HIGH-TEMPERATURE CRACK GROWTH IN DISSIMILAR MEDIA

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ABSTRACT

The microcrack growth behavior in dissimilar media is investigated with an aim at estimating service life of advanced ceramic composites under creep-rupture conditions. The crack is assumed to grow along an interface normal to a remote applied tensile stress via a coupled surface and grain-boundary diffusion under steady-state creep conditions. The tensile stress distribution along the interface ahead of the moving crack tip is solved, and it is found that a new length parameter exists as a scaling factor for which the solution becomes identical to the single-phase media when plotted on the nondimensional physical plane. In contrast to the elastic stress solution which shows singularity at the tip and oscillatory character away from the tip, the creep stresses have a peak value away from the tip due to a wedging effect and interfacial sliding eliminates stress oscillation resulting in a decoupling between mode I and mode II stress fields. This solution ties the far-field loading parameter to the crack-tip conditions in terms of the unknown crack velocity to give a specific V-K_I relationship. It is shown that an exponent of 12 in the conventional crack growth power law emerges at higher applied stresses.

KEYWORDS

Bimaterial Interface; Ceramic Composites; Crack Growth; Creep Rupture; High-Temperature Fracture; Interfacial Crack; Lifetime Predictions; Mass Transport; Residual Stress

NOMENCLATURE

D_b, D_s	grain-boundary, surface diffusivity	x	nondimensional X (=X/L)
E	Young's modulus	κ_{tip}	surface curvature adjacent to the crack tip
J_{tip}	matter flux at the crack tip	ν	Poisson's ratio
K, K_I	mode I stress intensity factor	α	materials parameter describing σ_o'
K_G	stress intensity for Griffith cracks	β	Dundur's parameter
K_{th}	threshold stress intensity	λ	bimaterials elastic constant
k	nondimensional K (=K/K_G)	μ	shear modulus
kT	thermal energy per atom	κ	elastic constant
L	characteristic length along the interface	γ_b, γ_s	boundary, surface free energy
ℓ	reference length for the crack thickness	δ_b, δ_s	zone width for g.b. and surface diffusion
V	steady-state crack-tip velocity	Ω	atomic volume
V_{min}	minimum V	σ	normal stress (=σ_{yy})
v	nondimensional V (= V/V_{min})	σ_o	crack-tip stress (i.e. σ_{tip} or $\sigma(x=0)$)
X	Cartesian coordinate along the interface	b	Burger's vector

INTRODUCTION

The present paper is concerned with the crack growth behavior in an interface between two dissimilar phases subjected to long-term sustained loading conditions at elevated temperatures. In recent years, advanced ceramic composites reinforced with ceramic fibers have attracted considerable interest because of their potential advantages over conventional materials in high-temperature, load-bearing applications. Those advantages include enhanced strength and toughness and high resistance to corrosion and wear in severe service environment. Yet, their reliability under long-term sustained loading conditions remains to be quantitatively established so that lifetimes can be estimated to assure reliable service and avoid premature failure. Experimental observations of the microstructure of the creep-ruptured specimens indicate that the formation and propagation of microcracks along interfaces between fiber/matrix and matrix/matrix constitute major damage, and final coalescence of these microcracks is responsible for creep fracture [1-3]. This type of rupture mode is in sharp contrast with the conventional short-term fracture mode wherein fiber bridging and fiber pull-out are the prevailing phenomena[4-7]. Because most of the microcracks are found at interfaces oriented in the direction normal to the principal tensile stress axis, the role that mass transport plays in crack growth and linkage must be an important aspect of the creep-rupture process. Diffusion-induced crack growth in single-phase media has been considered previously [8]. The present work extends the treatment to a bimaterial system. The major goal is to solve the crack-tip velocity in the steady-state creep stage as a function of thermal-mechanical loading parameters.

At elevated temperatures in excess of about one third of the homologous temperature, mass transport becomes activated along high diffusivity paths such as interfaces and internal free surfaces. Because it takes less energy to form a void at an interface than in the bulk, creep cavities are predominantly observed at interfaces, rather than inside grains. Moreover, of all boundaries, cavitation seems to favor those normal to the principal tensile stress axis. These observations suggest that stress-driven diffusion around the cavity periphery plays an important role in the cavity growth process. One possible and convincing mechanism leading to cavity growth involves a coupled process of transportation of species along cavity surfaces towards the apex via surface diffusion and, from there, driving of atoms further away along the interface via grain-boundary diffusion. In this manner, the cavity tip is allowed to advance in a steady-state fashion. A direct proof showing this mechanism is indeed operative was provided by Varma and Dyson[9] for a nickel-base alloy. Based on this specific growth kinetics, many diffusional cavity growth models have been proposed[10-21] of which the diffusional crack growth model is particularly relevant to the present case. Two separate systems will be discussed, namely, crack growth in a single-phase system and in a dissimilar medium. The former aims at modeling matrix/matrix interfaces whereas the latter aims at fiber/matrix interfaces, taking the dissimilarity into account.

CRACK GROWTH IN SINGLE-PHASE MEDIA

In order to acquire a final solution relating crack velocity to applied stress intensity, K, a precise analysis of the tensile stress distribution at the interface ahead of the moving crack tip must be performed. In the absence of diffusion, the applied K induces the well-known elastic stress field with a characteristic $X^{-1/2}$ type singularity, where X is the distance away from the crack tip. However, as matter diffuses from the crack surfaces and deposits along the interface, the so called "wedging" effect is produced which alleviates the stress concentration at the crack tip. Chuang[8] has formulated this problem using the concept of infinitesimal edge dislocations to evaluate the residual stresses induced by mass transport along the interface. The result is an integral equation for the unknown stress distribution, $\sigma(x)$:

$$L^2 \cdot \sigma''(x) = \int_0^{\infty} \frac{\sigma(x')}{x-x'}\, dx' \qquad (1)$$

where prime denotes d/dx and $x = X/L$ is a nondimensional boundary coordinate scaled by L :

$$L = \sqrt{\frac{\pi}{4} \cdot \frac{E D_b \delta_b \Omega}{(1-\nu^2) V k T}} \qquad (2)$$

where E and ν are Young's modulus and Poisson's ratio, respectively; $D_b\delta_b$ is the grain-boundary diffusivity; Ω is the atomic volume and V is the crack velocity. The magnitude of L is typically in the order of a micrometer [21].

A complete solution for the integral equation was obtained numerically [8]. It was found that the stress distribution is dependent on a parameter α which, in turn, is a function of V. Typical stresses for $\alpha = 0, 1, 5, 10$ and 20, respectively, are plotted in Fig.1. Here a hatch on the top of a parameter denotes nondimensionalization. For the sake of comparison, the elastic stresses for the case of $\alpha = 20$ are given by a dashed line. It is seen from this plot that the stress singularity at the crack tip has been eliminated by diffusion and the peak stresses now occur at around X = 0.9 L instead. The sizes of the diffusion zone are estimated to be approximately 4L, beyond which the influence of diffusion becomes insignificant. The undisturbed far-field stresses are solely controlled by the applied K, which can then be related to $\sigma(0)$ and $\sigma'(0)$ at the crack tip by the numerical solution. This relationship can be cast in the following form:.

$$K = 0.75\, \sigma(0)\, L^{1/2} + 0.60\, \sigma'(0)\, L^{3/2} \qquad (3)$$

This is a significant result as this equation appears to have a widespread applicability, not only to the current model of diffusive crack growth in a single-phase elastic system, but also to a future model of bimaterial systems as will be discussed later. In addition, Cao *et al.*[22] applied this equation to a problem of high-

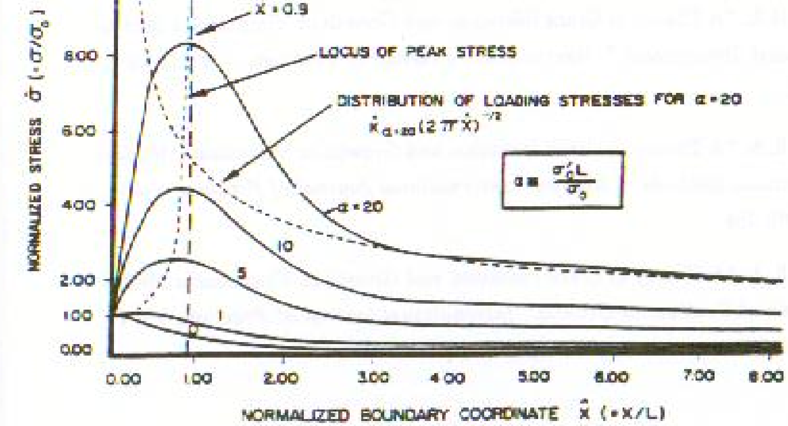

Figure 1. Solutions of tensile stresses at the interface ahead of the crack tip.

temperature, stress-corrosion cracking of ceramics in which corrosive liquid product filled the crack-like cavity and served as a major species in the mass transport process.

The final goal of expressing K in terms of V, or vice versa, can be achieved by substituting the stress conditions at the crack tip (namely, $\sigma(0) \propto V^{1/3}$; $\sigma'(0) \propto V^{2/3}$) in Eqn.(3). The result is a V-K relationship in non-dimensional form: $k=0.845\,(v^{-1/12}+v^{1/12})$ where $k= K_I/K_G$ and $v=V/V_{min}$. Here K_G[1] is the critical K-value based on the Griffith theory and V_{min} is a materials parameter depending on temperature, diffusivity and elastic constants. It can be shown that if the physically inadmissible branch of the equation is discarded, a one-to-one relationship between the crack-tip velocity, V, and the applied stress intensity in mode I, K_I, can be established. Solving v in terms of k, one finally arrives at the following expression:

$$v = \left(0.59\,k + \sqrt{0.35k^2-1}\right)^{12} \tag{4}$$

It is seen that for a creep crack growing in a single-phase *elastic* material by diffusion, a unique V-K_I relationship emerges, irrespective of materials species or temperature. However, as indicated in Eqn.(4), the V-K_I relationship, in general, can not be cast in a simple form of a power-law equation which is conventionally adopted as an empirical expression. Fig.2 plots the equation in a double log space, and a few remarks are in order. First, there exists a threshold stress intensity, $K_{th} = 1.69\,K_G$, below which the applied stress is not sufficient to drive the crack and, as a result, the crack ceases to propagate and sintering may actually occur. Secondly, at higher K values, say $K>4K_G$, a power-law equation, $V=(\text{constant}) \cdot K^{12}$ becomes an asymptote and is a fair representation of Eqn.(4). Finally, for intermediate K-levels in the range $4K_G > K > 1.69K_G$, the theory predicts that the stress exponents vary from 12 to infinity.

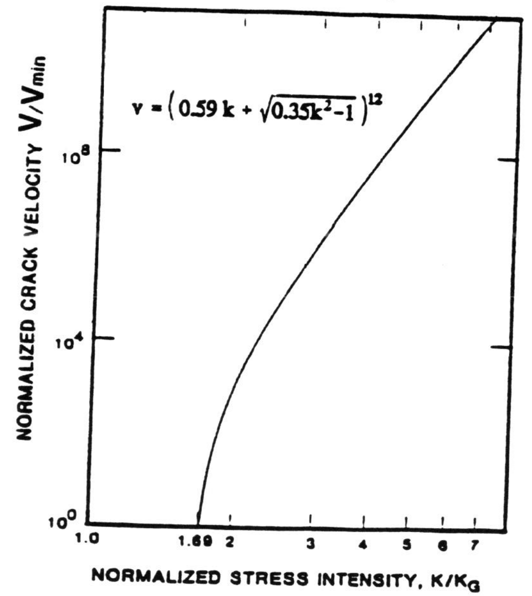

Figure 2. V-K relationship predicted by the diffusive crack growth theory.

[1] K_G is related to true surface energy and should not be confused with K_{IC} which is, in general, about one order of magnitude higher.

CRACK GROWTH IN DISSIMILAR MEDIA

In the case of a crack growing along a fiber/matrix interface under creep conditions, what effects of dissimilarity, if any, on the crack growth behavior between the fiber and the matrix, in terms of differing physical and mechanical properties, becomes an interesting subject and deserves a thorough investigation. As the geometry dictates, the crack appears to be much thinner than the grain size or fiber diameter. Plane-strain conditions should prevail and this case can be modeled as a two-dimensional, two-phase solid containing an interfacial crack, phase 1 representing the fiber and phase 2 the matrix. The problem now is to solve the crack growth rate for a given stress and temperature, and again this can be divided into two parts.

Crack-tip Conditions For the first part of the problem, Chuang *et al.*[24] have recently investigated the crack-tip morphology that could be developed by surface diffusion-controlled crack growth along an interface between the two dissimilar phases. In contrast to the symmetric case developed in single-phase systems, four asymmetric cases are possible for two-phase media depending on the degree of dissimilarity in surface free energy and diffusivity of the two adjoining phases. Excluding the physically inadmissible case of the tip morphology, there are three possible cases of crack-tip morphology where the upper and lower cavity surfaces are no longer symmetrical with respect to the tip owing to differing properties of the dissimilar phases. A tip morphology map in the space of surface free energies, γ_1 versus γ_2, can be constructed to demonstrate prospective areas where each case applies. The near-tip shape again can be uniquely described by a logarithmic function as in the case of single-phase materials. However, the maximum half-thickness is now 2.0 L instead of 1.41 L. These results yield a more complex expression for the matter flux and the root radii (or the surface curvatures) at the crack tip. These expressions involve physical properties of the two dissimilar phases. However, the velocity dependence remains the same, namely, the curvature at the tip, $\kappa_{tip} \sim V^{1/3}$, and the matter flux at the tip, $J_{tip} \sim V^{2/3}$. Accordingly, the boundary conditions at the moving crack tip have the following relationships: $\sigma(0) \sim V^{1/3}$ and $\sigma'(0) \sim V^{2/3}$. Of course, the proportionality constants will have lengthy expressions in terms of materials constants and temperature. From Eqn.(1), the stress solution $\sigma(x)$ or in tensorial term, $\sigma_{yy}(x)$ along the interface will relate the far-field stress to the normal stress at the moving crack tip. In order to solve Eqn. (1), initial conditions of stress at the crack tip have to be formulated. Expressions of the crack-tip stress and its first derivative in terms of the unknown, *a priori*, crack velocity, V, will allow the stress solution to yield the ultimate V-K_I relationship we desired.

The chemical potentials at the crack surface and at the interface can be expressed in the following forms, respectively: $\mu_b = -\sigma\Omega$ and $\mu_s = \Omega\gamma\kappa$ where κ here is surface curvature. The chemical potential at the crack tip where the two crack surfaces join the interface must have a unique value, otherwise there will be an unbounded flux there. This means that $\sigma_{tip} = -\gamma\kappa$. But $\kappa = -\sqrt{2(1-F)}/\ell$ where $F \equiv \sin\theta$ is the sine of the tip angle, and ℓ is a length parameter defined by

$$\ell = \left(\frac{D_s\delta_s\gamma_s\Omega}{VkT}\right)^{1/3}. \tag{5}$$

Here D_s is surface diffusivity; δ_s, the thickness of the effective surface diffusion layer; γ_s, the surface energy. Normally δ_s is in the order of $\Omega^{1/3}$. Let $\Delta_s = (D_{s1}\delta_1\Omega_1)/(D_{s2}\delta_2\Omega_2)$. Furthermore, it can be shown that the tip shape at the upper surface has the form

$$F_1 = 1 - \frac{\hat{\gamma}_2^{1/3}(\hat{\gamma}_1+\hat{\gamma}_2-1)}{\hat{\gamma}_1(\hat{\gamma}_2^{1/3}+\Delta_s^{2/3}\hat{\gamma}_1^{1/3})} \tag{6}$$

where subscript 1 means phase 1. For surface energy the normalizing parameter is γ_b. Finally, we are able to express the crack-tip stress, σ_{tip}, as a function of the crack velocity and other physical parameters of the two adjoining phases 1 and 2.

$$\sigma_{tip} = \frac{\sqrt{2}\gamma_b}{\ell_1}\sqrt{\frac{\hat{\gamma}_1\hat{\gamma}_2^{1/3}(\hat{\gamma}_1+\hat{\gamma}_2-1)}{\hat{\gamma}_2^{1/3}+\Delta_s^{2/3}\hat{\gamma}_1^{1/3}}} \quad . \tag{7}$$

Note that it can also be shown that when the crack-tip stress is expressed in terms of the phase 2 parameters, the result is identical to Eqn.(7). Note also that because both ℓ_1 and ℓ_2 are inversely proportional to $V^{1/3}$ (see Eqn.(5)), the stress at the crack tip is a function of V to the power of one third.

The first derivative of the crack-tip stress, $\sigma'(0)$ can be derived by a combination of grain-boundary diffusion equation , conservation of mass and steady-state conditions at $x=0$. The result is

$$\sigma'(0) = \sqrt{2}\sqrt{1-F_1}\,\frac{(D_{s1}\gamma_s)^{1/3}}{\Omega_1^{1/3}D_{b1}\delta_b}\,(VkT)^{2/3} \tag{8}$$

where D_{b1} is the grain-boundary diffusivity for phase 1. It is shown that the first derivative of the crack-tip stress is in direct proportion to V to the power of two third.

<u>Stress Analysis</u> To formulate the desired V-K_I relationship, a stress analysis at the interface ahead of the moving crack tip must be carried out. The elastic stress field, in the absence of creep, in a bimaterial containing an interfacial crack has been investigated by many authors[25-28]. The results show many characteristics distinct from the single phase case. (1) Uniaxial stressing results in a mixed mode response. Thus, a far-field applied tensile stress will produce shear stresses at the interface. (2) The normal stresses at the interface ahead of the crack tip induced by K_I show oscillatory decaying with increasing distances from the crack tip; and (3) The elastic crack opening displacements show an unrealistic interpenetration behind the crack tip, although the overlapping zone size is of atomic dimensions.

The above elastic behavior is indeed a fair representation of the short-term material responses. However, under long-term creep conditions where mass transport processes are active, it can be argued from physical principles that surface diffusion will prevent interpenetration and grain-boundary diffusion will relax the shear stresses produced at the interface resulting in grain-boundary sliding. This latter phenomenon is widely observed during high-temperature creep especially for ceramic systems where a liquid phase often exists between the grains. Based on the assumption that the interface ahead of the crack tip cannot resist shear, it was found that mode I and II stress fields become decoupled. In addition, residual stresses induced by diffusion must be taken into account. To formulate the residual stress, consider an edge dislocation situated at an arbitrary location $x=x_o$ ahead of the semi-infinite crack tip; then, the stress field generated by this dislocation has the form

$$\sigma_{yy} + i\,\sigma_{xy} = \sigma_\infty - \lambda(b_y + i\,b_x)\left[\frac{1}{x-x_o} + \pi i\beta\,\delta(x-x_o)\right] \tag{9}$$

where i is the unit of imaginary number, b_y and b_x are the Burgers vectors in the y and x directions, respectively, $\delta(x)$ is the Dirac delta function, λ and β are elastic constants defined by

$$\lambda = \frac{\mu_1\mu_2[\mu_1(1+\kappa_2)+\mu_2(1+\kappa_1)]}{\pi(\mu_1+\mu_2\kappa_1)(\mu_2+\mu_1\kappa_2)} \quad ; \tag{10}$$

β is a Dundur's parameter[28] defined by

$$\beta = \frac{\mu_1(\kappa_2-1)-\mu_2(\kappa_1-1)}{\mu_1(\kappa_2+1)+\mu_2(\kappa_1+1)} \tag{11}$$

where μ in these expressions is the shear modulus; and $\kappa=3-4\nu$ for plane-strain conditions, ν being Poisson's ratio. Equation (9) is a result with no crack-dislocation interactions under the remote tensile loading conditions. In addition to the normal stresses generated at the interface, shear stresses are produced there which would not be there under a symmetrical remote loading conditions were the system monolithic rather than dissimilar.

Now consider an arbitrary distribution of dislocations along $y=0$ under the action of a remote stress σ_∞ such that $db=\delta' dx$. The stresses at any point x can be integrated to yield the following forms:

$$\sigma_{yy}(x) = \sigma_\infty - \lambda\int_{-\infty}^{\infty}\frac{\delta_y'(x_o)}{x-x_o}dx_o + \lambda\pi\beta\,\delta_x'(x) \tag{12}$$

and

$$\sigma_{xy}(x) = -\lambda\int_{-\infty}^{\infty}\frac{\delta_x'(x_o)}{x-x_o}dx_o - \lambda\pi\beta\,\delta_y'(x) \; . \tag{13}$$

Now, it may be argued that there exists a physical requirement that the interface cannot resist shear stresses because of high-temperature creep. This is especially so in the case of advanced ceramics where flat boundaries are often coated with thin films of liquid phases that are introduced as sintering aids. This means that $\sigma_{xy}=0$ at the interface ahead of the crack tip. In addition, there is a relationship between δ_x and δ_y, namely,

$$\delta'_y(x) = \frac{-1}{\pi\beta}\int_{-\infty}^{\infty}\frac{\delta'_x(x_o)}{(x-x_o)}\,dx_o \tag{14}$$

and Hilbert transformation leads to

$$\delta'_x(x) = \frac{\beta}{\pi}\int_{-\infty}^{\infty}\frac{\delta'_y(x_o)}{(x-x_o)}\,dx_o \tag{15}$$

Substituting Eqn.(15) into Eqn.(12), we have

$$\sigma_{yy}(x) = \sigma_{\infty} - \lambda(1-\beta^2)\int_{-\infty}^{\infty}\frac{\delta'_y(x_o)}{(x-x_o)}\,dx_o \qquad (16)$$

Again, by applying the Hilbert transformation, we finally arrive at

$$\pi^2\lambda(1-\beta^2)\delta'_y(x) = \int_0^{\infty}\frac{\sigma_{yy}(x_o)}{x-x_o}\,dx_o \qquad (17)$$

because in the traction-free crack plane ($y=0$, $x<0$) $\sigma_{yy}=0$. Using grain-boundary diffusion equation to eliminate δ'_y in Eqn.(17) yields the following integro-differential equation for the unknown $\sigma_{yy}(x)$:

$$L^2 \cdot \sigma''(x) = \int_0^{\infty}\frac{\sigma(x_o)}{x-x_o}\,dx_o \qquad (18)$$

where a new L is defined by

$$L = \sqrt{\frac{\pi^2\lambda(1-\beta^2)\langle D_b\Omega\rangle\delta_b}{VkT}} \qquad (19)$$

where $\langle D_b\Omega\rangle = (D_{b1}\Omega_1 + D_{b2}\Omega_2)$ is the sum of the two-phase properties. It was found, after comparing Eqn.(18) with (1) that the tensile creep stresses have the same solutions as indicated in Fig.1, if a new length parameter, L, defined in Eqn.(19) is used as a scaling parameter.

As a result, the V-K_I law for bimaterial systems has the same form as the single-phase systems depicted by Eqn.(4), except that the normalizing parameters V_{min} and K_G will have different expressions involving the materials constants associated with the two-phase properties from Eqns. (3), (7) and (8). It should be noted that when the properties of phase 1 are identical to that of phase 2, then $\beta=0$ according to Eqn.(11) and from Eqn.(10) $\lambda=2\mu/\pi(1+\kappa)=E/4\pi(1-\nu^2)$; thus, L in Eqn.(19) reduces to L of the single-phase case expressed in Eqn.(2). Because data on those emerging materials are lacking, a direct comparison between the theory and experiment cannot be made at this time. However, preliminary data on the creep-rupture lifetime of silicon nitride reinforced with silicon carbide whiskers seemed to support the stress dependence on the crack-tip velocity [30].

CONCLUSION

We have derived the V-K_I relationship for diffusion-induced microcrack growth in dissimilar media under sustained loads at elevated temperatures. The functional dependence of the tip velocity on the applied K_I is similar to the case of single phase media except for the two new materials constants, V_{min} and K_G, which have different expressions in terms of the elastic and physical properties of the adjoining phases.

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