INFLUENCE OF PHYSICAL-MECHANICAL CHARACTERISTICS OF ROCK THICKNESS, BORE-SOLUTION DENSITY AND TECTONIC STRENGTH VALUE ON STRESSED STATE AROUND DEEP BOREHOLE

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ABSTRACT

The influence of tectonic strengths and disconnected affect of a bore-solution on the elastic-plastic stressed state and on the boundary between elastic region and zone of nonelastic deformations around deep borehole was studied in this paper. The results of calculation are presented in form of table.

KEYWORDS

Deep borehole, tectonic strenghts, bore-solution, stressed state, elastic region, nonelastic deformation zone (NDZ), boundary between elastic region and NDZ (contour)

For investigation of the tectonic strength influence on streesed state around deep borehole the solution of two-dimensional elastic-plastic problem made by Galin L.A. can be used as it was carried out in work (Alimzhanov A.M.,1993). The Galin's solution was executed for the occasion of uniformity of material properties in nonelastic deformation zone (NDZ). However when drilling deep borehole in a sedimentary thickness the NDZ arises near its contour which included the places with droped mechanical characteristics.
Thus, out of the zone with droped mechanical characteristics which directly joined to borehole contour it is situated the zone of nonelastic deformations with uniform physical-mechanical properties. This zone in its turn borders on elastic region. All these zones consistently surround each other.

Let the deep vertical borehole with the radius R_0 was bored in a rock thickness. It's chosen the borehole section perpendicular to borehole axis on the depth h from daily surface. This section is presented as a weghtless infinite plane with the opening of radius R_0 compressed along axises X and Y On an plane infinity the following strengths affect:

$$\left.\begin{aligned}\sigma_x^{\infty} &= \lambda T\\ \sigma_y^{\infty} &= T\end{aligned}\right\},$$

where T is tectonic strength, $\lambda < 1$. (1)

It's considered the two-dimensional elastic-plastic problem with use of the polar coordinate system $\rho = r/R_0$, θ, where r is the flowing coordinate, ρ is the nondimensional flowing coordinate, θ is the polar angle.

The hydrostatic pressure of a bore-solution is applied on a borehole contour:

$$\sigma[\rho] = \gamma_B h, \quad \text{when } \rho = 1,$$

where γ_B is the bore-solution density. (2)

It's proposed the rock near borehole turns into nonelastic deformed state owing to an influence of tectonic strengths and a disconnected affect of bore-solution. The Mohr-Coulomb's plasticity condition is used as a condition of transition of rock into plastic state. The investigation of stressed-deformed state around deep borehole with the Saint-Venant's plasticity condition was carried out in paper (Alimzhanov A.M., 1993).

As a result of the generation of micro and macrocracks and the disconnected affect of a bore-solution on borehole walls the inner zone I arises around borehole contour within nonelastic deformation zone (NDZ) II. This inner zone is the disconnected zone characterized by a heterogeneity of rock mechanical properties.

Character of this heterogeneity can be described by the following function (Alimzhanov M.T., 1992):

$$K(\rho) = \begin{cases} K + \dfrac{K_0 - K}{\beta^n - 1}\left[\left(\dfrac{\rho_0}{\rho}\right)^n - 1\right], & \text{when } 1 \le \rho \le \rho_0 \\ K = const, & \text{when } \rho_0 < \rho \le \rho_L \end{cases} \quad (3)$$

where ρ_0 is the radius of disconnection zone I, ρ_L is the radius of homogeneous NDZ II, K is the value of cohesion coefficient of safe indestructed rock massif, K_0 is the minimum value of cohesion coefficient on a borehole contour when ρ_0 reaches the maximum value β, n is the approximation parameter.

Out of the NDZ (when $\rho > \rho_L$) there is an elastic region in which the components of stressed-deformed state submit to Hoek's Law relations. The material of an elastic region is considered as an homogeneous and isotropic.

At first the stress components in I and II zones, must be defined for decission of problem. The Mohr-Coulomb's plasticity condition is noted as follows:

$$(\sigma_\theta - \sigma_\rho)^2 + 4\tau_{\rho\theta}^2 = \sin^2\varphi\left[\sigma_\rho + \sigma_\theta + 2K(\rho)\,ctg\varphi\right]^2, \quad \text{when } 1 \le \rho \le \rho_0, \quad (4)$$

$$(\sigma_\theta - \sigma_\rho)^2 + 4\tau_{\rho\theta}^2 = \sin^2\varphi\left[\sigma_\rho + \sigma_\theta + 2K\,ctg\varphi\right]^2, \quad \text{when } \rho_0 < \rho. \quad (5)$$

Here $\sigma_\rho, \sigma_\theta, \tau_{\rho\theta}$ are stress components, φ is the inner friction angle, $K(\rho)$ is the variable cohesion coefficient described by the formula (3).

The stresses $\sigma_\rho, \sigma_\theta, \tau_{\rho\theta}$ when $1 \le \rho < \infty$ must comply to equilibrium equations which in polar coordinate system are noted in following form:

$$\begin{aligned} &\frac{\partial \sigma_\rho}{\partial \rho} + \frac{1}{\rho}\frac{\partial \tau_{\rho\theta}}{\partial \theta} + \frac{\sigma_\rho - \sigma_\theta}{\rho} = 0, \\ &\frac{\partial \tau_{\rho\theta}}{\partial \rho} + \frac{1}{\rho}\frac{\partial \sigma_\theta}{\partial \theta} + 2\frac{\tau_{\rho\theta}}{\rho} = 0. \end{aligned} \quad (6)$$

The equation of deformation compatibility can be resulted to the such form:

$$\left(\frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial \theta^2}\right)\left(\frac{\partial^2 F}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial F}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 F}{\partial \theta^2}\right) = 0, \quad (7)$$

where F is till unknown stress function. If the function F was somehow defined, then the stresses $\sigma_\rho, \sigma_\theta, \tau_{\rho\theta}$ is found through simple differentiation:

$$\sigma_\rho = \frac{1}{\rho}\frac{\partial F}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 F}{\partial \theta^2}; \quad \sigma_\theta = \frac{\partial^2 F}{\partial \rho^2}; \quad \tau_{\rho\theta} = -\frac{\partial}{\partial \rho}\left(\frac{1}{\rho}\frac{\partial F}{\partial \rho}\right). \quad (8)$$

The formulae (8) may be used for determination the stress function. Owing to symmetry of borehole naturally to suppose the function F doesn't depend from polar angle θ in NDZ. Consequently the derivative from θ is equal nought, that is

$$\frac{\partial F}{\partial \theta} = \frac{\partial^2 F}{\partial \theta^2} = 0. \tag{9}$$

According to the formula (9) the Mohr-Coulomb's plasticity condition can be presented as follows:

$$\sigma_\theta - \alpha_1 \sigma_\rho = \beta_1 K(\rho), \tag{10}$$

where $\alpha_1 = \frac{1+\sin\varphi}{1-\sin\varphi}$, $\beta_1 = \frac{2\cos\varphi}{1-\sin\varphi}$.

Using the relations (8) in the formula (10) the initial equation is received for definition stress functions in I and II zones:

$$\frac{d^2F}{d\rho^2} - \alpha_1 \frac{1}{\rho}\frac{dF}{d\rho} - \beta_1 K(\rho) = 0. \tag{11}$$

Deciding this equation the expressions of stress function are defined for I and II zones accordingly:

$$F_1 = C_1 \rho^{1+\alpha_1} + \frac{\beta_1(K-D)}{2(1-\alpha_1)}\rho^2 + \frac{\beta_1 D \rho_o^n}{(2-n)(1-\alpha_1-n)}\rho^{2-n}, \tag{12}$$

$$F_2 = C_2 \rho^{1+\alpha_1} + \frac{\beta_1 K}{2(1-\alpha_1)}\rho^2, \tag{13}$$

where $D = \frac{K_o - K}{\beta^n - 1}$.

Using the expressions (8) and (12) the formulae for determination of stress components are received in I zone:

$$\sigma^I_{[\rho]} = C_1(1+\alpha_1)\rho^{\alpha_1-1} + \frac{\beta_1(K-D)}{1-\alpha_1} + \frac{\beta_1 D}{1-\alpha_1-n}\left(\frac{\rho_o}{\rho}\right)^n,$$

$$\sigma^I_{[\theta]} = C_1\alpha_1(1+\alpha_1)\rho^{\alpha_1-1} + \frac{\beta_1(K-D)}{1-\alpha_1} + \frac{\beta_1 D}{1-\alpha_1-n}\left(\frac{\rho_o}{\rho}\right)^n. \tag{14}$$

The constant C_1 is determined from the boundary condition (2):

$$C_1 = \frac{\gamma_{B}h}{1+\alpha_1} - \frac{\beta_1(K-D)}{(1-\alpha_1)(1+\alpha_1)} - \frac{\beta_1 D}{(1-\alpha_1-n)(1+\alpha_1)}\rho_o^n. \tag{15}$$

Using the expressions (8) and (13) the formulae for determination of stress components are received in II zone:

$$\sigma^{II}_{[\rho]} = C_2(1+\alpha_1)\rho^{\alpha_1-1} + \frac{\beta_1 K}{1-\alpha_1},$$

$$\sigma^{II}_{[\theta]} = C_2\alpha_1(1+\alpha_1)\rho^{\alpha_1-1} + \frac{\beta_1 K}{1-\alpha_1}. \tag{16}$$

The constant C_2 is found from stress joined condition on the boundary ρ_o between I and II zones, that is from $\sigma^I_{[\rho]} = \sigma^{II}_{[\rho]}$, when $\rho = \rho_o$:

$$C_2 = -\frac{\beta_1 D}{(1+\alpha_1)(1-\alpha_1)}\rho_o^{1-\alpha_1} + C_1. \tag{17}$$

So the stress components for both the disconnection zone and uniform NDZ are defined.

Now the stress state in an elastic region will be considered. For arbitrary angle of inner friction φ the approximate solution exists only (Alimzhanov M.T., 1982).
For further solution of problem this approximate method is used taking into account the heterogeneity of mechanical properties in NDZ.
In an elastic region the stress function F must answer to the compatibility equation (7) and boundary condition (1). All the necessary conditions will be satisfied if to set the stress components in following form:

$$\begin{aligned} \sigma_{(\rho)} &= T\left[\lambda_2(1-a\rho^{-2}) - \lambda_1(1-2b\rho^{-2}+c\rho^{-4})\cos 2\theta\right], \\ \sigma_{(\theta)} &= T\left[\lambda_2(1+a\rho^{-2}) + \lambda_1(1+c\rho^{-4})\cos 2\theta\right], \\ \tau_{(\rho\theta)} &= \lambda_1 T(1+b\rho^{-2}-c\rho^{-4})\sin 2\theta, \end{aligned} \tag{18}$$

where $\lambda_1 = \frac{1}{2}(1-\lambda)$, $\lambda_2 = \frac{1}{2}(1+\lambda)$, a, b, c the arbitrary constants are subjected to definite from conditions of contour L, which divided NDZ and elastic region. The equation of contour L can be presented as follows:

$$\rho_L = \rho_1 + \lambda_1 \rho_2(\theta). \tag{19}$$

Here λ_1 is took as a small parameter. So long as λ_1 is a small value, then in expansion ρ_L on degrees λ_1 it can neglect the terms with λ_1^2 and with more. Hence the following necessary expressions are for the further solution:

$$\begin{aligned} \rho_L^{-2} &\approx \rho_1^{-2}\left[1 - 2\lambda_1\rho_1^{-1}\rho_2(\theta)\right], \\ \rho_L^{-4} &\approx \rho_1^{-4}\left[1 - 4\lambda_1\rho_1^{-1}\rho_2(\theta)\right], \\ \rho_L^{\alpha} &\approx \rho_1^{\alpha}\left[1 + \alpha\lambda_1\rho_1^{-1}\rho_2(\theta)\right]. \end{aligned} \tag{20}$$

Inserting the values (20) into formulae (16) and (18) the expressions for stress components on the contour L are got. For NDZ the stresses on the contour L will be

$$\begin{aligned} \sigma^L_{[\rho]} &= \frac{\beta_1 K}{1-\alpha_1} + (1+\alpha_1)C_2\rho_1^{\alpha_1-1} + (1+\alpha_1)(\alpha_1-1)C_2\lambda_1\rho_1^{\alpha_1-2}\rho_2(\theta), \\ \sigma^L_{[\theta]} &= \frac{\beta_1 K}{1-\alpha_1} + \alpha_1(1+\alpha_1)C_2\rho_1^{\alpha_1-1} + \alpha_1(1+\alpha_1)(\alpha_1-1)C_2\lambda_1\rho_1^{\alpha_1-2}\rho_2(\theta), \end{aligned} \tag{22}$$

and in a region of elastic deformations

$$\begin{aligned} \sigma^L_{(\rho)} &= \lambda_2 T(1-a\rho_1^{-2}) - \lambda_1 T(1-2b\rho_1^{-2}+c\rho_1^{-4})\cos 2\theta + 2\lambda_1\lambda_2 T a\rho_1^{-3}\rho_2(\theta), \\ \sigma^L_{(\theta)} &= \lambda_2 T(1+a\rho_1^{-2}) + \lambda_1 T(1+c\rho_1^{-4})\cos 2\theta - 2\lambda_1\lambda_2 T a\rho_1^{-3}\rho_2(\theta), \\ \tau^L_{(\rho\theta)} &= \lambda_1 T(1+b\rho_1^{-2}-c\rho_1^{-4})\sin 2\theta. \end{aligned} \tag{23}$$

Comparing the stress values on the contour L the system of equations is received for determination a, b, c, ρ_1 and $\rho_2(\theta)$:

$$1+b\rho_1^{-2}-c\rho_1^{-4}=0,$$

$$\frac{\beta_1 K}{1-\alpha_1}+(1+\alpha_1)C_2\rho_1^{\alpha_1-1}-\lambda_2 T(1-a\rho_1^{-2})=0,$$

$$\frac{\beta_1 K}{1-\alpha_1}+(1+\alpha_1)\alpha_1 C_2\rho_1^{\alpha_1-1}-\lambda_2 T(1+a\rho_1^{-2})=0, \tag{24}$$

$$(1+\alpha_1)(\alpha_1-1)C_2\rho_1^{\alpha_1-2}\rho_2(\theta)+T(1-2b\rho_1^{-2}+c\rho_1^{-4})\cos 2\theta-2\lambda_2 Ta\rho_1^{-3}\rho_2(\theta),$$

$$\alpha_1(1+\alpha_1)(\alpha_1-1)C_2\rho_1^{\alpha_1-2}\rho_2(\theta)-T(1+c\rho_1^{-4})\cos 2\theta+2\lambda_2 Ta\rho_1^{-3}\rho_2(\theta).$$

After execution of necessary transformations the typical parameters are found:

$$\rho_1^{\alpha_1-1}=\frac{2\lambda_2 T-2A}{C_2(\alpha_1+1)^2};\quad C=3\rho_1^4;$$

$$a=\frac{(\alpha_1-1)(\lambda_2 T-A)}{(\alpha_1+1)\lambda_2 T};\quad b=2\rho_1^2; \tag{25}$$

$$\rho_2(\theta)=\frac{2\rho_1 T\cos 2\theta}{(\alpha_1-1)(\lambda_2 T-A)},\quad \text{where } A=\frac{\beta_1 K}{1-\alpha_1}.$$

If to insert the found values ρ_1 and $\rho_2(\theta)$ from (24) into formula (19) then the complete equation of the contour L (the boundary between NDZ and elastic region) is got:

$$\rho_L=\left(\frac{2(\lambda_2 T(1-\alpha_1)-\beta_1 K)}{C_2(1-\alpha_1)(1+\alpha_1)^2}\right)^{\frac{1}{\alpha_1-1}}\left[1+\lambda_1\frac{2T\cos 2\theta}{\beta_1 K-(1-\alpha_1)\lambda_2 T}\right]. \tag{26}$$

We will consider an example.
Let the deep vertical borehole with the radius $\rho=1$ was drilled in rock massif subject to influence of tectonic strength T with the following mechanical characteristics: the mean rock density $\gamma=2{,}5\ t/m^3$; $T=\gamma h,\ 3/2\gamma h,\ 2\gamma h$; $K=200\ t/m^2$; $K_0=2\ t/m^2$; $\beta=3$; $n=1$
The bore-solution density $\gamma_B=1{,}4\ t/m^3;\ 1{,}6\ t/m^3;\ 1{,}8\ t/m^3$.
We will find the NDZ boundary on the depth $h=2000m$ with $\lambda=0{,}9$ for $\theta=0,\ 30°,\ 60°$ and $90°$.
The results of calculation according to the formula (26) for $\varphi=19°30'$ and $\varphi=30°$ are presented in table.
In the table for comparison the values of NDZ boundary are adduced when rock massif is subjected to stress geostatic field that is when $\lambda=1$. The results show the NDZ boundary essentially depend from the tectonic strength value, the bore-solution density, physical-mechanical characteristics of rock thickness and drilling depth.

TABLE

Influence of physical-mechanical characteristics of rock thickness, bore-solution density, tectonic strength value on boundary between elastic region and NDZ

φ	γ_B, t/m^3	$T, \gamma h$; $\gamma h = 5000\frac{t}{m^2}$	$\lambda=1$	ρ_L, $\lambda=0,9$, for polar angle θ: 0	30°	60°	90°
19°30'	1,4	1	1,941	2,124	2,032	1,850	1,758
		1,5	2,808	3,082	2,945	2,671	2,535
		2	3,676	4,041	3,858	3,493	3,311
	1,6	1	1,750	1,914	1,832	1,667	1,585
		1,5	2,531	2,778	2,655	2,408	2,284
		2	3,313	3,642	3,477	3,148	2,984
	1,8	1	1,592	1,742	1,667	1,518	1,443
		1,5	2,304	2,528	2,416	2,191	2,079
		2	3,015	3,315	3,165	2,865	2,716
30°	1,4	1	1,244	1,305	1,275	1,214	1,183
		1,5	1,506	1,582	1,544	1,469	1,431
		2	1,729	1,817	1,773	1,685	1,641
	1,6	1	1,175	1,233	1,204	1,146	1,117
		1,5	1,422	1,494	1,458	1,387	1,351
		2	1,633	1,716	1,674	1,591	1,550
	1,8	1	1,116	1,171	1,143	1,088	1,061
		1,5	1,351	1,419	1,385	1,317	1,283
		2	1,551	1,630	1,590	1,512	1,472

REFERENCES

Alimzhanov A.M.(1993): On tectonic strength influence and disconnected affect of bore-solution on stressed-deformed state around deep borehole. Herald of National Academy of Sciences of Kazakhstan, N 1, P. 78-83.
Alimzhanov M.T.(1982): Stability of Body Equilibrium and Problems of Rock Mechanics: Alma-Ata, "Nauka", P. 272.
Alimzhanov M.T. (1992): Methods of Continuum Nonlinear Mechanics in Problems of Rock Pressure Control at Drilling Deep Heles.Int.J. Advances in Mechanics, v.15, N 3/4, 41-69.