C* field, two regimes of creep crack growth have been confirmed. One, when void growth is unconstrained, occurs at low diffusion rates, and the crack velocity varies with C* to the index 1/(n+1). The other regime occurs at higher diffusion rates, and the crack velocity varies with C* to the index n/(n+1). Here void growth is constrained by the surrounding creeping material. A transition region has been observed in which the index of C* changes between these extreme values.

The modelling described in this paper has been confined to situations where the damage growth is controlled by grain-boundary diffusion. The techniques described here can, however, be readily extended to include other mechanisms of damage growth within the damage zone.

REFERENCES

Bassani J.L. and McClintock F.A. (1981). Creep relaxation of stress around a crack tip. Int.J.Solids Struc. Vol.17, 479-492.
Cocks A.C.F. and Ashby M.F. (1982a). On creep fracture by void growth. Prog.in Mat.Science Vol.27,189-244.
Cocks A.C.F. and Ashby M.F. (1982b). The growth of a dominant crack in a creeping material. Scripta Met. Vol.16, 109-114.
De Voy J. and Cocks A.C.F. (1992), to appear.
Hawk D.E. and Bassani J.L. (1986). Transient crack growth under creep conditions. J.Mech.Phys.Solids Vol.34, 191-212.
Hughes T.J.R. (1980). Generalization of selective integration procedures. Int.J.Num.Methods Eng. Vol.15, 1413-1418.
Hughes T.J.R. (1987). The finite element method. Prentice-Hall, London.
Hutchinson J.W. (1968). Singular behaviour at the end of a tensile crack in a hardening material. J.Mech.Phys.Solids Vol.16, 13-31.
Landes J.D. and Begley J.A. (1976). A fracture mechanics approach to creep crack growtn. In: Mechanics of Crack Growth, ASTM STP 590 128-148.
Li F.Z, Needleman A. and Shih C.F. (1988a). Characterization of near tip stress & deformation fields in creeping solids. Int.J.Frac. Vol.36, 163-186.
Li F.Z, Needleman A. and Shih C.F. (1988b). Creep crack growth by grain boundary cavitation. Int.J.Frac. Vol.38, 241-273.
Needleman A. and Rice J.R. (1980). Plastic creep flow effects in the diffusive cavitation of grain boundaries. Acta Met. Vol.28, 1315-1332.
Needleman A. and Shih C.F. (1978). Finite element method for plane strain deformations of incompressible solids. Computer Methods Appl.Mech.Eng. Vol.15, 223-240.
Rice J.R. and Rosengren G.F. (1968). Plane strain deformation near a crack tip in a power-law hardening material. J.Mech.Phys. Solids Vol.16, 1-12.
Riedel H. (1980). The extension of a macroscopic crack at elevated temperature by the growth and coalesence of microvoids. In: Creep in Structures, IUTAM Symposium, 504-519, Springer-Verlag, Berlin.
Riedel H. (1987). Fracture at high temperatures. Springer-Verlag, Berlin.
Saxena A. (1991). Creep crack growth in high temperature ductile materials. Eng.Frac.Mech. Vol.40, 721-736.
Shih C.F. and Needleman A. (1984). Fully plastic crack problems: solutions by a penalty method. J.Appl.Mech.ASME Vol.51, 48-56.
Thouless M.D. (1988). Modelling creep crack growth in ceramic materials. In: Mechanics of Creep Brittle Materials I, (A.C.F. Cocks & A.R.S. Ponter) 50-62, Elsevier, London.
Tvergaard V. (1985). Effect of grain boundary sliding on creep constrained diffusive cavitation. J.Mech.Phys.Solids Vol.33, 447-469.
Wang T.C, Shih C.F. and Needleman A. (1991). Crack growth by grain boundary cavitation in the transient & extensive creep regimes. Int.J.Frac. Vol.52, 159-189.
Witts N.P. (1992) PhD Thesis, University of Leicester.

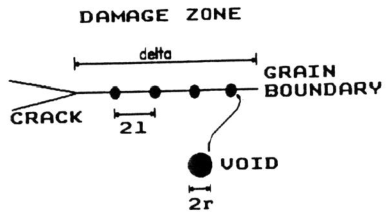

Fig.1: Crack tip with damage zone of size delta.

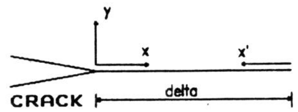

Fig.2: Growing crack tip coordinate systems.

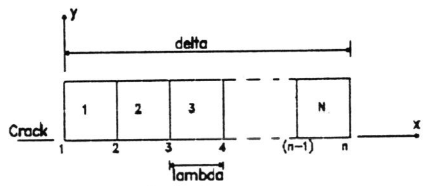

Fig.3: Finite elements and nodes adjacent to the damage zone.

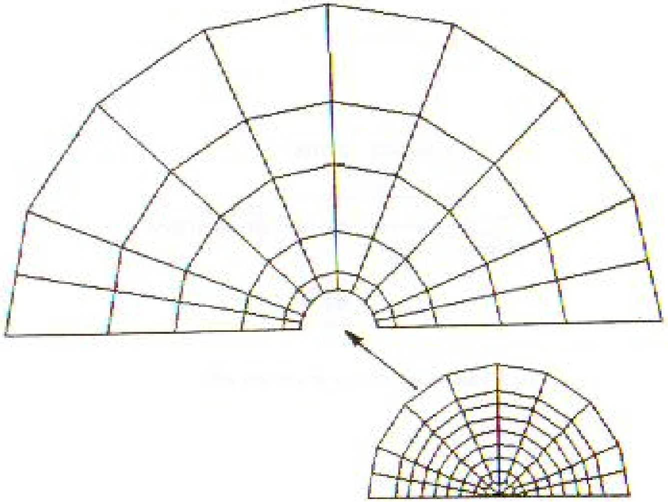

Fig.4: Finite element mesh, with crack-tip at semi-circle centre, and C* field tractions applied at semi-circular boundary.

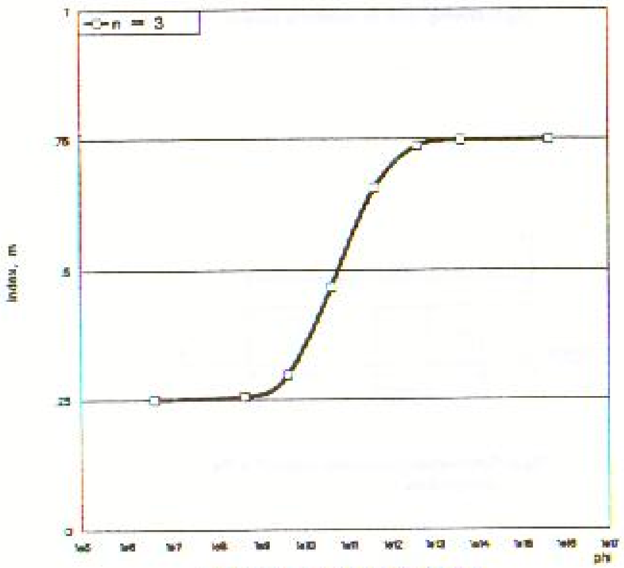

Fig.5: Variation of index m in eqn(14) from unconstrained to fully constrained growth, m = 1/(n+1) to n/(n+1).

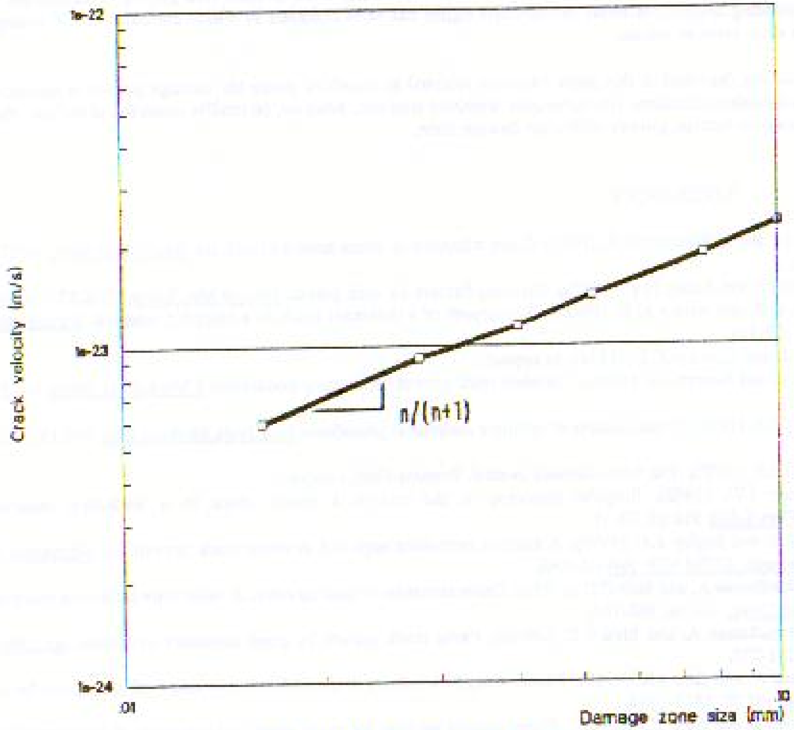

Fig.6: Variation of crack velocity with damage zone size for n = 3.0 & phi = 2.084E-12. Gradient = n/(n+1).