FORECASTING METALLIC MATERIAL CREEP AND FAILURE UNDER STATIC LOADING

R.G. TETERUK

T.G. Shevchenko Kiev University,
Glushkov Str. 6, Kiev, Ukraine

ABSTRACT

The problem of forecasting the long-term strength of materials under stationary loading is considered. The applicability of the G.Genki-N.Hoff viscous flow concept and its modifications for solving problems of such class is analyzed for the case of of metallic materials. The paper also considers the creep model basing on the isochrone diagrams similarity principle and describing all the three stages of the process without involving the damageability function. A new procedure of determining the rheological constants is proposed for the given model, using the spline function. It is shown that the given model can be used for forecasting the fatigue life of materials which fail by the brittle and mixed mode ($\varepsilon_R < 30$ %) and the Hoff model - for materials failing by the tough fracture ($\varepsilon_R \geq 30$ %).

KEYWORDS

Creep, time-to-failure, viscous flow, isochrone diagrams, spline function, momentary stress-strain diagram.

INTRODUCTION

Two fundamentally different approaches to solving the problem of forecasting the long-term strength have by now been formed in mechanics of materials. The first approach is empirical and based on the processing of experimental data derived from the long-term strength testing.

The second approach is a theoretical one, and based on using models, whose structure of fundamental equations and coefficient magnitudes are found from independent experiments, not connected with plotting the long-term strength curves. The most widely known concept is that of N.Hoff (1953), which is a further development of H.Henki's idea (1925) of loading. It was used for forecasting the time-to-failure of discs, pipes and other structural members. However, a sufficient experimental check of the model even in a uniform stressed state was not carried out.

The present work considers the applicability of N.Hoff concept to the problem of long-term strength forecasting, as well as the applicability of the fundamentally different creep model, allowing for the medium strain ageing and basing on the isochrone diagram similarity principle.

ONE-DIMENSIONAL CONCEPT OF TOUGH FRACTURE

Let us consider, in terms of N.Hoff's concept (1953), the problem of a delayed fracture of a cylindrical rod being stretched with the specified force P and being in the creep state. As the initial evolution equation let us use the exponential law of the steady-state creep in the form of:

$$\dot{\varepsilon}^c = B\sigma^n \qquad (1)$$

where ε^c is creep flow, σ is actual (current) value of stress, B, n are experimentally determined coefficients (B>0, n ≥ 1) Proceeding from the condition of material incompressibility, σ is determined by the following relationship:

$$\sigma = \sigma_0 l/l_0 \qquad (2)$$

where σ_0, l are initial stresses and rod length, l is current value of its length. For high levels of strain and strain rate, using logarithmic measures, we shall have

$$\varepsilon^c = \ln(l/l_0) \qquad (3)$$

Simultaneously solving the (1) and (3) equations and allowing for (2) and initial conditions l=l at t=0, we shall have

$$l/l_0 = \{1 - Bnt\sigma_0^n\}^{-1/n} \qquad (4)$$

Using, similar to H.Henky (1925), the $l \to \infty$ fracture criterion at $t \to t_R$, we shall determine the tough fracture time t from the relationship:

$$t_R = \frac{1}{Bn\sigma_0^n} \qquad (5)$$

because at $l \to \infty$ from (4) it is necessary that $(1 - Bnt\sigma_0^n) \to 0$

The results of calculations (dashed lines) by the equation (5) for some of reseaches materials are compared with the experimental data (dots) in Fig. 1.

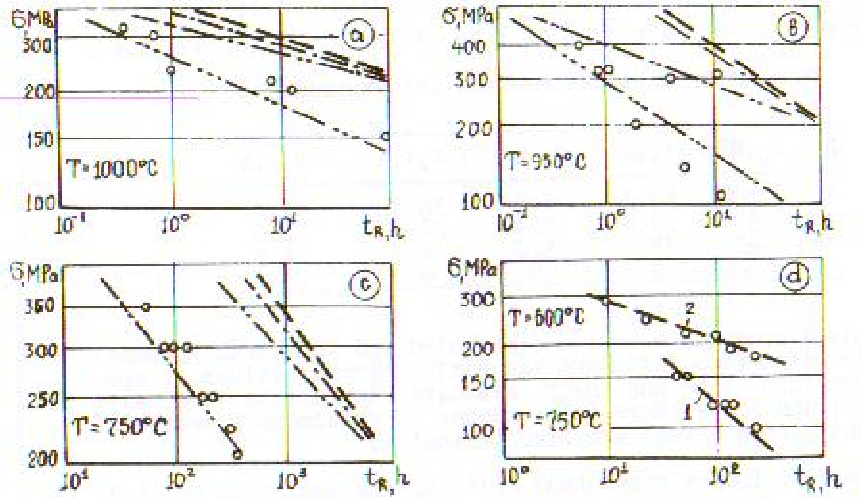

Fig. 1. Calculated (lines) and experimental (dots) curves of long-term strength of ВЖЛ12У(a),ЭИ867(b),ЭИ437Б(c) alloys and ХН55МВЦ(d, curve 1), 08Х18Н9(d, curve 2) steels: (--) calculation by the N.Hoff model (5); (-.-) calculation by the model (7); (-..-) calculation by the model (8); (-...-) calculation by the model (15).

It can be seen that for not highly ductile materials (Fig.1 a,b,c) the calculation by the N.Hoff model yields considerably overestimated results as compared to the experimental assessment of the time-to-failure. In some cases the error amounts to two orders and practically does not depend on the stress level. Similar estimates were also derived when comparing the calculated (dashed lines) and experimental (dots) creep curves (Fig. 2). The strains were calculated from equation (4).

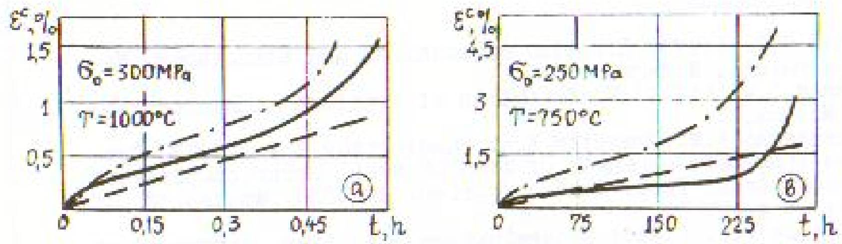

Fig. 2. Curves calculated by the N.Hoff model (dashed lines) and model (15) (dash-and-dot line), and experimental curves (continuous lines) of creep of ВЖЛ12У(a) ЭИ437Б(b).

Let us consider the problem of forecasting the long-term strength on the basis of some modifications of the NHoff model

Model Allowing for Elastic Deformation and Unsteady-state Creep Flow. Let us use the improvement proposed by Odquist (1966) in order to take into account the elastic deformation and unsteady-state creep flow. Then, the determinant creep equation (1) can be expressed by

$$\dot{\varepsilon}^c = \dot{\sigma}/E + Am\sigma^{m-1}\dot{\sigma} + B\sigma^n \qquad (6)$$

where E is elasticity modulus, A, m are experimentally determined coefficients. Using the (2) and (3) relationships for σ and, $\dot{\varepsilon}^c$ respectively, and considering, as before, that at the moment of failure the rod acquires an infinitely large elongation, from equation (6) we shall have

$$t_R' = \frac{1}{Bn\sigma_0^n}\left\{1 - \frac{n}{n-1}\frac{\sigma_0}{E} - A\sigma_0^m\frac{mn}{n-m}\right\} \qquad (7)$$

As can be seen from the analysis of the relationship(7), t_R' will be smaller than t_R, determined from the basic N.Hoff model (5). Hence, the results derived in this case should correlate better with the experiment. In reality, however, this improvement is quite negligible (see Fig. 1, dash-and-dot line) and is of a symbolic nature.

Model Allowing for Limited Stress Range. Since in reality at the moment of fracture the stresses are limited, and do not tend to infinity, the material ultimate strength σ_B can be assumed to be the permissible stress limit. Then from the relationship (4) allowing for (2) for time-to-failure we'll have

$$t_R'' = \frac{1}{Bn\sigma_0^n}\left\{1 - (\sigma_0/\sigma_B)^n\right\} \qquad (8)$$

The results of calculations performed using the equation (8) are shown in Fig.1 by a dashed line with two dots. It can be seen that an essential improvement of the calculation results as compared to the basic model was achieved, although the relative error compared to the experiment still reaches 60% and more.

Evaluation of the Applicability of the Tough Fracture Concept. The tough fracture mechanism described above is the first attempt at a theoretical analysis of creep rupture. The qualitative evaluations of the applicability of such a mechanism are well-known. They are connected with certain

classes of materials, with short durations of loading, with absence of inner damage accumulation, etc. For quantitative evaluations let us use the results of comparing the experimental and calculated data derived in the present work. For this purpose let us establish the dependence of relative error $\Delta\delta$ between calculation and experiment on the values of strain ε at the moment of fracture and the extent of strain ε corresponding to the ultimate tensile strength. The graphic interpretation of the appropriate dependences is shown in Fig. 3.a, and covers the data for all the studied materials, temperatures and stresses.

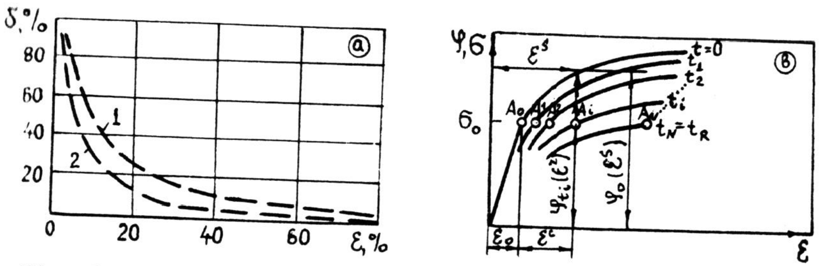

Fig. 3. Dependence of relative error between the calculated by the N.Hoff model (5) and experimental values of fatigue life on the extends of strains ε (a, curve 1) and ε (a, curve 2). Scheme of indentification of non-linear creep and instantaneous non-linearity (b).

It can be seen that with the higher "deformability" of the material the difference between the theory and experiment is quickly reduced. At values equal to ε =20-25% (ε =30-40%) the error $\Delta\delta$, as can be seen, is 10-15%, itbeing quite acceptable for engineering calculations. It confirms for the satisfactory correlation of the calculated data with the experiment for ХН55МВЦ and 08Х18Н9 steels (Fig. 1,b), for which ε_B is 25% and 29%, respectively.

Thus, the N.Hoff creep concept has rather serious limitations as to its aplicability, and can be used for calculation of the time-to-failure of only highly ductile materials . In order to describe creep and calculate the time-to-failure for the other material class, let us consider another approach allowing for the strain ageing of the medium.

CREEP MODEL BASING ON THE ISOCHRONES SIMILARITY PRINCIPLE

Processing of experimental data on creep indicated that in most cases the insochrone diagrams are similar (Rabotnov, 1966). The similarity of the creep process in " σ - ε " coordinates to the instantaneous deformation process allows to use a different approach to the creep model construction. Let us write the similarity law in the following form(Golub, 1989):

$$\varphi_0(\varepsilon^s) = \varphi_t(\varepsilon^r)\cdot\Big[\ 1 + G(t)\ \Big] \qquad (9)$$

where $\varphi_0(\varepsilon^s)$ is the function describing the instantaneous deformation; ε^s is scleronomic deformation (Fig. 3,b); ε^r is rheonomic deformation, including ε_0 initial deformation and creep deformation ε^c($\varepsilon^r=\varepsilon_0 + \varepsilon^c$) ; φ is function of this deformation assigning the isochrone, corresponding to (fixed) time t ; G(t) is similarity function selected so that at t=0 $\varphi_t(\varepsilon^r)$ coincided with $\varphi_0(\varepsilon^s)$. With the fixed magnitude of σ_0 stress in " $\sigma - \varepsilon$ " coordinates the creep process can be represented as a $\{A_i\}$ set points with (σ_0, ε^r_i), $i = \overline{1,N}$, coordinates (Fig. 3, b), belonging to different isochrones (here, ε^r_N is the magnitude of deformation by the moment of failure). As the (10) condition is fulfilled for the entire family of isochrones, it will be also valid for the $\{A_i\}$ set of points. Hence, since $|\varphi_{t_i}(\varepsilon^r_i)| = |\sigma_0|$ for $\forall\ i$, from (9) we shall have

$$\varphi_0(\varepsilon^s_i) = \sigma_0\Big[\ 1 + G(t)\ \Big] \qquad (10)$$

Taking into account the arbitrary selection of t_i values for the isochrone family and noting that at $\forall\ i$ $|\varepsilon^s_i| = |\varepsilon^r_i|$ (Fig. 3, b) from (10) we shall have

$$\varphi_0(\varepsilon^r) = \sigma_0\Big[\ 1 + G(t)\ \Big] \qquad (11)$$

The (11) equation is, essentially, the creep equation written in the non-explicit form.The main idea of (11) representation, apparently, is that the development of the creep process in the " $\sigma - \varepsilon$ " plane is identified with the nature of instantaneous deformation. Let us consider the possibility of transforming the equation (11) into the creep equation written in the explicit form. For this purpose, by differentiating both sides of (11) with respect to t and by isolating the rheonomic deformation accumulation rate $\dot{\varepsilon}^r$, we shall have the fundamental creep equation in the following form :

$$\dot{\varepsilon}^r = \frac{d}{dt}\Big\{\ \sigma_0\Big[\ 1 + G(t)\ \Big]\Big\}\ \left[\frac{\varphi_0(\varepsilon^r)}{d\varepsilon^r}\right]^{-1} \qquad (12)$$

where the $\dfrac{\varphi_0(\varepsilon^r)}{d\varepsilon^r} := g_0(\varepsilon^r)$ value characterizes the intensity of initial strain ageing of the elastic-creep medium. As

$\varepsilon^r=\varepsilon_0 + \varepsilon^c$, the $\dot{\varepsilon}^c$ creep rate equation will completely coincide with (12). The feature of the fundamental equation(12) consists in that the $\dot{\varepsilon}^c$ creep rate is assigned as the function of the g_0 instantateous tangent modulus, depending on the current value of the creep flow, and variable operator of similarity 1+G(t). In many cases the G(t) function which correlates well with the experiment, is assigned as the exponential dependence $G(t)=at^b$ (a>0; 0<b≤1).

Determination of Model Parameters. To calculate the creep strain and time-to-failure within the framework of assumed model, coefficients a and b have to be defined in advance. It can be done using some different techniques. However, the most preferable techniques is one with the momentary stress-strain diagram and two isohron curves being used as source data. Then, the coefficients are determined from the following system of equations:

$$\lg a + b \lg t_1 = \lg\left(\frac{\varphi_0(\varepsilon^s_j)}{\varphi_{t_1}(\varepsilon^r_j)} - 1\right) ,\ j = \overline{1,m}$$

$$\lg a + b \lg t_2 = \lg\left(\frac{\varphi_0(\varepsilon^s_j)}{\varphi_{t_2}(\varepsilon^r_j)} - 1\right) ,\ j = \overline{m+1,p} , \qquad (13)$$

where $\varphi_{t_1}, \varphi_{t_2}$ are functions assigning the isochrones, corresponding to the time t_1 and t_2 , $|\varepsilon^s_j| = |\varepsilon^r_j|$, $j = \overline{1,p}$ In order to approximate the momentary stress-strain diagram in the $[\varepsilon^0, \varepsilon^s_N]$($\varepsilon^0$=0, $\varepsilon^s_N \le \varepsilon_s$), let us use the interpolation cubic spline which in the $\varepsilon^s_i, \varepsilon^s_{i+1}$ ($\varepsilon^s_i=\varepsilon^0+ih$, $i=\overline{0,N-1}$, $h=(\varepsilon^s_N-\varepsilon^0)/N$) nodes takes the σ_i, σ_{i+1} values, respectively, in the $[\varepsilon^s_i, \varepsilon^s_{i+1}]$ partial segment is desribed by the following expression:

$$\varphi_0(\varepsilon^s) = \frac{(\varepsilon^s_{i+1} - \varepsilon^s)(2(\varepsilon^s - \varepsilon^s_i) + h)}{h^3}\sigma_i + \frac{(\varepsilon^s_{i+1} - \varepsilon^s)^2\ (\varepsilon^s - \varepsilon^s_i)}{h^2}m_i +$$

$$+ \frac{(\varepsilon^s - \varepsilon^s_i)(2(\varepsilon^s_{i+1} - \varepsilon^s) + h)}{h^3}\sigma_{i+1} + \frac{(\varepsilon^s - \varepsilon^s_i)^2\ (\varepsilon^s - \varepsilon^s_{i+1})}{h^2}m_{i+1} \qquad (14)$$

where $m_i = \varphi'_0(\varepsilon^s_i)$ is the spline slope in the ε^s point. They are determined from the condition of the second derivative continuity which comes to the solving the linear equations system.

Fig. 1,a shows the creep flow calculation by the model (12) (dash-and-dot line) using the obtained coefficients. On the whole, as can be seen, the model describes all the three creep stages and correlates quite satisfactorily with the experiment.

<u>Calculation of the Long-term Strength of Materials.</u> In terms of the constructed model the moment of the loss of deformation stability occurs under the condition $g_0(\varepsilon^r) \to 0$, which in real material develops at the deformations equal to ε_B. Hence, from (12) we shall have $\dot{\varepsilon}^0 \to \infty$. In this case creep is completed by the accelerated stage, and fracture occurs with necking. Thus, by assuming the value of deformation accumulated by the moment of time t_R to be $\varepsilon_R^r \simeq \varepsilon_B$, we shall have

$$t_R = \left(\frac{\varphi_0(\varepsilon_B) - \sigma_0}{a\sigma_0} \right)^{1/n} = \left(\frac{\sigma_B - \sigma_0}{a\sigma_0} \right)^{1/n} \qquad (15)$$

The results of calculation performed by the (15) model for some of the studied materials are compared with the experiment in Fig. 1 (dashed line with three dots). On the whole, as can be seen, the correlation is quite satisfactory.

CONCLUSIONS

The N.Hoff creep concept can be used for forecasting the long-term strength of materials and calculation of the time-to-failure of the structural members in case of the highly ductile materials with ε_B = 20 % and higher.

In case of the low ductility materials (ε_B < 20%) it is rational to apply the creep model basing isochrones diagrams similarity principle, wich describes all the typical stages of creep (up to failure), and allows with the required degree of accuracy to reconstruct the creep curves by the results of the short-term tests at the stage of the unsteady-state creep.

REFERENCES

Golub V.P. (1989). To the theory of creep of isotropic initially strengthening media. J. Soviet Appl. Mech., Vol. 25, N 2, p. 90-100 (in Russian).

Hencky H. (1925). Uber langsame stationare Stromungen in plastischen Massen mit Rucksicht auf die Vorgange beim Walzen. Pressen und Ziechen von Metallen, Bd 25, H. 2, s.115-124.

Hoff N.J. (1953). The necking and rupture of rods subjected to constant tensile loads. J.Appl.Mech., Vol.20,N 1.p.105-108.

Odqwist F.K. (1966). Mathematical theory of creep and creep rupture. Clarendon Press,Oxford.

Rabotnov Yu. N. (1966). Creep of structural members. Nauka, Moscow (in Russian).