MICROMECHANICAL MODEL OF CERAMIC MATRIX COMPOSITE REINFORCEMENT BASED ON DISCRETE FIBER DISTRIBUTION

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ABSTRACT

This paper addresses the fracture resistance mechanism in fiber reinforced ceramics, and focuses attention on the specific effects associated with the nonlinear nature of the fiber pullout mechanism. The model is based on a consideration of the representative boundary value problem typical for the bridging process. The theoretical solution includes an accurate account of the nonlinear fiber matrix friction. An explicit incorporation into the analysis of a discrete fiber distribution and formulation of an exact solution to the corresponding problem are distinct features of this model. The developed approach allows consideration of several types of nonlinear fiber pullout - force dependence. The distinct features of the nonlinear process demonstrate that, contrary to the linear case, the universal fracture resistance curves cannot be developed in cases with significant nonlinear contribution in the fiber friction law. The resulting resistance curves strongly depend on the absolute values of the matrix fracture toughness. On the other hand, these distinct patterns may be used for identification of the particular friction law and determination of the friction parameters.

KEYWORDS

Ceramics, composites, fiber reinforcement, micromechanics, fracture, fracture resistance, bridging.

INTRODUCTION

A significant technological effort is devoted to development of ceramic-based compositions, such that the high temperature performance qualities of ceramics will be preserved, and the undesirable brittleness will be reduced. In relation to this, several models have been introduced in the literature. A literature survey, along with a complete description of the mechanical processes taking place and the main efforts in modeling these processes, are given by Aveston, Cooper and Kelly (1971), Rose (1987). The common feature in the development of the models (Budiansky, Hutchinson and Evans (1986), Budiansky and Amazigo (1988 and, 1989), Nemat-Nasser and Hori (1987)) is a substitution of the action of discrete fibers by a distribution of forces which, supposedly, produces a similar toughening effect. Usually, the analysis of the model is based on formulation and numerical solution of a singular integral equation which reflects the force-displacement relation in the process zone. The analytical approach presented in this study departs from this well-established fracture mechanics scheme. The analysis outlined below is based on the consideration of a two dimensional model which captures the main features of the fiber reinforcement process. The subject of the analysis is microprocesses occurring within the process zone formed ahead of a growing crack, and, therefore, the small scale framework is used. The two dimensional formulation is chosen to represent a plane perpendicular to the crack front which passes through the array of reinforcing fibers. The load is assumed to be aligned with the fibers, and the analysis is concentrated on a crack growing in transverse direction to the fibers. The crack growth resistance mechanism in this material is based on formation of the bridging zone, the process zone in this case, where the cracked matrix is held by the remaining intact fibers behind the crack front. Thus, the high intensity stress field typical for the vicinity of the crack tip is distributed among these bridging fibers and the leading crack tip arrested by the matrix. The energy release rate associated with the crack growth under the described conditions consists of two parts: the energy absorption rate due to the friction on the fiber-matrix interface associated with the crack advance, and the energy release rate due to the crack advance within the matrix.

The fracture resistance R of the composite may be represented as an applied load required to maintain the crack growth as a function of the crack advancement from the instant prior to formation of the bridging zone. Thus, assuming that for the crack advancement the leading stress intensity factor acting on the matrix K_L has to be maintained at the critical value for the matrix K_{IC}, the material resistance can be represented as

$$R = K_{IC}\frac{K_\infty}{K_L}. \tag{1}$$

The ratio K_∞/K_L (dimensionless fracture resistance) characterizes here the fiber reinforcement effect, that is, a relative reduction of the local, leading, stress intensity factor as compared with the stress intensity factor on the macroscale.

The model described here is based on the following assumptions: The elastic properties of the fibers are assumed to be very similar to the properties of the matrix with no significant difference in values of elastic constants. The difference between the strain magnitude in the fiber and in the matrix is insignificant at a finite distance from the crack surface, $|y| > 0$. The local debonding at the fiber-matrix interface does not influence to

a significant degree the resulting stress field at locations remote from the interface, and, therefore, for purposes of this analysis, no debonding is allowed. *The bridging zone is represented as an array of microcracks between the fibers with misfits on the ligaments (fibers) equal to the amount of the fiber pull-out.* The profile of the matrix separation along the fibers has to retain axial symmetry, and, thus, the misfit at each ligament corresponding to a fiber has to have a constant value along that ligament. The magnitude of the misfit on each ligament is different and is controlled by the fiber pullout - force relationship.

ANALYSIS

This analysis is aimed to evaluate the principal fracture mechanics parameters associated with the processes taking place during the bridging zone development ahead of the macrocrack. The crack size is assumed to be significantly larger than the bridging zone size, and, therefore, the small scale approach may be used. In the framework of this analysis, the applied load is represented through the remote stress intensity factor which controls the outer stress field of the process zone. The fiber thickness, here is taken as a, the period of fiber spacing p, and in case of the first microcrack formation after the first fiber, the length of the microcrack is b-a. The methodology of the analysis was developed by Rubinstein (1985, 1987) and Rubinstein and Xu (1990). The basic relationships of the linear plane theory of elasticity in terms of the complex potentials ϕ and ψ are employed here using standard notations, Muskhelishvili (1975). Limiting our attention to the Mode I loading, so the direction of applied tension is parallel to the direction of fibers, the symmetry condition on $y=0$ can be stated as

$$\sigma_{12}(z=x) = 0 = \mathrm{Im}(\bar{z}\phi''(z) + \psi''(z)). \tag{2}$$

Functions ϕ and ψ are analytic in the plane with cuts along $y=0$, and, therefore, they may be considered as analytic in the upper half plane. Using condition (2) and applying the principle of analytical continuation, one obtains the relationship between the analytic potentials, which is true up to a real constant,

$$\psi'(z) = -z\phi''(z). \tag{3}$$

The constant is dropped since both sides of (3) have to vanish as $z\to\infty$. With relation (3), the expressions for the normal stress and displacement components along $z=x$ become

$$\sigma_{22} = 2\mathrm{Re}\phi'(x), \qquad u_2 = \frac{\kappa+1}{2\mu}\,\mathrm{Im}\phi(x). \tag{4}$$

Thus, only one analytic function ϕ has to be determined, and the boundary conditions can be written in terms of this function. The condition at infinity states that function ϕ' has to match the applied stress field, which should be given in terms of a remote stress intensity factor K_∞ (we consider Mode I loading only),

$$\phi'(z) \longrightarrow \frac{K_\infty}{2\sqrt{2\pi z}} \quad \text{as} \quad z \rightarrow \infty . \tag{5}$$

The boundary value problem for the unique determination of the physically suitable function ϕ' will be completed with the addition of statements of the traction free surface on the main crack and microcracks, and statements specifying constant displacements on the ligaments corresponding to the bridging fibers, that is

$$\begin{aligned} &\mathrm{Re}\,\phi'(x)=0 \quad \text{on} \quad x<0 \quad \text{and} \quad a+pk < x < p(k+1) \\ &\mathrm{Im}\,\phi'(x)=0 \quad \text{on} \quad pk < x < a+pk \quad \text{and} \quad pN < x , \\ &k=0,1,2,\ldots,N-1. \end{aligned} \tag{6}$$

The physically suitable analytical function which satisfies all conditions stated above is chosen by Rubinstein and Xu (1990) as

$$\phi'(z) = \frac{K_\infty}{2\sqrt{2\pi z}} \frac{\prod_{k=0}^{N-1}(z-d_k)}{\prod_{k=0}^{N-1}(z-a-pk)^{\frac{1}{2}}(z-p(k+1))^{\frac{1}{2}}} . \tag{7}$$

The branch of the square root function is chosen with the condition that for $z=x>pN$ the result of the square root is real and positive. N real constants d_k have to be determined from the conditions on the fibers. Assuming that the constants d_k are found, the stress intensity factors are determined by taking the appropriate limits; for the leading microcrack tips the results are

$$K_{pj} = K_\infty \frac{\prod_{k=0}^{N-1}(pj-d_k)}{\sqrt{p^N j(p-a)} \prod_{k=0,k\neq j+1}^{N-1}(j-k+1)^{\frac{1}{2}}(p(j-k)-a)^{\frac{1}{2}}} . \tag{8}$$

$j=1,2,\ldots,N$

Case $j=N$ determines the stress intensity factor at the leading end of the bridging zone, the value acting on the uncracked matrix. This value determines the resistance of the material to the bridging zone extension. A complete set of expressions for all crack tips is given by Rubinstein and Xu (1990).

The bridging zone initiation may be analyzed by taking $N=1$; the final expressions for the stress intensity factors acting at the main crack K_0, at the leading tip of the microcrack crack K_b, and at the trailing tip of the microcrack K_a are

$$K_0 = K_\infty \frac{d}{\sqrt{ab}} , \quad K_a = K_\infty \frac{d-a}{\sqrt{a(b-a)}} , \quad K_b = K_\infty \frac{b-d}{\sqrt{b(b-a)}} . \tag{9}$$

The constants d_k correspond to locations of maximal crack opening of each microcrack.

As mentioned above, the necessary set of equations for determination of these constants should be given by the friction or fiber pullout - force relationship on the fibers. The pullout displacement B_k on a fiber k consists of the cumulative displacements on fibers ahead of fiber k plus a misfit at the microcrack immediately in front of it ΔB_k. The force F_k acting on a fiber k is determined from the given stress distribution (7) with (4). The friction law relating the fiber pullout to the acting force is not completely understood. The general form of this relationship may be written as

$$\lambda H(F_k - f) = B_k^\alpha , \quad k=0,1,2,\ldots,N-1. \tag{10}$$

Here f is a threshold force, and α is a parameter determining the power of this relationship and λ is a constant. Most commonly these parameters are assumed as $f=0$ and $\alpha = 1$ (linear relationship), to simplify the analysis. Two cases will be discussed below; linear relationship and parabolic relationship, $f=0$, $\alpha = \frac{1}{2}$.

The energy absorbed by the fiber pull-out process due to the bridging zone extension, G_f, can be evaluated by employing Rice J-integral. Thus, in the case of one fiber link

$$K_0^2 - K_a^2 + K_b^2 = K_\infty^2 = G_f + K_b^2 \tag{11}$$

or

$$G_f = K_0^2 - K_a^2 = K_\infty^2 \frac{2bd - d^2 - ba}{b(b-a)} . \tag{12}$$

The energy absorbed by the fiber pull-out process will produce a positive contribution if

$$b > d > b - \sqrt{b(b-a)} . \tag{13}$$

The negative contribution is physically possible because of the restriction on fiber bending which is implicitly assumed in the formulation. This simple analysis of the energy relationships cannot be used for the general case inasmuch as d is not an independent parameter; it depends on a friction law and a/b ratio.

LINEAR FORCE - PULL-OUT DISPLACEMENT RELATIONSHIP.

The linear case solution was obtained by Rubinstein and Xu (1990) in closed form for the case $N=1$ and numerically for an increasing bridging zone up to $N=18$. The nondimensional friction coefficient is introduced as

$$\Lambda = 2\frac{\lambda\mu}{\kappa+1} . \tag{14}$$

The constant d, for the case N = 1, is

$$\frac{d}{b} = \frac{E\left(1-\frac{a}{b}\right) + \Lambda\left[K\left(\frac{a}{b}\right) - E\left(\frac{a}{b}\right)\right]}{K\left(1-\frac{a}{b}\right) + \Lambda \cdot K\left(\frac{a}{b}\right)} \qquad (15)$$

$K(m)$ and $E(m)$ are complete elliptic integrals of the first and second kind. The case $N=1$ characterizes the bridging zone initiation process. During the development of the microcrack the fiber not only restrains the separation of the matrix but also restrains the shape of the matrix at the fiber - matrix interface; namely, it restrains the rotation of this ligament. The three dimensional surface corresponding to these data demonstrates that the relatively small microcrack, with respect to a fiber thickness, is unlikely to exist for intermediate, non-zero, values of Λ. The small microcrack will represent an unstable situation. The fiber spacing in the matrix becomes an important factor for optimal reinforcement. This spacing has to accommodate this unstable microcrack growth before it reaches the next fiber; otherwise, this unstable matrix failure will extend through the array of fibers. There exists an optimal combination of the fiber -spacing ratio and the fiber pull-out parameter Λ when values of the leading stress intensity factor are minimal, and thus the material resistance is maximal. The equation (11) suggests that the optimum reinforcement will take place when $K_a = 0$. A special case $K_o = K_b = 0.7071K_\infty$ takes place at $a/b=0.5$, $\Lambda = 1.0$, $K_a = 0$.

The natural expectation is that with elongation of the bridging zone, the load on the fiber separating the bridging zone from the main crack (first fiber) will increase. The first fiber is experiencing the maximal load in the array, but this load, in most cases, is reached after development of a few microcracks. Thus, long bridging zones may develop, and a primary limitation on this length is the leading stress intensity factor, which acts on the uncracked matrix, and the length of the fiber available to be pulled out.

The resistance curves obtained on the basis of the described analysis show a significant spread of resistance values over the given values of Λ and dependence on the fiber spacing aspect ratio. The high values of Λ allow significant matrix separation in the bridging zone, and that contributes to the load redistribution, which causes higher values of the leading stress intensity factor; i. e., lower fracture resistance. These data support the conclusion from the exact solution describing the bridging zone initiation that *there is an optimal combination of the fiber spacing ratio and the fiber pull-out parameter at which the maximal fracture resistance may be achieved.*

NONLINEAR FORCE - PULL-OUT DISPLACEMENT RELATIONSHIP.

The nonlinear case brings several interesting aspects into the process which may require redesigning the experimental procedure for composite evaluations. Therefore, to obtain a better understanding of the nonlinear phenomenon, only the one fiber link case is analyzed here. The method of solution of the problem can be easily applied to any rational power of the force - pull-out displacement. After setting $\alpha=0.5$ and $f=0$ in

equation (10), taking the square of both sides, and substituting the following expressions for the force and displacement, which are obtained by integration of the stress function (7), the quadratic equation for the ratio d/b is obtained. An important aspect of the obtained result has to be pointed out. In addition to the dimensionless friction Λ given by (14), which includes the interface property λ and material constants μ and ν, the nonlinear case includes the loading parameter

$$K^* = \frac{K_\infty(\kappa+1)}{\mu}\sqrt{\frac{a}{2\pi}}. \qquad (16)$$

Thus, contrary to the linear case the *composite resistance curve pattern depends on the matrix toughness in addition to the fiber spacing aspect ratio and fiber-matrix interface friction.* The dimensionless resistance parameter based on the ratio K_∞/K_L cannot be used for the nonlinear case. It is clear from the derivation, that this is a general property for any nonlinear case. On the other hand, one can use the experimental resistance curve obtained for this simple geometry as data for the inverse problem, and, thus, the friction law can be accurately determined. The final equation for d/b is

$$A_1(d/b)^2 + A_2(d/b) + A_3 = 0 \qquad (17)$$

$$A_1 = K^2(a/b), \quad A_2 = 2K(a/b) + K(1-a/b)/\Lambda^2K^*(a/b)^{\frac{1}{2}}$$

$$A_3 = -E(1-a/b)/\Lambda^2K^*(a/b)^{\frac{1}{2}} + [E(a/b) - K(a/b)]^2.$$

The equation (17) has two real roots one of which corresponds to the position of the maximal microcrack opening. The loading curves associated with development of the microcrack were obtained for different Λ and dimensionless matrix toughness $K_b(\kappa+1)(a/2\pi)^{½}/\mu$. The interesting feature of this nonlinear case is that the composite formed with the matrix with lower toughness has a higher resulting toughness enhancement. The nonlinearity of the force-displacement relation (10) is a significant factor in terms of the composite toughening. The fiber spacing is more critical for the optimal toughness than in the linear case. The region of matrix weakening due to the fiber inflexibility is observed here as well as in the linear case; however, the region and intensity of this effect are different. The region and intensity of unstable matrix cracking are increased significantly as compared with the linear case.

CONCLUSIONS

The micromechanical toughening model for the fiber reinforcement of brittle matrix was presented. The analysis of the model is based on discrete fiber distribution and addresses such important aspects as fiber spacing ratio and fiber flexibility.

Two types of the fiber pull-out displacement - force laws were considered. Exact closed form solutions are given for the bridging zone initiation in the cases of the linear and square root force - displacement relationships.

In the case of the nonlinear force-displacement relationship, the experiments conducted on a laboratory composite cannot be directly transferred to other types of composites. The patterns of the resistance curves strongly depend on the friction parameter and the matrix toughness. To be reliable, these experiments must be carefully designed.

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REFERENCES

Aveston, J., Cooper, G.A. and Kelly, A., 1971, *Conference on The Properties of Fiber Composites, National Physical Laboratory,* Guildford, Surrey: ICP Science and Technology Press. pp. 15-26

Budiansky, B., Hutchinson, J.W. and Evans, A.G., 1986, *J. Mech. Phys. Solids,* 34, 167-189.

Budiansky, B. and Amazigo, J. C., 1988, "Small-Scale Bridging and the Fracture Toughness of Particulate-Reinforced Ceramics," *J.Mech. Phys. Solids,* 36, 167-187.

Budiansky, B. and Amazigo, J. C., 1989, "Toughening by Aligned, Frictionally Constrained Fibers," *J.Mech. Phys. Solids,* 37, 93-109.

Nemat-Nasser, S. and Hori, M., 1987, "Toughening by Partial or Full Bridging of Cracks in Ceramics and Fiber Reinforced Composites," *Mechanics of Materials,* 6, 245-269.

Muskhelishvili, N.I., 1963, *Some Basic Problems of the Theory of Elasticity,* Noordhoff, Groningen, Holland.

Muskhelishvili, N.I., 1972, *Singular Integral Equations,* Noordhoff, Groningen, Holland.

Rose, L.R.F., 1987, "Crack reinforcement by distributed springs," *J. Mech. Phys. Solids,* 35, 383-405.

Rubinstein, A.A., 1985, "Macrocrack Interaction with Semi-infinite Microcrack Array," *International Journal of Fracture,* 27, 113-119.

Rubinstein, A.A., 1987, "Semi-Infinite Array of Cracks in a Uniform Stress Field," *Engineering Fracture Mechanics,* 26, 15-21

Rubinstein, A.A. and Xu, K., 1990, "Micromechanical Model of Crack Growth in Fiber Reinforced Ceramics." *Journal of the Mechanics and Physics of Solids.* Vol. 40, No. 1, pp. 105-125.