MIXED MODE DYNAMIC FRACTURE OF A CERAMIC

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Abstract : An original tensile split HOPKINSON bar previously designed for polymers testing was used for convenient biaxial fracture tests on a ceramic material. The equivalent fracture toughness including a crack tip finite radius correction was calculated using the maximum circumferential stress fracture criterion. The classical WEIBULL statistical analysis has been successfully extended to these mixed mode conditions and the results show that the probabilistic equivalent dynamic fracture toughness of this brittle material is independant of crack orientation.

INTRODUCTION

Many high technology industrial applications are interested in the use of ceramics owing to their high resistance to elevated temperature and corrosive environment, their friction properties and many other qualities. Unfortunately, the mechanical behaviour of this material limits applications in structural components. Together with many efforts in the developement of new materials investigations are conducted towards appropriate design methods.

These methods have to include particular features of the mechanical behaviour such as dispersed ultimate properties and specially fracture toughness. This scatter may be attributed to high concentration in structural defects associated to a brittle fracture process. Therefore statistical analysis of fracture data are widely used to classify these materials.

In this work, an experimental investigation of dynamic mixed mode fracture toughness is performed and values are presented in a WEIBULL analysis which has been modified in order to take crack angle into account

EXPERIMENTAL INVESTIGATION

Dynamic loading of ceramic samples is provided by the use of a tensile split HOPKINSON bar apparatus. The stress intensity factor rate achieved on this apparatus, depending on the tested material, is about 10^5 MPa$\sqrt{m}$. s^{-1} for ceramics. The original experimental set-up is shown on figure 1.

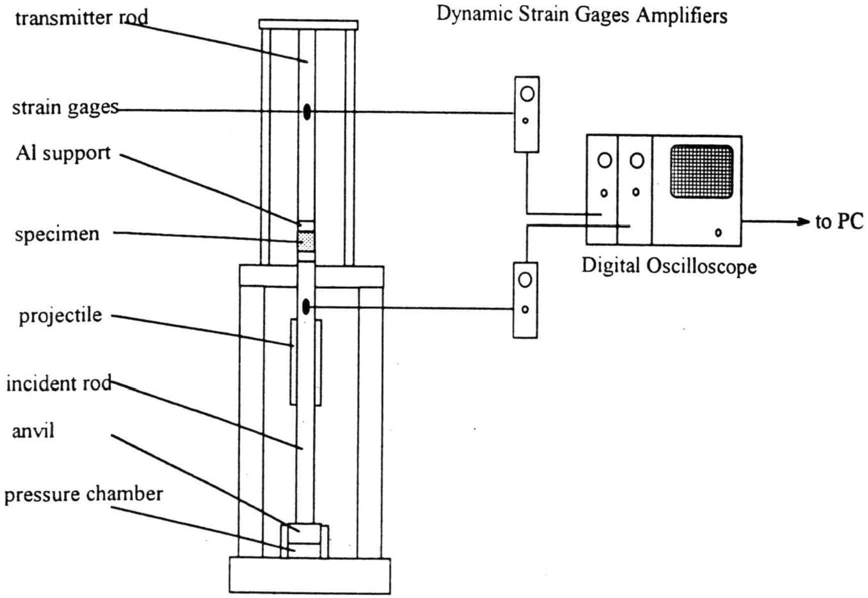

figure 1 : experimental apparatus

The tensile stress wave is produced by the impact of a tubular projectile on an anvil screwed at the extremity of the incident bar. Both the incident and transmitter bars are made out of a low modulus commercial aluminium alloy leading to measurable strains under rather small stresses by the way of semi-conductor strain gages. The detail for the calculation of the dynamic load deflection diagram recorded from this apparatus may be found in reference [1]. For the calculation of the dynamic stress intensity factor of fragile materials such as ceramics the required measure is the specimen stress, σ_s (t), given from the record of the transmitted strain signal, ε_t (t), versus time by :

$$\sigma_s (t) = \frac{A_b}{A_s} . Eb . \varepsilon_t (t) \qquad (1)$$

were A_b and A_s are respectively the bar and specimen cross sections, and E_b the modulus of the bars' material (measured by the way of an ultrasonic technique and confirmed by strain waves speed measurements).
The specimen dimensions and geometry are depicted on figure 2.

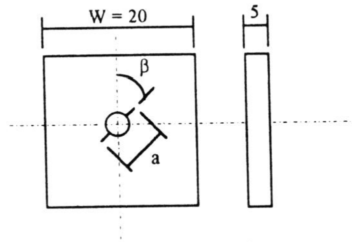

figure 2 : sample dimensions

The cracks were machined using a thread saw with a thread diameter of 0.1 mm. All samples were previously glued on aluminium supports using a two components epoxy glue (polymerization time 48 h) and the whole fixed on the bar using a cyanoacrylatic glue for the Al/Al interface. This experimental protocol reduced the manipulation time down to 5 min and it was verified that the additional interfaces did not disturb the measures.
The specimen thickness, B=5 mm, has been chosen in order to respect plain strain conditions at crack tip since :

$$B > 2.5 \left(\frac{K_{Ic}}{\sigma_y}\right)^2 \qquad (2)$$

with K_{Ic} about 4 MPa$\sqrt{m}$ and σ_y about 90 MPa. The different crack angles tested were 30°, 45°, 60° and 90°. A schematic of the recorded incident, reflected and transmitted waves is shown on figure 3.

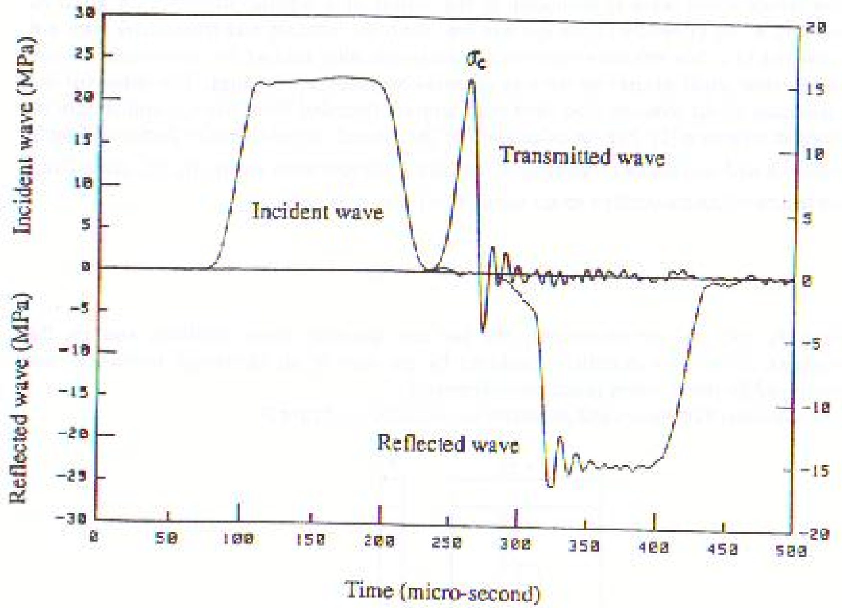

figure 3 : recorded stress waves

From this records the fracture stresses were extracted in order to compute the critical stress intensity factor for all conditions. Unfortunately, the fixation of the samples by the mean of glued interfaces could not achieve the load required for fracture at the lowest crack angle value (30°) and the reported tests only concern the highest angles. Table 1 remind fracture stresses for these different angles.

Angle β	Fracture stress [MPa]						
90°	27.2	32.1	30.2	31.5			
60°	42.5	38.8	36.3	37.7	35.5	35.5	36.7
45°	50.7	48.8	45.4	47.6	40.1		

Table 1 : Fracture stresses for different crack angles

STRESS INTENSITY FACTOR COMPUTATION

For the specimen geometry depicted in figure 2, the mode I and mode II stress intensity factors are given from simple trigonometric analysis as :

$$K_1 = \sigma_c \sqrt{\pi a}\ \sin^2\beta \ . \ Z(\frac{a}{W}) \qquad (3a)$$

$$K_2 = \sigma_c \sqrt{\pi a}\ \sin\beta \ . \ \cos\beta \ . \ Z(\frac{a}{W}) \qquad (3b)$$

where σ_c is the remove stress, a the crack length, W the specimen width and Z the classical finite width correction for central cracked tension plate given in refrence [2] by BROWN and SRAWLEY :

$$Z(\frac{a}{W}) = 1.77\ [\ 1 - 0.1\ (\frac{2a}{W}) + (\frac{2a}{W})^2\ ] \qquad (4)$$

The circumferential stress at the crack tip due to the mode I and mode II stress intensity factors with the correction for crack tip radius is :

$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r}}\ [\cos\frac{\theta}{2}\ (\ K_1 \cos^2\frac{\theta}{2} - \frac{3}{2} K_2 \sin\theta\ ) + \frac{\rho}{2\ r}\quad (K_1 \cos\frac{\theta}{2} + K_2 \sin\frac{\theta}{2})] \qquad (5)$$

where ρ is the crack tip radius and r,θ the polar coordinates. Taking ρ = r and following the maximum circumferential stress mixed mode criterion [3], the equivalent fracture toughness is calculated as:

$$K_{ceq} = \sigma_{\theta\theta max} \sqrt{2\pi r} = K_{1c} \cos\frac{\theta_o}{2} (\cos^2\frac{\theta_o}{2} + \frac{1}{2}) + \frac{1}{2} K_{2c} \sin\frac{\theta_o}{2} (1 - 6\cos^2\frac{\theta_o}{2}) \qquad (6)$$

where K_{1c} and K_{2c} are the critical values of expressions (3a) and (3b), and θ_o is the crack initiation angle which can be calculated from :

$$\tan\beta \sin\frac{\theta_o}{2}\ (6\cos^2\frac{\theta_o}{2} + 1) - \cos\frac{\theta_o}{2}\ (4 - 9\cos\theta_o) = 0 \qquad (7)$$

For the given values of crack orientation angle β=90°, 60° and 45°, the calculated initiation angle θ_o is equal to 0°, -34.02° and -48.8° respectively. For the experiments carried out in the present work, the value of this angle was determined by the mean of image analysis and correspond quite well to the theoritical predictions (from 40 to 45° for β=45°). The table II reminds the experimental results in terms of the fracture stresses, and the computed fracture toughnesses.

$\beta = 90°$

σ_c [MPa]	K_{1c} MPa√m	K_{2c} MPa√m	K_{ceq} MPa√m
27.2	3.4	0	3.4
32.1	4.0	0	4.0
30.2	3.8	0	3.8
31.5	3.9	0	3.9

$\beta = 60°$

σ_c [MPa]	K_{1c} MPa√m	K_{2c} MPa√m	K_{ceq} MPa√m
42.5	3.9	2.2	5.2
38.8	3.5	2.0	4.8
36.3	3.3	1.9	4.4
37.7	3.4	2.0	4.7
35.5	3.2	1.9	4.4
35.5	3.2	1.9	4.4
36.7	3.3	1.9	4.5

$\beta = 45°$

σ_c [MPa]	K_{1c} MPa√m	K_{2c} MPa√m	K_{ceq} MPa√m
50.7	3.0	3.0	5.3
48.8	2.9	2.9	5.1
45.4	2.7	2.7	4.8
47.6	2.8	2.8	5.0
40.1	2.4	2.4	4.2

Table II : Equivalent stress intensity factors for different crack angles

The experimental values are in the range from 4.8 to 6.7 MPa√m. Such a scatter (about 30%) in the fracture experiments was expected because of a very brittle fracture behaviour associated to a high concentration in structural defects in ceramics. It is then convenient to present the results from a statistical point of view, and refering to the previous works of WALLIN [4] a WEIBULL analysis was chosen.

PROBABILISTIC FRACTURE MECHANICS

Introduced by WEIBULL [5] for fracture stress of brittle materials, the analysis is started from a set of two main assumptions :

i) the material is statistically homogeneous and isotropic, i.e. the probability to find a flaw of a given "severity" is the same everywhere in the volume.

ii) initiation of a single flaw leads to the complete failure of the volume.

Then the volume may be seen as a monodimensional chain in which the fracture probability for a particular link out of a total number of N links to fail under a given stress level σ is supposed to be $f(\sigma)$. Thus for the all volume of N elements the fracture probability is :

$$F_N(\sigma) = 1 - [1 - f(\sigma)]^N \quad (8)$$

assuming N is large, an approximate solution may be writen as :

$$F_N(\sigma) = 1 - \exp\{-Nf(\sigma)\} \quad (9)$$

From the work WALLIN the number of links, N, is proportional to the fracture process volume, V, and the failure probability takes the form :

$$P_f = 1 - \exp\{V\,\Phi(\sigma)\} \quad (10)$$

In the original work of WEIBULL the $\Phi(\sigma)$ function is taken as :

$$\Phi(\sigma) = 0 \qquad \text{for } \sigma < \sigma_u$$

$$\Phi(\sigma) = \left\{\frac{\sigma - \sigma_u}{\sigma_o - \sigma_u}\right\}^{m_1} \qquad \text{for } \sigma > \sigma_u \quad (11)$$

where σ_u is a threshold stress under which no fracture is observed, σ_o a normalisation factor with no physical significance and m_1 a shape parameter of the distribution related to the scatter and commonly named the WEIBULL modulus.

It is obvious that such an expression is unable to take into account flaws in the material or alternatively that the enventual flaws are assumed to be of the same "severity". Avoiding this problem WALLIN improved the relation by subtituing the fracture toughness to the ultimate strength to give :

$$P_f = 1 - \exp\left\{\frac{K_{1c} - K_u}{K_o - K_u}\right\}^{m_2} \tag{12}$$

where the volume is no more apparent since it can be shown [4] :

$$V = Cte \, . \, K_I^4 \tag{13}$$

The exponent m_2 is different from m_1 since it represents the scatter in K_{1c} datas and has a theoritical value equal to 4 when the number of tests is large enough (>20) which is not the case in the present study.

MIXED MODE PROBABILISTIC FRACTURE APPROACH

For the mixed mode tests we have performed, an attempt was made to modify relation (11) in order to take into account the flaw orientation. The modified relation is :

$$P_f = 1 - \exp\left\{\frac{K_{ceq} - K_u}{K_o - K_u}\right\}^{m_2} \tag{14}$$

where K_{ceq} is calculated using relation (6). In fact, for simplicity, the K_u parameter was set to zero, and the two remaining parameters of the WEIBULL distribution are deduced from the plot of :

$$\ln\ln\left(\frac{1}{1 - P_f}\right) \text{ versus } \ln(K_{ceq})$$

which from relation (14) is expected to be a straigth line which slope is m_2 and intercept with ordinates is the K_o factor. This plot is shown on figure 4

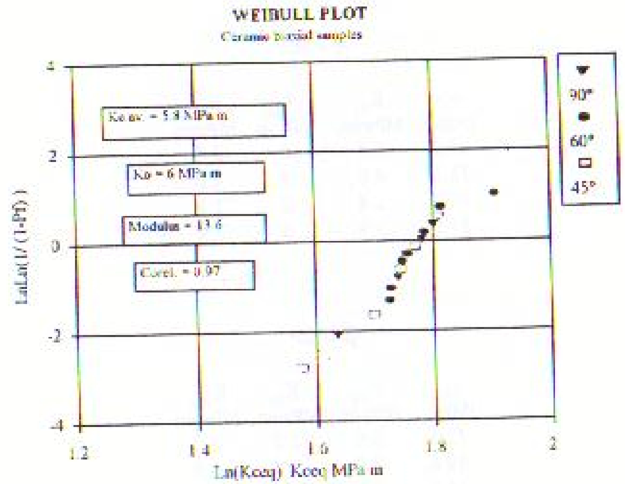

figure 4 : determination of WEIBULL parameters

The average value of K_{ceq} = 6 MPa$\sqrt{m}$ is coherent with values found in literature for this kind of materials [6] but the modulus value of 13.6 is quite high probably because of the insufficient number of experiments. However it should be pointed out that regardless of the crack angles, the results are homogeneously distributed along the line which indicates that the probabilist equivalent stress intensity factor is indeed independant of crack orientation.

CONCLUSIONS

A mixed mode dynamic fracture toughness investigation on a ceramic material has been performed and 16 specimens were successfully tested. The dynamic loading conditions have been achieved on a tensile split HOPKINSON bar apparatus which was previously designed for polymers testing. An expected large scatter, inherent to ceramic ultimate properties, is observed and is treated by the way of a WEIBULL statistical analysis of fracture. The previous approach of WALLIN has been improved to become crack angle independant.

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