INFLUENCE OF MODE II STRESS INTENSITY FACTOR ON THE FRACTURE TOUGHNESS OF CONCRETE

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ABSTRACT

The process of crack propagation in concrete beams under bending has been studied by means of photoelastic coating method. Isochromatic fringe patterns have been utilized in numerical analysis of the stress field parameters at the vicinity of a crack tip. The elastic degradation of concrete in the microcracking region is described by the „fictitious" modulus of elasticity for concrete. It seems, that critical stress intensity factors should not be considered separately for each mode of crack propagation in concrete. In every case both stress intensity factors K_I and K_{II} exist together forming the critical parameter K^*. Fringe patterns observed during crack propagation are typical for opening mode, while in the moment preceding crack advancement they are under mixed-mode loading conditions.

KEYWORDS

Concrete, fracture mechanics, mixed-mode loading, photoelastic coating method, crack.

INTRODUCTION

One of the major points of interest in fracture mechanics of concrete is the mixed-mode crack propagation problem. The term „mixed-mode" used herein implies that the crack is subjected to combined stresses, so the deformation at the crack tip may have both, mode I (opening) and mode II (sliding) components. In terms of mixed-mode crack propagation, the effects of specimen dimensions and test configuration act together with the influences of structural inhomogeneity, as in the case of concrete. Many different test geometries have been used in experimental investigations. They have been summarized in a comprehensive study by Carpinteri and Swartz (1991). Experimental results for fracture toughness parameters vary substantially depending on test condition and crack propagation criteria used (Jeng and Shah, 1988). Some conclusions formulated on the basis of experimental data seem to be contradictory. It pertains to the vital problems of the existence of shear fracture of concrete and crack propagation process. Bazant and Pfeiffer (1986) have found out, that mode II fracture exists and follows the size effect law of blunt fracture, which implies the presence of a large fracture process zone at the fracture front. Considering the partition of the fracture energy, they emphasized that fracture energy for mode II is about 25 times larger than the tensile (mode I) fracture energy. In contrast Ballatore at al. (1990) reported the mixed-mode I fracture energy G_F. Even this superficial analysis of the mixed-mode crack propagation problem gives rise to some important questions, that are crucial for the investigation of the fracture process in concrete:

- is it possible in practice to separate mode I and II in concrete,
- is the crack growth due to a mode I deformation mechanism only,
- is the magnitude of mixed-mode fracture energy the same or different from mode I fracture energy.

The photoelastic coating method has been used for the investigation of the fracture process in concrete beams subjected under mode I loading conditions. Isochromatic fringe patterns connected with the modes I and II have been observed in every test. Stress intensity factors K_I and K_{II} have been evaluated numerically.

EXPERIMENTAL PROCEDURE

The experiments were carried out on concrete beams of the two types. Their dimensions and loading scheme are shown in Fig. 1.

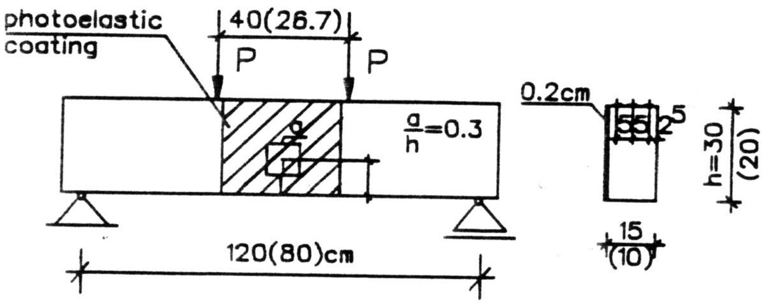

Fig. 1. Dimensions of the specimens and loading conditions

Each series, A (h = 0.3 m) and B (h = 0.2 m), consisted of 3 elements. In the midsection of beams an artificial notch of relative length a/h = 0.3 was moulded. Notches were formed by thin steel plate (0.001 m) covered with a silicon separating agent, inserted while casting. To decrease the influence of shrinkage, specimens were stored in water for 90 days. After drying of concrete beams, the lateral surfaces were ground (outer layers 0.005 m) and the photoelastic coating was glued on one lateral surface at the midspan.
A set of strain gauges was fixed on the opposite side of beams. The strain gauges indicated the strains in the compression and tension area of the beams and at the position of the neutral axis. Isochromatic fringe patterns were recorded at the loading levels of 0.5 P_u and 0.9 P_u (P_u - ultimate load).

Six additional elements were investigated with the aid of a high-speed camera (HYCAM), at the rate of 100 frames/second. Details of the experimental procedure and materials characteristics are given elsewhere (Jankowski and Styś, 1990). In Fig. 2 the experimental set-up for the recording of isochromatic fringe patterns is presented.

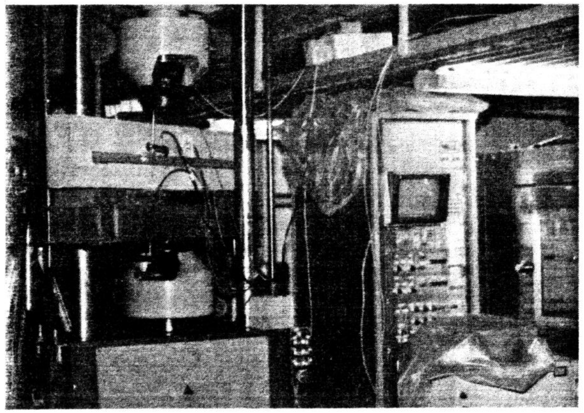

Fig. 2.

Experimental set-up for the acquisition of the photo-elastic data

NUMERICAL PROCEDURE FOR THE ANALYSIS OF PHOTOELASTIC DATA

With the assumption of elastic-brittle, isotropic model, the isochromatic fringe distortions entailed by structural irregularities and boundary-loading effects were expressed in terms of the two Wetsergaard type stress function (Rossmanith, 1979). The functions are connected with modes I and II according to formulae:

$$\Phi_I(z) = A_0 z^{-0.5} + A_1 z^{0.5} \tag{1}$$

$$\Phi_{II}(z) = A_0 z^{-0.5} + B_1 z^{0.5} \tag{2}$$

After introducing SIF:

$$K_I = A_0(2\pi)^{-0.5}; \qquad K_{II} = B_0(2\pi)^{0.5} \tag{3}$$

and higher order terms in stress functions:

$$\beta_1 = A_1(A_0)^{-1} r_s; \qquad \beta_2 = B_1(B_0)^{-1} r_s \tag{4}$$

where r_s - is a scalling factor, one may characterize the stress tensor at the crack tip by parameters $K_I, K_{II}, \beta_1, \beta_2$. It is also reasonable to include uniform stress field σ_{ox} parallel to the crack. Henceforth the stresses are described in the following form:

$$\sigma_x = \mathrm{Re}\,\Phi_I(z) + 2\,\mathrm{Im}\,\Phi_{II}(z) - y\left[\mathrm{Im}\,\Phi'_I(z) - \mathrm{Re}\,\Phi_{II}(z)\right] + \sigma_{ox} \tag{5}$$

$$\sigma_y = \mathrm{Re}\,\Phi_I(z) + y\left[\mathrm{Im}\,\Phi'_I(z) - \mathrm{Re}\,\Phi_{II}(z)\right] \tag{6}$$

$$\tau_{xy} = \mathrm{Re}\,\Phi_{II}(z) - y\left[\mathrm{Im}\,\Phi'_{II}(z) - \mathrm{Re}\,\Phi'_I(z)\right] \tag{7}$$

When dealing with cement matrix heterogeneous composites, the problem of a crack tip localization arises. For reasons such as, crack front curvature, microcracking and stochastic scatter of though gravel inclusions, the crack tip in the photoelastic coating does not always coincide with the crack tip in the concrete specimen. An additional correction procedure has been introduced for proper crack tip localization. Two independent parameters x_0, y_0 are the cartesian coordinates of the crack tip in reference to the coordinate system which can be easily positioned (Sanford, Dally, 1979). Finally, the basic equation of photoelastic coating method depends on 7 parameters:

$$f_k(K_I, K_{II}, \beta_1, \beta_2, \sigma_{ox}, x_o, y_o) = \left[N f_\sigma E_s^*(1+\nu_c)\right]\left[2E_c(1+\nu_s^*)\right]^{-2} \tag{8}$$

where: N - is the fringe order, f_σ, E_c and ν_c characterize properties of the photoelastic coating. E_s^* and ν_c^* are „fictitious" modules of elasticity and Poisson ratio for a concrete specimen respectively. They take into account the elastic degradation due to microcracking in the crack tip region in concrete elements. The „fictitious" elastic modulus was derived on the basis of strain loci in the cross - section containing a notch (Fig. 3). The function of stress $\sigma_y(x)$ in the equilibrium equations is given as a product of the strain function $\varepsilon_y(x)$ and $E_s(x)$:

$$\sigma_y(x) = \varepsilon_y(x) * E_s(x) \tag{9}$$

Equilibrium equations and restrictions imposed on the range of variability of $E_s(x)$ function allow $E_s(x)$ to be defined as a second degree polynomial. In the next step, one can calculate the mean value E_s^*, which characterizes material properties in the microcracking region:

$$E_s^* = \frac{1}{d}\int_0^d E_s(x)\,dx \tag{10}$$

where d - is the extent of the microcracking zone. Assuming the strength hypothesis (Jankowski and Styś, 1990) it is possible to define the extent of fracture process zone boundary on the basis of boundary fringe order N_b.

It is a matter of convenience to formulate a hypothesis, which depends on material characteristics attainable by simple laboratory tests. The criterion for microcracks formation was formulated in terms of ultimate strain $\varepsilon_u = 1.1 \times 10^{-4}$.

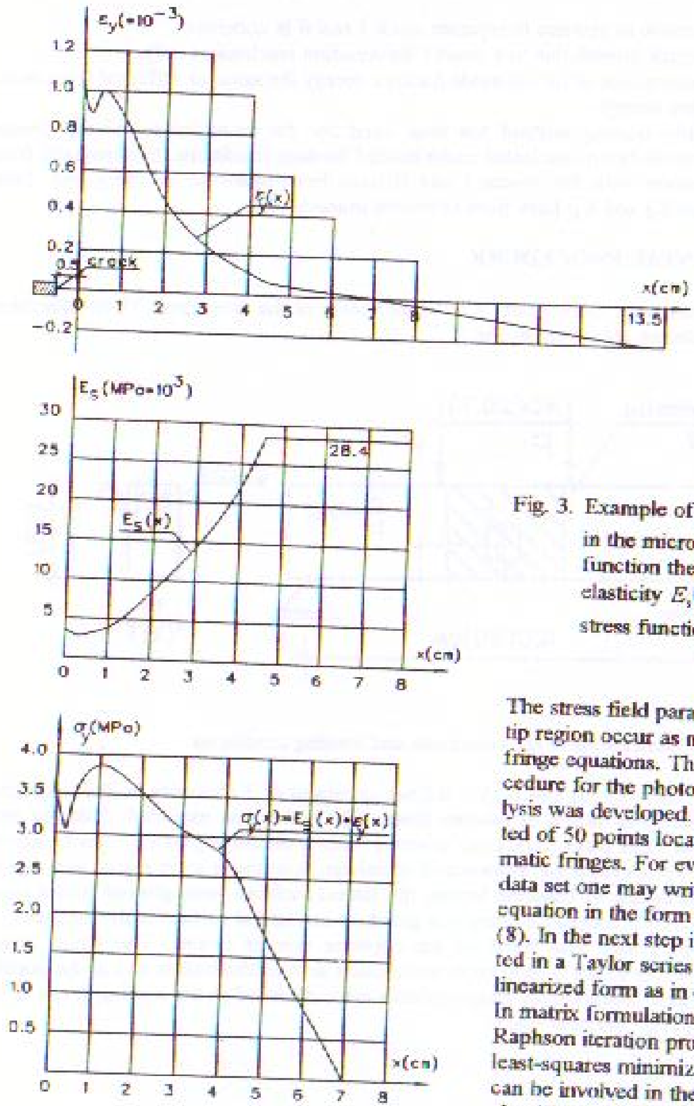

Fig. 3. Example of strains $\varepsilon_y(x)$ in the microcracking zone (a) function the modulus of elasticity $E_s(x)$ - (b), and stress function $\sigma_y(x)$ - (c)

The stress field parameters in a crack tip region occur as non-linear in the fringe equations. The numerical procedure for the photo-elastic data analysis was developed. Data set consisted of 50 points located on isochromatic fringes. For every point of the data set one may write the fringe equation in the form of expression (8). In the next step it can be expanded in a Taylor series and given in a linearized form as in equation (11). In matrix formulation, Newton-Raphson iteration procedure and the least-squares minimization process can be involved in the numerical algorithm (Sanford and Dally, 1979)

$$f_{k_{(i+1)}} = f_{k_i} + \left(\frac{\delta f_k}{\delta K_I}\right)_i \Delta K_I + \left(\frac{\delta f_k}{\delta K_{II}}\right)_i \Delta K_{II} + \left(\frac{\delta f_k}{\delta \beta_1}\right)_i \Delta \beta_1 + \left(\frac{\delta f_k}{\delta \beta_2}\right)_i \Delta \beta_2 + \\ + \left(\frac{\delta f_k}{\delta \sigma_{ox}}\right)_i \Delta \sigma_{ox} + \left(\frac{\delta f_k}{\delta x_o}\right)_i \Delta x_o + \left(\frac{\delta f_k}{\delta y_o}\right)_i \Delta y_o \quad (11)$$

where: i - is the number of consecutive iteration step and $\Delta K_I, \Delta K_{II}, \Delta\beta_1, \Delta\beta_2, \Delta\sigma_{ox}, \Delta x_o, \Delta y_o$ are correction factors for each parameter, obtained in the i^{th} iteration. For n points it fields in the matrix form:

$$[f]_{nx1} = [A]_{nx7}[\Delta]_{7x1} \quad (12)$$

$$[f] = [-f_1 \ldots -f_n]^T \quad (13)$$

$$[A] = \begin{bmatrix} \frac{\delta f_1}{\delta K_I} & \frac{\delta f_1}{\delta K_{II}} & \frac{\delta f_1}{\delta \beta_1} & \frac{\delta f_1}{\delta \beta_2} & \frac{\delta f_1}{\delta \sigma_{ox}} & \frac{\delta f_1}{\delta x_o} & \frac{\delta f_1}{\delta y_o} \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \frac{\delta f_n}{\delta K_I} & \frac{\delta f_n}{\delta K_{II}} & \frac{\delta f_n}{\delta \beta_1} & \frac{\delta f_n}{\delta \beta_2} & \frac{\delta f_n}{\delta \sigma_{ox}} & \frac{\delta f_n}{\delta x_o} & \frac{\delta f_n}{\delta y_o} \end{bmatrix} \quad (14)$$

$$[\Delta] = [\Delta K_I \Delta K_{II} \Delta\beta_1 \Delta\beta_2 \Delta\sigma_{ox} \Delta x_o \Delta y_o]^T \quad (15$$

and

$$[A]^T_{7xn}[f]_{nx1} = [A]^T_{7xn}[A]_{nx7}[\Delta]_{7x1} \quad (16)$$

$$[S]_{7x7} = [A]^T_{7xn}[A]_{nx7} \quad (17)$$

$$[\Delta]_{7x1} = [S]^{-1}_{7x7}[A]^T_{7xn}[f]_{nx1} \quad (18)$$

After the matrix of correction factors $[\Delta]$ is computed, we may improve the estimates of the unknown parameters and calculate the new elements of the matrix [f] for the next iteration. The process is repeated until $[\Delta]$ becomes acceptably small. The points of the data set were located at the isochromatic fringes of the order N = 1.0 ; 1.5 ; 2.0. Maximal radius enclosing the area of the disturbances entailed by the proximity of a crack tip was evaluated as 0.005 m.

CRACK PROPAGATION UNDER MIXED MODE CONDITIONS

Bazant and Gambrova (1980) formulated an empirical stress - displacement relation for concrete, applying energy balance approach and implemented it in finite elements procedure. They claimed, that the first displacement on the crack had to be normal and the slip would be able to occur only after some finite opening had already been achieved. Important conclusion coming from this finding is, that mode I parameters control crack propagation in concrete even under mixed mode conditions. Some experimental results confirm this hypothesis indirectly (Ballatore et al., 1990). Using high-speed (100 frames per second) picture recording of the isochromatic fringe patterns, it was possible to study the process of crack propagation in concrete. The analysis of cinematograms recorded on concrete beams at the different time instants of the crack evolution revealed, that when the crack moved from point to point the isochromatic fringe patterns were characteristic of mode I loading. This phenomenon was typical for all recordings and does not depend on the fringes configuration at the moment preceding crack propagation. In Fig. 4 the examples of dynamic recording of the isochromatic fringe patterns are presented. Steady-state configuration is strongly influenced by mode II loading and boundary effects.

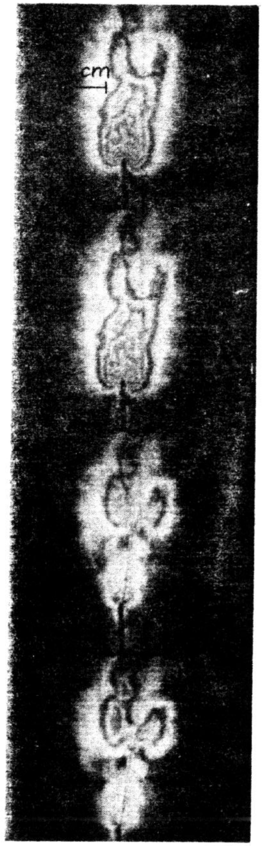

Fig. 4. Example of the dynamic recording of the isochromatic fringe patterns during crack propagation and in the moment preceding crack advancement

RESULTS OF NUMERICAL ANALYSIS

Results of numerical calculations are summarized in Table 1. These are fracture parameters and „fictitious" elastic moduli for concrete. One may notice considerably large values of SIF K_{II} , which are of the same order as SIF K_I. The global stress intensity factor K*, was evaluated according to the formula:

$$K* = \left(K_I^2 + K_{II}^2\right)^{0.5} \quad (19)$$

Effective fracture energy of concrete G* is related to K*:

$$G* = (K*)^2\left(E_s^*\right)^{-1} \quad (20)$$

Table 1. Results of numerical calculations

Beam No.	Fictitious modulus of elasticity E_r^*	K_I	K_{II}	β_1	β_2	σ_{ox}	K*	G*
	[MPa]	$MNm^{-1.5}$	$MNm^{-1.5}$			MPa	$MNm^{-1.5}$	N/m
A1	8832	0,487	0,636	25,737	0,571	-7,007	0,801	72,64
A2	8595	0,863	0,351	20,454	0,711	-7,690	0,932	101,06
A3	9453	0,752	0,485	18,643	0,906	-6,522	0,895	84,71
B1	10740	0,995	0,317	8,192	0,926	-8,577	1,044	101,48
B2	11404	0,986	0,485	15,637	0,533	-8,090	1,099	105,88
B3	9806	0,844	0,504	10,508	0,282	-7,160	0,983	98,54

CONCLUSIONS

From the considerations presented above, the following conclusions may be drawn:

1. It is hardly possible to separate mode I and mode II loading conditions during tests performed on concrete specimens.
2. The global value of the fracture toughness is usually obtained in experiments. In the case of stress intensity factor it contains a part of K_I and K_{II} and both coupled SIF should be taken into account.
3. During crack propagation in concrete beams opening mode (I) prevails. When the crack moves from point to point the isochromatic fringe pattern is of the mode I type, while in the moment preceding rupture the configuration of isochroms is influenced by sliding mode (II) and boundary effects.

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