THREE-DIMENSIONAL PROBLEMS ON CRACK GROWTH ON MECHANICAL AND DIFFUSION LOADING

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ABSTRACT

We introduce numerical treatment method of crack growth in solids. The loading of media with crack is controlled by process of gas diffusion into crack. We consider quasistationary kinetic crack growth regime with material's crack resistance characteristic being some function - the kinetic dependence of crack growth velocity (V) from stress intensity factor (K) on crack contour.Such problems model constractions fracture phenomena in conditions of aggressive media influence. They also model the processes of crack growth in media filled with gas, for example in geophysical media.

In those problems we take into account the change of pressure in the crack under gas diffusion as well as kinetics of crack growth under this pressure. Thus we solve the problem of diffusion theory and elasticity theory for media with cracks under some additional conditions which ensure the interaction between those problems.

We consider an infinite media with a crack occupying an arbitrary domain in plane. The three-dimensional diffusion theory and elasticity theory problems are reduced to two-dimensional integrodifferential equations in the crack domain. which are then solved by Boundary Element Method.

KEYWORDS

Crack growth, gas diffusion, kinetics, stress intensity factor, boundary element method.

BASIC EQUATIONS

We consider the problem on quasistationary normal crack growth. The crack occupies the domain G in plane z=0 and appears in moment t=0. (Fig.1).

The velocity of crack propagation V in each point of crack contour is considered to depend on stress intensity factor N: V=f(N) (Fig.2). We consider the crack in unboundary elasticity media which is saturated with gas of concentration c_0.

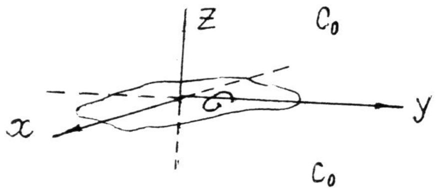

Fig.1

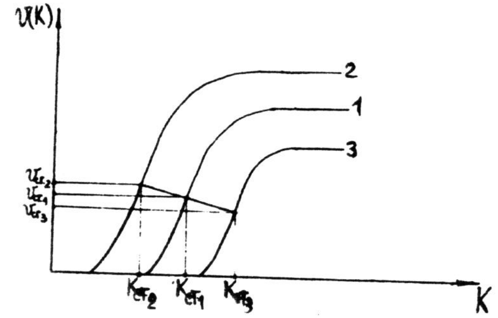

Fig.2

The crack is modeled of ideal flowing (far from equilibrium state). The crack propagation velocity is believed small in comparison on velocity of transition processes establishment. So the flow into crack is founded from stationary diffusion problem solution in each time moment t. The boundary conditions for the symmetric problem are

$$\Delta c = 0$$

$$c|_{z=0} = 0 , \quad (x,y) \in G$$

$$\partial c/\partial z|_{z=0} = 0 , \quad (x,y) \notin G$$

$$c|_{z=0} = C_0$$

Where c_0- gas concentration. We want to find the diffusion flow density $q(x,y)= \partial c/\partial z|_{z=0}$, $(x,y) \in G$.

The elasticity problem is the problem on normal crack G, which surfaces are applied on loading p - gas pressure. Gas pressure p is depend on growing crack volume V and pushed gas mass n. The gas is believed to be ideal and the processes are isothermic.

We reduce the elasticity and diffusion problems to boundary integral equations by standard meaning and obtain to solve the equations system in each time step t

$$C_0 = \frac{1}{2\pi}\iint \frac{q(x',y')\,dx'dy'}{\sqrt{(x-x')^2+(y-y')^2}} \tag{1}$$

$$p(t) = -\frac{E}{4\pi(1-\nu^2)}\Delta_{x,y}\iint_{G(t)} \frac{u(x',y')\,dx'dy'}{\sqrt{(x-x')^2+(y-y')^2}} \tag{2}$$

$$pV = nRT \tag{3}$$

$$V(t) = \iint u(x,y)\,dx\,dy \tag{4}$$

$$Q = -D\iint q(x,y)\,dx\,dy \tag{5}$$

$$n(t+\Delta t) = n(t) + Q\Delta t \tag{6}$$

$$u(\xi,s,t) = \frac{4(1-\nu^2)}{E} N(s,t)\sqrt{\xi} \tag{7}$$

where u - crack aperture, $Q=\partial n/\partial t$ - gas flowrate into the crack, R - universal gas constant, T - gas temperature, E and n - Young modulus and Puasson koefficient, the equations (7)-(9) are used for calculations of stress intensity factor N and new crack contour.

NUMERICAL PROCEDURE

We employ the boundary element method to solve equations (1) and (2). To determine u(x,y) and q(x,y) the original domain G is partitioned into equal squares with side h, and it is approximated by a net domain G(h). The solutions are sought in the form of the expansion

$$q(x,y) = \sum_{p_1p_2} Q_{p_1p_2}\Psi_{p_1p_2}(x,y,h), \quad u(x,y) = \sum_{p_1p_2} U_{p_1p_2}\Psi_{p_1p_2}(x,y,h)$$

$$\Psi_{p_1p_2}(x,y,h)=\begin{cases}(1-|x/h-p_1|)(1-|y/h-p_2|), & (x,y)\in g^{2h}_{p_1p_2}\\ 0, & (x,y)\notin g^{2h}_{p_1p_2}\end{cases}$$

$$g^{2h}_{p_1p_2}=\{x,y:|x/h-p_1|,|y/h-p_2|\le 1\}$$

where the unknown coefficients $Q_{p_1p_2}$ and $U_{p_1p_2}$ to be determined coincide with the values of $q(x,y)$ and $u(x,y)$ at the points of net. $Q_{p_1p_2}$ are founded from correspondent functional minimization

$$\min J_1(h)=\sum_{p_1p_2}\sum_{q_1q_2}a'_{p_1p_2q_1q_2}Q_{p_1p_2}Q_{q_1q_2}+2\sum_{p_1p_2}U_{p_1p_2}b'_{p_1p_2}$$

$$a'_{p_1p_2q_1q_2}=a_{|p_1-p_2|,|q_1-q_2|}=\frac{1}{C'(2\pi)^2}\iint_{-\infty}^{+\infty}\frac{1}{|\xi|}\Psi_{p_1p_2}(\xi,h)\overline{\Psi_{q_1q_2}(\xi,h)}d\xi$$

and $U_{p_1p_2}$ are founded from correspondent equations system solving (Balueva at al.,1989)

$$\sum_{q_1q_2}a_{p_1p_2q_1q_2}U_{q_1q_2}+b_{p_1p_2}=0$$

$$a_{p_1p_2q_1q_2}=\frac{1}{C(2\pi)^2}\iint|\xi|\Psi_{p_1p_2}\Psi_{q_1q_2}d\xi$$

In each time step t gas pressure p was determined at first. The crack aperture $u_1(x,y)$ was founded under p=1 for the crack contour G(t), then the. crack volume V_1 under p=1 was determined in formula (4) and p is equal $p^2=nRT/V_1$.
After this q(x,y), u(x,y) under p(t) were determined, new crack volume V(t), gas mass n(t), the stress intensity factor N(s,t), velocities V(s,t) in contour points s and new contour G(t+Dt) were founded. The new contour founding is special calculation problem (Goldstein at al., 1984).

RESULTS AND DISCUSSIONS

The series of models calculations was madefor penny-shaped cracks growth on different external conditions: the gas concentration c_0, kinetic dependence V(K). The crack growth time t_m from one radii R (R=1) to another (R=2) was calculated (Fig.3). The dependence of crack growth time t on gas concentration c_0 for different kinetic curves (Fig.2) is shown on Fig.4.

It is evident from Fig.4 that the growth time t_m is the smaller than the gas concentration c_0 is the bigger and the kinetic curve is the more left (on Fig.2).

We also made calculations for elliptic cracks with equalsquare but different $\lambda=a/b$ growth. For example, the elliptic crack growth with $\lambda=a/b=2$ is shown on Fig.5.

The crack growth time t_m as a function of $\lambda=a/b$ isnonmonotonic. The smallest time t_m is for elliptical cracks with $\lambda=a/b=2$. It is connected with nonmonotonic dependence of max stress intensity factor K_{max} (in points of crossing of smallest semiaxes with contour) from λ: K_{max} have maximum for the elliptic crack with $\lambda\sim 2$. It is established from numerical results that the crack growth regime becomes stationary with const velocity after some time. It is consequence of balance of crack sizes increasing and gas pressure decreasing (to be connected with crack volume increasing but particularly compensating gas flow into it). The stationary regime establishment is proved analytically for circular cracks and formula for the crack propagation velocity is given (Balueva, 1989).

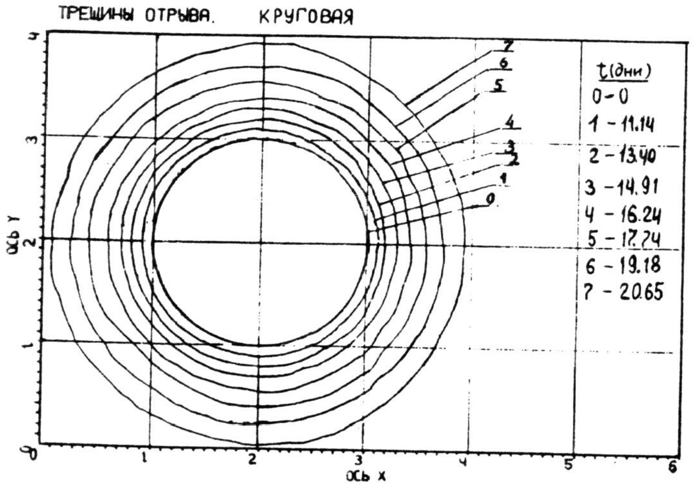

Fig.3.

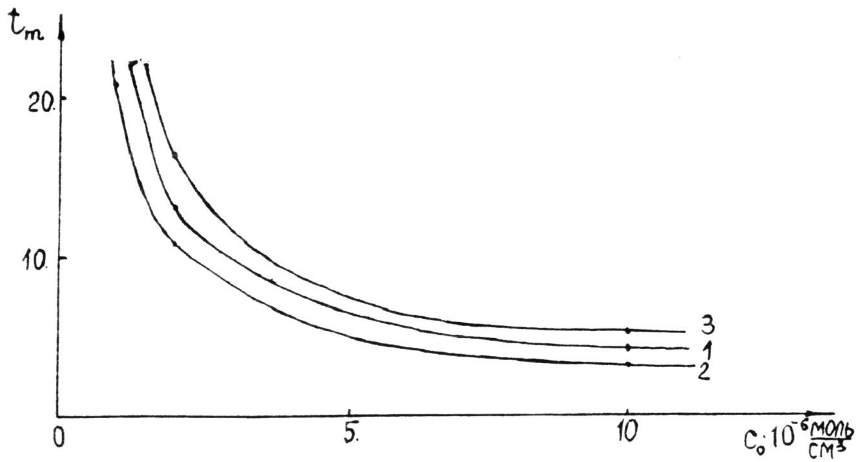

Fig.4.

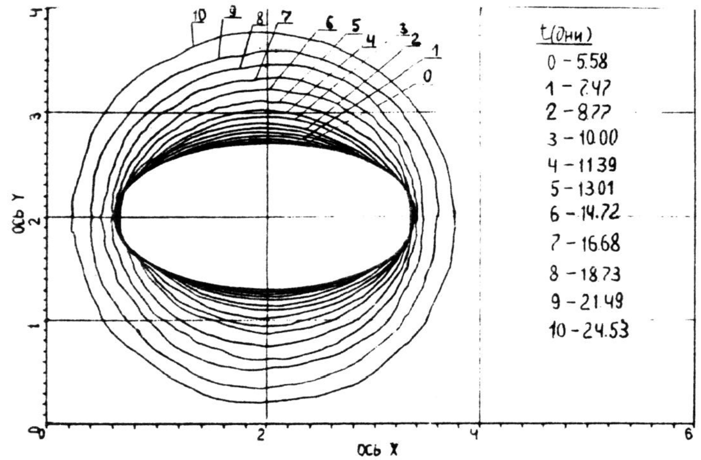

Fig.5.

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