ANALYTICAL INVESTIGATIONS OF FRACTURE AND ACCOMPANYING THERMAL EFFECTS IN SUPERCONDUCTORS UNDER LORENTZ FORCES

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ABSTRACT

The mechanical disturbances in type I and II superconductors (SC) under a magnetic field and currents and some thermal effects as a result of local failure at low temperatures are studied. General statements are given by macroscopic approaches to superconductivity and brittle fracture. As for particular problems we focus our attention to stress concentration sites (the top of an angle, a contact zone,...). Besides we find the solution for a source of heat into SC and use it for the calculation of temperature around a small quickly arisen defect.

KEYWORDS

Superconductor, current, magnetic field, fracture, heat

INTRODUCTION

The discovery of superconductivity in the metaloxyder at nitric temperatures by Bednorz and Müller (1986) has stimulated the new explosive of attention to this phenomenon. The most values of current density $\sim 10^{11}$ A/m^2 have been recorded in very thin films. They predict fantastic values of critical magnetic field of some hundred tesla. Nevertheless, low temperature superconductivity remains in good while the critical field ~ 10-10^2T and the current $\sim 10^9$-10^{10}A/m^2 only. It keeps such kind of adventures as more technologies of materials. The large Lorentz forces created by the interaction between currents and magnetic field can cause stresses within SC more than limit stresses. Especially, the stress concentration points can be dangerous. In turn, the mechanical failure and fracture (plastic deformation, cracking,...) prevents to achieve the desirable currents and magnetic field because of heat emission. However, the stress-strain states of different SC specimens have not been studied enough. It is important not only for the prediction of fracture but also for the analysis of the SC-phase-stability because the critical temperature and magnetic field depend on the deformation (Gurevich et al., 1987). Just the present paper is devoted to failure in SC under Lorentz forces. The possible original stress fields induced, for example, by cooldown of composite SC can be taken into account by means of superposition. In contrary with the known stress analyses of SC we propose a general consideration without going into detail. Note should be taken, many parameters are not stable and until now the standard theory of both SC and fracture is absent. Therefore we restrict simple approaches. Such kind of type I SC as an angle, a cylinder, a sphere, a plate with an ellipse hole are considered. These results hold also for common conductors (winding) under skin-effects. The prediction of fracture near the edge of a SC film strip placing onto the boundary of an elastic dielectric angle and near the contact zone of type II SC cylinder pressed by Lorentz force to elastic substrate are obtained. The ultimate aim is to plot the surface of fracture in the space of magnetic induction-current. The temperature increments in the result of defect appearance is of great interest for SC stability. We receive some evaluations of it by using an automodel solution for a special nonlinear equation of thermoconductivity.

TYPE I SUPERCONDUCTOR

Let SC occupy a domain $\Omega \in \mathbb{R}^n$, $n=2,3$, $\partial\Omega=\Gamma$ and be surrounded by a matrix or vacuum $(\vartheta=\mathbb{R}_n-\bar{\Omega})$ with a background magnetic field B_a. Each media in mechanical aspect is elastic, brittle and homogeneous. According to the known approach to superconductivity of ideal type I SC the operating values of current j, temperature T and field B satisfy the conditions $j=0$ $(\Omega+\vartheta)$, $B=0$ (Ω), $B_\Gamma<B_c(T)$, $T<T_c$ where T_c and B_c are critical values. In other words the current concentrates onto Γ only, so that the complete magnetic field B eliminates within Ω (the Meissner effect). To point out that it is true under skin-effect for a common conductor (winding) too. The general problem separates into two more simple ones. The former is electro-magnetic one. After the solution of Von Neuman problem for Laplace equation with respect to the potential of the disturbed field $B-B_a$, Lorentz pressure p and j are found by formulae $\mu\mu_0 j=B_\Gamma$, $2\mu\mu_0 p=B_\Gamma^2$, where μ and μ_0 are magnetic permeability and constant. The latter problem consists in the study of the stress state under full contact conditions on Γ besides the jump of normal stress $[\sigma_n]=p$. In regular stress points in $\vartheta+\bar{\Omega}$ we consider usual criterion of strength including limit tension and shear stress parameters σ_* and τ_*. In singular points we make the same stress analysis but at the distance ρ_0 from these points, where ρ_0 is a core region radius. The radius is comparable with the characteristic size of structure to predetermine the damage (for metals $\rho_0 \sim 10^{-3}-10^{-4}$m). Of course, the other energy or force criteria can be used in case of need. Now we proceed to some particular cases assuming that the matrix is much more weak in comparison with SC.

The Field $B=(0,0,B_z)$ and a Plane Domain $\Omega(x,y)$. Pressure is constant on Γ. Therefore, in case of finite Ω the state of SC is hydrostatic and safety. Now let SC-plate contain an elliptic hole. Then the tension stresses appear and the limit value $B = (2\sigma_*\mu\mu_0\sqrt{2\rho_0/a})^{1/2}$ if $\rho_*=b/a \ll \rho_0^2$ for thin ellipse (a and b is half axes), $B=(2\mu\mu_0\sigma_* b/a)^{1/2}$ if $\rho_* \gg \rho_0$. As known the large tension stresses are produced within the winding under skin-effect.

The Field $B=(B_x,B_y,0)$. The former problem is equivalent to that of non cavitation infinite flow around rigid contour Γ by ideal incompressible liquid. Only compressive pressures are on Γ and shear stresses are dangerous for convex forms of SC. By known solution their small maximums for sphere and cylinder are reached in the center. For a SC angle β the asymptotics are

$$B \sim \rho^{\alpha}, \quad p \sim \rho^{2\alpha}, \quad \alpha=(\beta-\pi)/(2\pi-\beta) \qquad (\Gamma)$$

where ρ is the distance at the angle top. At $0<\beta<\pi$ the unlimited growth of B near the top lead to fracture of superconductivity. At $\beta>\pi$ p and B vanish at $\rho\to 0$ but within SC stretching singular stresses appear ($\sigma \sim \rho^{-\kappa}$, where $0<\kappa\le 1/2$). The calculations are shown that even in case of special points and real value ρ_0, B_c, σ_* breaking magnetic field B_f seems to be much more in comparison with common operating fields (which is well known for common points). Therefore, the above consideration is of more practical interest for skin-effect.

TYPE II SUPERCONDUCTOR

The following physical assumptions concerning the behaviour of the rigid type II SC can be advanced (Gurevich et al., 1987):

1. $\mu=1$, $B_{c1}<B<B_{c2}$, where $B_{c1,2}$ are the upper and lower critical magnetic induction; at the operating field $B>B_{c2}$ we deal with a normal conductor.
2. According to the concept of critical state by Bean and London a volt-ampere characteristic of rigid SC is $j=j_c E/E$, where E is the any small intensity of electric field and j_c is a constant at T=const (Bean concept).
3. As a rule, even in high pulsed magnetic field the quasistatics with respect to stress fields takes place. It can be broken only for very thin plates and shells.
4. Within a monocrystal oxide film onto a dielectric substrate the normal and shear stresses are distributed according to

a linear law through thickness, if the inequality $\mu_0 j_c h \ll B_a$ takes place. On the contact surface they achieve the values $(\vec{\tau},\sigma)=\mathbf{J}\times\mathbf{B}$, where $J=h\cdot j_c$, h is the thickness, **B** is a constant across the film section. We can ignore elasticity of the film because usually $h\le 10^{-4}$m. Hence, these stresses are the boundary conditions for an elastic problem for the dielectric.
5. In composite and microcomposite type II SC the average equations coincide with those for a homogeneous SC of effective electric and elastic parameters. So, j_c needs to be replaced by $j_s=\kappa_s\cdot j_c$ where κ_s is a volume contents of the SC component.
6. The mentioned brittle fracture approach is attracted below.

<u>Mathematical Model.</u> According to physical assumptions 1-6 we give a general statement of problem to determine magnetic and stress fields inside or/and outside type II SC (Ω). Currents j flows within partial volume $\Omega_j\subset\Omega$ and J on $\Gamma_J\subset\Gamma$. In both general case of SC-film and SC-volume the complete magnetic induction **B** can be represented as a sum of a particular solution of the Maxwell equations in the form of Biot-Savart law (contributions of currents) and the background field B_a=const.

$$\mathrm{div}\Sigma + \mathbf{F}=0, \quad \Sigma=\mathbf{C}\mathbb{E}, \quad \mathbb{E}=1/2\ \{\vec{\nabla}\mathbf{u}+(\vec{\nabla}\mathbf{u})^*\}, \quad \mathbf{F}=\mathbf{j}\times\mathbf{B} \quad (\Omega) \qquad (1)$$

$$\mathbf{B}(\mathbf{x},t)=\mathbf{B}_a+\frac{\mu_0}{\pi\varepsilon_n}\left\{\int_{\Omega_j}\frac{\mathbf{j}\times\mathbf{r}}{r^n}d\Omega_y+\int_{\Gamma_J}\frac{\mathbf{J}\times\mathbf{r}}{r^n}d\Gamma_y\right\}, \qquad \mathbf{r}=\mathbf{x}-\mathbf{y} \qquad (2)$$

$$\Sigma\cdot\mathbf{n}=\mathbf{J}\times\mathbf{B}+\mathbf{f}\ (\Gamma), \quad \mathbf{C}=\{C_{ijkl}\}, \quad \varepsilon_n=2,4\ (n=2,3), \quad \mu_0=4\pi 10^{-7}H/m \qquad (3)$$

Here $\mathbf{C},\Sigma,\mathbb{E},\mathbf{u},\mathbf{F}$ are the matrix of elastic moduli, the stress and deformation tensors, the vector of displacements and volume Lorentz force, the upper star denotes the transporniere operation, **n** is the outward normal to Γ, **f** is a vector of external loads, for example, contact balancing forces. If j and J are unknown functions then additional Maxwell equations and different relations for superconductivity need to be attracted. In this case we have a connected complex problem. However, often we can account j=const in Ω_j, j=const on Γ_J. Then the system (1)-(3) is complete and, moreover, divided. Relations (2) give the solution for the magnetic field, then we have to solve an elastic problem. The latter contains the monotype conditions if vector **f** is known. After all we can utilize the fracture criterion and find the fracture surface in space of parameters B_a,j,J.

<u>Elastic Dielectric Angle or Semi-Space with SC Film Strip.</u> Consider dielectric elastic angle $\rho\ge 0$, $0\le\theta\le\beta<\pi$ in the magnetic

field B_a. Part of its surface $\theta=\beta$, $0<\rho<1$ is covered by a SC film strip with high current J which is parallel to the edge line. Asymptotics of stresses at $\rho\to 0$ are completely defined by the local boundary conditions at $\beta\le\pi$ because any available eigenfunction (stress) as the solution of uniform problem disappears at $\rho\to 0$ as ρ^λ ($\lambda>0$ at $\beta\le\pi$). Guessing the particular solution of the singular problem, where non-zero conditions are the asymptotics of Lorentz tractions with the unusual ln singularity of shear stress, we find the main terms of stresses at $\rho\to 0$ with accuracy O(1):

$$\sigma_{r\theta}=(b\sin 2\theta+2a\sin^2\theta)\ln\rho, \quad \sigma_\theta=2\{a(\sin\theta\cos\theta-\theta)-b\sin^2\theta\}\ln\rho$$

$$a=-\tau_1\Delta\sin^2\beta, \; b=\tau_1\Delta(\beta-\sin\beta\cos\beta), \; \tau_1=J^2\mu_0/2\pi, \; \Delta^{-1}=2\sin^2\beta-\beta\sin 2\beta$$

If $\beta>\pi$ then $-1/2\le\lambda<0$ and the major eigenfunction with the coefficient depending on the problem as a whole must be added. The magnetic field **B** also has the same singularity. The character of singularities in stresses at $\beta\to 0$ are the same as in classical theory of thin plates. On the other hand at $\beta=\pi-\delta$, $\delta\to+0$ we see $b\sim\tau_1/2\delta$, the limits at $\delta\to+0$ are not regular and $\ln^2\rho$ appears at $\beta=\pi$ for the case of semi-space or near the other film edge:

$$\pi\sigma_{\rho\theta}\sim -1/2\ \tau_1\sin 2\theta\ \ln^2\rho, \quad \pi\sigma_\theta\sim\tau_1\sin^2\theta\ \ln^2\rho, \quad \rho\to 0, \quad 0\le\theta\le\pi$$

The fracture begins at having achieved one of the equalities $\sigma_\theta=\sigma_*$, $\sigma_{r\theta}=\tau_*$ ($\rho=\rho_0$, $0\le\theta\le\beta$) in case $h\ll\rho_0$. Else the above results are external expansions for future local analysis. The fracture surface depends on $\rho_0,\beta,\sigma_*,\tau_*$ for an angle and it is difficult to give general analysis. If one of the above limit equalities fulfills for semi-space then opening crack can be produced along the normal to the surface or shear crack at $\theta=\pi/4$, $3\pi/4$.

<u>Hertz Contact Problem for SC Cylinder.</u> Let a type II SC long elastic cylinder (cable of unit radius) rest on a dielectric elastic homogeneous semi-space. The rectangular coordinate system xy is connected with the center of the contact zone and the polar one ρ,θ with the center of the cylinder. The background magnetic field is $\mathbf{B}_a=(B_a,0,0)$ and the transport current flows with a constant density j through the whole section of the cylinder. Then Lorentz forces by volume $f_y=-B_a j$ press cylinder to dielectric. Another component $f_r=-1/2\ \mu_0\rho j^2$ from the current interaction directs to the center. Therefore, the main problem can be represented by superposition of two problems. The former consists in the determination of a stress state arising under the action of the internal forces f_r in the absence of any loading on the cylinder surface. It is similar to the problem of stress state in a rotating cylinder under the

inertia forces. Equilibrium equations in the latter problem should be solved in the absence of surface shear stresses and at non-zero normal stresses only in the contact zone. It is resolved by the method of asymptotic expansion matching at common assumption that the contact zone $|x|<a$ is small $(a\ll 1)$. So, in areas far from the contact zone it is similar to one of the known problem about the point force action on the self-weight balanced thin disk. Sum of the two solutions give a complete result. Stretching stresses are absent far from the contact strip, shear stresses are not very large but become the main at $B_a \to 0$, j=const. Both bodies near the contact zone can be considered as two elastic semi-planes. Then boundary stress and size of the contact zone are found from the solution of known Hertz problem. All of the stresses on the cylinder surface are known as well as the Green function for semi-plane. Consequently, the stresses near the small contact domain can be determined. Since the stresses in the plane problem are eliminated at the contact zone edge (contrary to 3-D case), only the solution in symmetrical plane x=0, where we expect maximum shear stresses, is demonstrated:

$$\sigma_x = -\frac{4P}{\pi a}\left(\eta_* - \eta - \frac{1}{2}\eta_*^{-1}\right), \quad \sigma_y = -\frac{2P}{\pi a \eta_*}, \quad \sigma_z = \nu(\sigma_x + \sigma_y)$$

$$\tau_1 = \frac{\sigma_y - \sigma_x}{2\tau_0} = -\gamma\eta\left(1 - \frac{\eta}{\eta_*}\right) + 1, \qquad \tau_2 = \frac{\sigma_y - \sigma_z}{2\tau_0} = -\gamma\left[\nu(\eta - \eta_*) + \frac{1}{2\eta_*}\right] + \nu, \qquad \tau_3 = \tau_2 - \tau_1$$

$$\eta = y/a, \quad \eta_* = \sqrt{1+\eta^2}, \quad \tau_0 = (1-2\nu)\mu_0 j^2/16(1-\nu), \quad v = (1-\nu^2)/E + (1-\nu'^2)/E'$$

$$\sigma_y(x,0) = -\frac{2P}{\pi a^2}\sqrt{a^2 - x^2}, \quad \gamma = \frac{16}{\mu_0}\frac{1-\nu}{1-2\nu}\sqrt{B_a/j^3 v}, \quad a = \sqrt{4B_a j v}, \quad P = \pi B_a j$$

Here E, E', ν, ν' are the elastic parameters of SC and substrate, τ_j are dimensionless complete shear stresses. At $\eta \sim 1$ the values of $|\tau_1|$, $|\tau_3|$ will be maximal. If γ is small then the maximums $|\tau_1|$ and $|\tau_3|$ are achieved at the center of contact strip. With increasing of γ the second extremum of function $|\tau_1(\eta)|$ placing at point η=0,8 will dominate.

TEMPERATURE FIELD BY FORMATION OF A DEFECT

It is well known the crack propagation gives rise to large growth of temperature near its tip. The increase of hundred degrees have been recorded. Especially, these effects is essential at low temperature because the degeneration of the heat parameters of materials at $T \to 0°K$. Namely, the heat capacity is described by the known latticed and electronic asymptotics:

$$C_v \approx \frac{12}{5}\pi^4 \frac{Nk}{A}\left(\frac{T}{\theta}\right)^3, \quad C_{el} \approx \frac{\pi^2}{2}\frac{Nk}{A}\frac{T}{T_F}$$

where N, A, k, θ, T_F is Avogadro number, atomic weight,

Boltzmann constant, Debye and Fermi temperatures. The simple estimation shows that at the adiabatic plastic deformation of metal beam (Al,Cu, $\varepsilon_p \sim 10^{-4}$-$10^{-2}$ and T_1=4,2°K) the increase of T is approximately $\Delta T \sim 10°$-$40°$. The mean increase of T through the plastic zone near a moving crack tip has the same order. The approximate increase of T near the suddenly formed defect within a material (crack, failure domain, plastic zone near a pore) can be found from the solution of problem about instantaneous concentrate (plane, line, point: n=0,1,2) heat source q. The value of q is equal the dissipate energy by the defect birth. For example, $q=\gamma$ for crack as a plane source and $q=2\gamma S$, where S is the square of produced crack, for crack as a point source. Let the media possess the nonlinear heat capacity $C_v = C_0 T^3$ and the known heat conductivity $\lambda = \lambda_0 T$. The initial temperature T_1 in comparison with increments of T is neglected. The automodel solution of the equation of thermocondutivity

$$\Phi_{,\tau} = x^{-n}(x^n \Phi^{1/2})_{,x}, \quad \tau = \kappa t, \quad \Phi^4 = T, \quad \kappa = 2\lambda_0/d_0 C_0, \quad n=0,1,2$$

where x, t is the coordinate and time, d_0 is the density, under conditions for the complete heat and the disappearance on the infinity is (Devjatkin and Simonov, 1992)

$$T^4 = A_n \tau^\eta \left(1 + \frac{A_n^{1/2} x^2}{(3-n)\tau^{2/(3-n)}}\right)^{-2}, \quad \eta = \frac{2n+2}{n-3}, \quad A_n = Q_n^{\frac{2}{3-n}} e_n^{-4}, \quad Q_n = \frac{4q}{C_0 d_0 \varepsilon_n}$$

$$e_0 = (\sqrt{3}\pi/4)^{1/3}, \quad e_1 = 1, \quad e_2 = \pi/4, \quad \varepsilon_0 = 1, \quad \varepsilon_1 = 2\pi, \quad \varepsilon_3 = 4\pi$$

$$T_*(x) = \frac{1}{2}\left[\frac{(3-n)^4 Q_n^2}{e_n^{6-2n}}\left(\frac{n+1}{x^2}\right)^{n+1}\right]^{1/8}, \quad x_*(T_*) = (n+1)^{\frac{1}{2}}\left[\frac{(3-n)^2 Q_n}{16\, e_n^{3-n}\, T_*^4}\right]^{\frac{1}{n+1}}$$

Some calculations of coordinate x_*, where $\max T = T_* = mT_1$ $(m>1)$ is a value of "dangerous" temperature by comparing with the initial temperature here, is represented below for typical epoxy resin in cryogenics.

T_1, r	1°, 1mm	1°, 0,1mm	0,1°, 1mm
m	x_*	x_*	x_*
3	3.8	0.83	83
5	1.9	0.43	42
10	-	-	17

Here r is radius of penny-shape crack as the point source of heat. If T_1=4.2° and (usually) T_*-T_1=0.1° than x_* can be found from classical fundamental solution of the linear equation with constant $C=C_0T_1^3$ and $\lambda=\lambda_0T_1$. It may be made because the zone of the great disturbances of T at $T_1\geq4^\circ K$ and $r_0<10mm$ localizes close by crack and $x_*(T_*=4.3^\circ) \gg r_0$. Then we calculate $x_*=1.5$, 7.0, 33 mm at $r_0=0.1$, 1, 10mm, i.e. really $x_*\gg r_0$. The above estimations corroborate our assumptions about the essential influence of heat at the microfracture of SC.

CONCLUSIONS

The main idea was to develop the theory of mechanical disturbances in different superconductors by combination of the general approaches from physics and mechanics. Then the different canonical and practical problems have been considered Some were suggested by physicists. The knowledge of the studied average stress fields are enough if a macroscopic failure theory is in force and are necessary before microscopic analysis for composites. The present results can be used for the choice of optimum configurations of SC, the estimations of their potential ability, the limit values of j and B and the temperature field near the small suddenly arisen defect and as the ground for future investigations.

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