DETERMINATION OF PARAMETERS FOR NUMERICAL MODELS OF CONCRETE FRACTURE

J.G.M. VAN MIER and E. SCHLANGEN

Delft University of Technology, Stevin Laboratory
P.O. Box 5048, 2600 GA Delft, The Netherlands

ABSTRACT

In the past decades many numerical models for concrete fracture have been developed. Important is that the model parameters can be determined in a simple and well defined manner. Because of the structural effects in fracture experiments, viz. the influence of specimen size, boundary conditions and loading sequence, a direct determination of macroscopic fracture parameters seems impossible. In the paper an alternative method of deriving fracture parameters is described. The use of micromechanical models seems a promissing tool. In such models the material structure is schematized directly. For concrete usually a two-phase material is modelled. The fracture law is very simple and realistic fracture response can be simulated. Because a small number of single valued fracture parameters suffices in the micromechanics models, tuning the model to experimental data is relatively easy. Important is that crack shapes and fracture mechanisms are simulated with a high degree of accuracy.

KEYWORDS

Concrete, Fracture, Softening, Numerical modelling, Material parameters, Testing, Lattice Model, Tension, Compression.

INTRODUCTION

Fracture of Concrete is a highly non-linear phenomenon. The stress-strain curve measured for example in tension shows an ascending branch, a peak at which the tensile strength of the material is defined, and a descending branch or softening branch. The post-peak behaviour can only be measured if a stiff servo-controlled testing machine is available. Experiments have shown that beyond peak localization of deformations occurs. This implies that a definition of strain becomes invalid in the post-peak regime. Hillerborg and co-workers[1976] were the first to appreciate this distinct post-peak behaviour of concrete in tension. They proposed a macroscopic fracture model for the material, viz. the Fictitious Crack Model, which is based on stresses and deformations rather than on stresses and strains, see Figure 1.

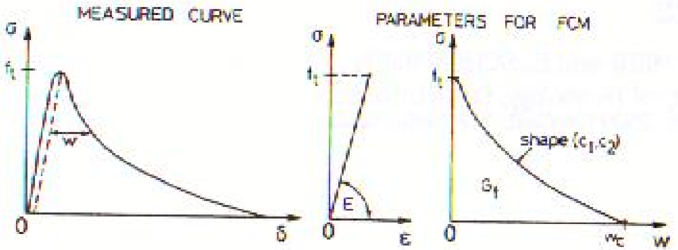

Fig. 1. Parameters for the Fictitious Crack Model, Hillerborg et al.[1976]

More recently it was shown that fracture in compression is a localized phenomenon . ell, see Van Mier[1984]. In principle this means that a similar reasoning might be followed for modelling fracture of concrete in compression. Hillerborg[1990] worked out the idea for compressive fracture and found favourable results for the rotational capacity of reinforced concrete beams. Decisive in such analysis is however that the input parameters can be measured to a high degree of accuracy.

The measurement of the model parameters may cause some problems. For determining the parameters needed in the Fictitious Crack Model, Hillerborg proposed to derive them from a stable displacement controlled uniaxial tension test. However, two stability criteria should be fulfilled before the measurement result can be regarded as a property of the material. In tension this seems to some extent possible, although uncertainties remain. In compression the same reasoning may lead to considerable problems. In this paper, the validity of parameters proposed in macroscopic models is debated.

A promising tool that may be used to overcome some of the experimental problems is the use of (numerical) micromechanics models. In such models, the concrete is modelled as a two-phase material (particles of material 1 are embedded in a matrix of material 2). Fracturing of the material can be modelled in a very simple and straightforward manner, and boundary condition effects and size effects become an integral part of the modelization, see for example Vonk et al.[1991], and Schlangen & Van Mier[1991]. The question remains however how the model parameters should be determined. An improvement to the macroscopic models seems that the fracture parameters are single valued, and do not depend anymore on the loading history. It seems that the models, if properly tuned, may serve as a tool to derive the parameters for a macroscopic fracture model. Important in deriving the parameters for the micromechanical models is that the fracture mechanisms and crack patterns can be simulated to a high degree of accuracy.

MACROSCOPIC FRACTURE MODELS

As mentioned, Hillerborg and coworkers were the first to derive a non-linear macroscopic fracture model for concrete. The model is sketched in Figure 1. Hillerborg recommended to

measure the relevant parameters in a stable displacement controlled uniaxial tensile test. The result should be separated in a pre-peak stress-strain curve, which is normally assumed to be linear with stiffness E, up to the tensile strength f_t. Beyond peak a stress-crack opening diagram is defined. The most important parameter is the shape of the diagram, which may for example be fitted using the following equation (Hordijk[1991]):

$$\sigma/f_t = \{ 1 + (c_1 * w/w_c)^3 \} * \exp(-c_2 * w/w_c) - w/w_c * (1 + c_1^3) * \exp(-c_2) \quad ...(1),$$

where f_t is the tensile strength of the material, w_c is the maximum crack opening when no stress is transfered anymore (at the end of the tail of the tensile softening diagram), and c_1 and c_2 are two empirical constants. In summary, the following parameters are needed for a full description of the *tensile* fracture relationship: the Young's modulus E, the tensile strength f_t, the shape of the descending branch (eq. (1)), the maximum crack opening w_c and two empirical constants c_1 and c_2. From the post-peak diagram of Figure 1, the fracture energy G_f can be derived following $G_f = \int \sigma dw$. This is assumed to be a material property. The various parameters change for different concrete compositions.
Many different models have been proposed for describing the tensile stress-deformation diagram of concrete. Another good example is the microplane model, developed by Bažant & Prat[1988] and Ožbolt & Bažant[1991]. In this model 13 empirical material constants are needed in order to describe the tensile stress-strain behaviour of concrete. No test method is described for determining these parameters, and in view of the abundance of parameters it is no surprise that the tensile stress-strain behaviour is described to a high degree of accuracy.

Up till now there seem to be no problems. However, the correctness of the above fitting procedures relies on the accuracy with which the tensile stress-deformation diagram can be measured. It should be mentioned that the same is true for compression, but in that case the situation is much more complicated because friction plays an important role.

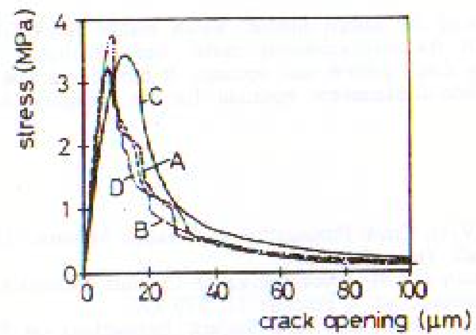

Fig. 2. Stress-crack opening diagrams for specimens of different size, Hordijk[1991].

For the measurement of the stress-deformation diagram, especially the post-peak part of the curve, two stability criteria must be fulfilled before the response curve approximates the 'true' behaviour of the material. First of all a stable servo-controlled test set-up is needed, and secondly a small specimen should be used in order to avoid, as much as possible, the non-uniform cracking of the specimen, see Van Mier[1991a] and Hordijk[1991]. Experiments have shown that non-uniformities in the region just beyond peak can be relatively large, depending on the rotational stiffness of the testing machine and the dimensions of the specimen. In Figure 2 some experimental results obtained by Hordijk[1991] are shown. Concrete prisms of different length and constant cross-sectional area (50 x 60 mm) were tested in uniaxial tension. The specimen length was 250, 125 and 50 mm for specimens A, B and C respectively. In addition to these tests, a long prism with reduced cross-section was tested as well (type D, 250 x 40 x 50 mm). In the specimens two 5 mm deep notches were sawn at half height, thereby reducing the effective cross-sectional area to 50 x 50 mm (for types A, B and C) and 40 x 40 mm for type D. A stable softening curve was measured by using the average deformation measured near the notched area over a measurement length of 35 mm as a control parameter. During a test, the loading platen were kept parallel to each other. Notably is the discontinuous shape of the post-peak curve for tests A, B and D. When the specimen length is reduced to 50 mm (type C), this 'bumpy' behaviour seems to disappear, but now a much larger curvature around the peak is obtained. The response of tests A, B and D can be explained from the fact that not the entire cross-section will fracture at the same moment, but rather the growth of a macroscopic crack through the specimens cross-section will start from one of the notches, even when the specimens are tested between non-rotating end-platens. Increasing the width of the specimen, while maintaining the same length, has a significant effect on the measured post-peak response as well, Van Mier[1991a]. The reason for this typical post-peak behaviour is that the specimen is loaded more and more eccentric when the crack zone traverses the specimen. Consequently a closing bending moment will act on the crack zone because the specimens ends are forced to remain parallel, see Van Mier[1991a]. This bending moment tends to arrest the propagating crack-zone. Note that the distinct pre-peak behaviour of test C in Fig. 2 cannot be explained with the Fictitious Crack Model. In this model pre-peak behaviour is assumed linear-elastic, see Fig. 1.

Clearly, the shape and size of the specimen affect the shape of the descending branch, and it does not seem straightforward to use the stress-deformation curve as a material property. The same holds for the tensile strength of the material. This parameter is highly sensitive to eccentricities in the test set-up (Zhou[1988]), non-uniform drying (Hordijk[1991]) and specimen geometry (Van Mier[1991a]).

Currently, efforts are made to extend the models for curvilinear crack growth due to combined tensile and shear loading (Hassanzadeh[1992]). However, recent tests have shown that as soon as the stress-state changes from uniaxial to biaxial, path dependent behaviour is obtained, see Van Mier et al.[1991]. This last point indicates strongly *that an unrestricted use of the so-called macroscopic fracture properties that are derived directly from experiments is not allowed.*

MICROMECHANICAL MODELLING OF FRACTURE

Numerical micromechanics may prove to be a helpful tool in understanding the fracture process of concrete in detail. The findings from such simulations might be used in the development of macroscopic fracture models. Following this procedure, *the specimen is regarded as a structure: the material structure is modelled in detail and the boundary conditions in the experiment are taken into account.* In this way the objections against fitting of macroscopic measurement results as described above is circumvented. The method seems however only useful if the parameters needed at the micro-level are more simple than those

at the macro-level. It should be mentioned that numerical micromechanics has become interesting only after the development of powerful computers. The computational effort is in general tremendous because the specimen is modelled in great detail.

Several different models have been proposed in the past few years. Two different types of models can be distinguished: (1) models based on finite element methods, and (2) lattice models. Examples of the first category are Roelfstra[1989], Willam et al.[1989], and Vonk et al.[1991]. Models in the second category were for example developed by Schorn and Rode[1989], Bažant et al.[1990] and Schlangen & Van Mier[1991].
Ingredients for all the models are a particle generator for simulating the two-phase structure of the concrete, and a simple fracture law. The models of the first category can generally be used for simulating tensile and compressive fracture, the lattice models are suited only for simulations of tensile fracture.

In the model developed by Vonk, the particle structure is derived from a regular grid of hexagonal aggregate particles embedded in a matrix with different properties. The hexagonal grid is subsequently distorted in a random manner. Interface elements determine the connectivity between the finite elements. Interfaces are defined in the matrix and at the boundary between aggregate particles and matrix. Simple interface elements were developed with two degrees of freedom. Fracturing of the composite is simulated using a simple fracture law for the interface elements. In Figure 3 a generated mesh and the fracture law for the interfaces are shown. The fracture law is a modified Mohr-Coulomb criterion. Tensile fracturing of the interface is governed by the tensile strength and a linear softening law. Shear fracture is governed by the cohesion c and also a linear softening law. The tensile softening is extremely brittle, w_c = 20 μm, for shear softening a larger maximum sliding deformation is selected, viz. in the order of 1 mm. Next to the parameters for the Mohr-Coulomb criterion, the elastic moduli for the aggregate and the matrix elements must be specified, the Poisson's ratio, and the initial stiffness of the interface elements. In total 14 parameters are needed, of which the parameters related to the linear softening are the most debatable. Introduction of softening at the micro-level leads to the same problems as mentioned before for the macroscopic models: the law cannot be determined in an experiment.

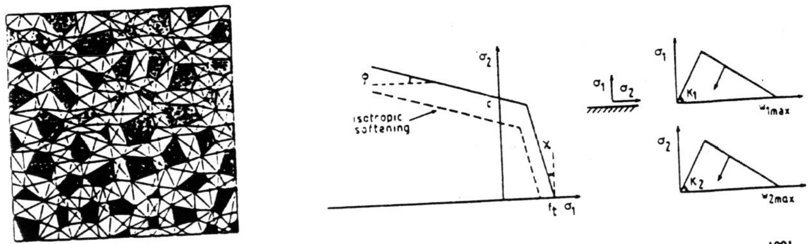

Fig. 3. Generated material structure and model parameters, after Vonk et al.[1991].

It seems that when the amount of detail in the generated particle structure of the concrete can be increased, that softening can be omitted altogether at the micro-level. In fact this was the reason for the development of a more refined, and as it turned out more simple, lattice model for concrete fracture, Schlangen & Van Mier[1991]. In this model a particle structure of the concrete is overlaid by a triangular network of purely brittle breaking beam elements. Different tensile strength and stiffness values have to be specified for beam elements that are situated inside an aggregate particle, in the matrix, or at the transition between particle and matrix as shown in Fig. 4. The fracture law for the beam elements is very simple. When the maximum stress in a beam is exceeded the beam is simply removed from the mesh and a new linear elastic analysis is performed. The stress in the beam is calculated following

$$\sigma = \beta \{ F/A + \alpha * (|M_i|, |M_j|)_{max}/W \} \quad ....(2),$$

where F is the normal force in the beam, M_i and M_j are the bending moments in the nodes i and j of the beam, A is the cross-section of the beam and $W = bh^2/6$. The factor α is introduced for selecting a failure mode where bending plays either a major or minor role. β is a scaling factor for the global maximum failure load. In summary, the parameters for the lattice model are the Young's moduli E_A, E_B and E_M for the aggregate, bond and matrix beams, the failure strengths σ_A, σ_B and σ_M for the respective beams, the cross-section of the beams b * h, and the factors α and β. The cross-sectional area of a beam element is determined from a linear-elastic analysis of the un-cracked lattice. The stiffness of the lattice should correspond to the stiffness of a concrete element of the same size. Thus, the number of parameters is relatively small. Only 8 parameters are needed, and most important of all, they are single valued. No softening is modelled at the micro-level.

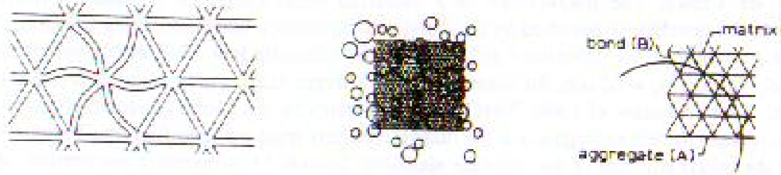

Fig. 4. Particle structure and overlay of beam elements in the lattice model.

In Figure 5 results of an analysis with the lattice model is shown. A uniaxial tension test is simulated, and the simulated crack patterns are compared with an experimental result obtained from a test on 2 mm mortar. More extensive comparisons are given in Schlangen & Van Mier[1991]. The result of Fig. 5 clearly shows that 'ductile' post-peak response is obtained by using a model with a purely brittle fracture law. The tail of the softening diagram can be explained from the development of intact ligaments connecting the crack faces.

The photograph in Fig. 5c is an example of crack face bridging in mortar at an average crack opening of 100 µm. The last crack stage in the simulation shows abundant crack face bridging as well. The amount of detail in the simulation can be increased by including smaller aggregate particles in the lattice. In that case the post-peak response will be less brittle. From the computational point of view this is however not very attractive. The main point is that with a simple fracture law at the micro-level, complex macroscopic response can be obtained. In Schlangen and Van Mier[1991] it is shown that the same model is capable of simulating curved crack growth in concrete as well, including the effect of boundary conditions on the fracture mechanism. This means that the path-dependent behaviour at the macro-level is circumvented, and is a direct consequence of the chosen modelization.

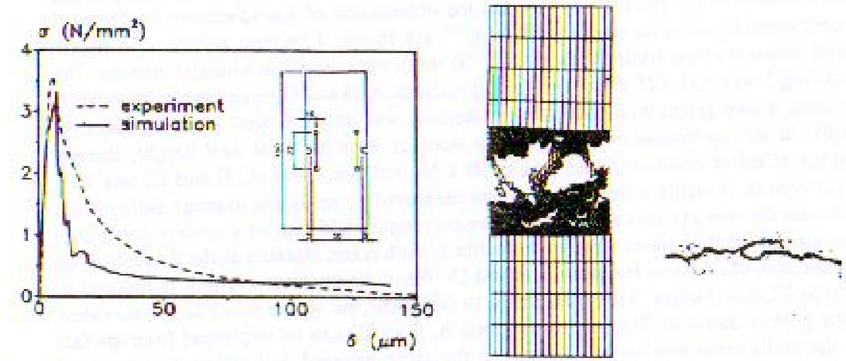

Fig. 5. Stress-crack opening diagrams (a), simulated crack pattern at the end of the stress-crack opening diagram (b), and example of crack face bridging in mortar (c).

The same type of crack face bridging is obtained with the micromechanics model developed by Vonk et al.[1991]. Moreover this model is capable of simulating correcly compressive fracture, including the effect of specimen size and boundary conditions. The stress-crack opening diagrams are more smooth in the latter simulations, probably because softening is introduced at the micro-level. Note that the simulations of Fig. 5 are two-dimensional representations of a three-dimensional crack process. It is the author's opinion that the stress-crack opening diagram will become more smooth in case of a full 3-D simulation.

In the micromodels, the material structure is modelled directly. For concrete this is relatively easy because it can be regarded as a two-phase material. The most important verification of the correctness of the simulations is the crack geometry, and its history. Consequently the problem is reduced to a geometrical problem. The determination of the initial geometry of the concrete material structure and the measurement of the crack geometry is in principle no problem.

CONCLUSION

In the paper the determination of parameters for macroscopic and microscopic fracture models for concrete is debated. In the macromodels no internal structure of the material is distinguished. In the micromodels, the internal structure of the concrete forms an essential ingredient. For the macromodels non-linear fracture laws are required, in the micromodels the fracture law can be described with a limited number of single valued parameters. The macromodels are path dependent under biaxial loading, whereas path dependency forms an integral part of the solution in the micromechanical approaches. Based on these observations, it can be concluded that the micromechanical models are suited very well for simulating the fracturing of heterogeneous materials like concrete in great detail. The outcome of these models may be used for tuning macroscopic fracture formulations, although the path-dependency under biaxial loading will remain a problem in the macromodels. A disadvantage of the microscopic models is that an enormous amount of computer time is needed. Hopefully this problem can be reduced in the future.

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