THEORETICAL STUDIES IN KINETICS OF FATIGUE SURFACE FAILURE

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ABSTRACT

A model has been developed to study the fatigue wear process of surfaces. The rate of damage accumulation as function of the load and depth is shown to determine the character of the process. In some cases there are two types of failure: surface wear and undersurface disruption which has a character of delamination. Rates of surface, undersurface and total wear, moments of delamination, thickness of separating layers are determined numerically. It is shown that for steady-state stage of process within the framework of the model integral evaluation of total wear rate may be obtained analytically if the rate of damage accumulation is a known function.

KEYWORDS

Fatigue failure, wear, discrete contact, stress field, damage accumulation, delamination.

INTRODUCTION

Wear is a process of gradual (repeated) failure of surface layers of bodies in contact. Thus for wear process understanding we must involve in consideration contact mechanics and failure mechanics. In analysis based on contact mechanics specific feature of problems including wear is the progressive diminishing of the body size and the change of their shape, which results in redistribution of contact pressure, controlling in turn the wear process.

Specific feature of wear if considered in frames of failure mechanics is repeated character of the surface disruption (in contrast with design failure where the propagation of crack is as a rule identical to catastrophe). That is why an investigation of different modes of surface failure (such as fatigue, erosive, abrasive wear) is based on ideas of failure mechanics but the peculiarities of surface failure necessitate their modification and development. For example the theory of fracture was applied (Kolesnikov and Morozov,1989) in consideration of erosive wear (arising in interaction of a stream of abrasive particles and a surface of solid). An analysis of fatigue wear can also be based on the general theory of fatigue (Collins, 1981).

Fatigue failure of material occurs as a result of damage accumulation process during cyclic loading. Failure takes place when damage reaches a threshold level. Such understanding of fatigue mechanism is generally accepted. Although there are a lot of different physical approaches to damage concept, in calculation an increment of damage per load cycle is assumed to depend on a characteristic of stress field.

In this parer an approach using the concept of fatigue is applied for investigation of surface failure. We investigate here not the fatigue failure as physical phenomenon but the kinetics of the fatigue wear, which is analyzed provided that the function of damage accumulation rate is known.

THEORETICAL MODEL

We consider a wear of the half-space which is acted by a cyclic surface load. Oscillating undersurface stress field causes a damage accumulation process. Suppose rate of damage accumulation q=dQ/dt≥ is a function of amplitude value of the load P(t) and the distance ζ from surface of the half-space to a given point. Since the stress field vanishes at infinity,

$$\lim_{\zeta\to\infty} q(\zeta,P)=0.$$

We use a fixed cartesian reference frame with the origin on the surface at $T=T_0$ and directing the z-axis inside the half-space, the x- and y-axes along the half-space surface.

It will be shown below that the surface coordinate in wear process is a piecewise continuous function of time $\xi(T)$, where $\xi(T_0)=0$. For each time interval $[T_n, T_{n+1}]$ (n=0,1,2,...), where $\xi(T)$ is continuous we can determine damage accumulation function by the equation $(z>\xi(T))$

$$Q(z,T)=\int_{T_n}^{T} q(z-\xi(t),P(t))dt+Q_n(z) \qquad (1)$$

where $Q_n(z)=Q(z,T_n)$, $0\le Q_n(z)<1$.

Failure takes place the a point z^* at the time instant T^* in which it is satisfied the failure criterion

$$Q(z^*,T^*)=1 \qquad (2)$$

Consider the wear process from the initial time T_0(n=0). It appears from the equation (1) that the failure process is determined by $q(\zeta,P)$ and $Q_0(z)$ which we treat as continuous. If $q(\zeta,P)$ and $Q(z)$ are monotone decreasing with depth, the condition (2) is correct only for $z=\xi(T)$ (on the surface of the half-space) after the time instant T_1 $(\xi(T_1)=0)$.

Continuous change of linear size of the body $\xi(T)$ we shall characterize by the term "surface wear".

If one of the function $q(\zeta,P)$ and $Q(z)$ (or both of them) is not monotone decreasing with z, for example if this function has a maximum on some depth, the condition (2) may become correct for an internal point $z=z_1$ of the half-space. In this case at the time instant T_1 undersurface failure (separation of a layer of some thickness $\Delta z=z_1$) takes place which is then followed by continuous surface wear ;at $T=T_2$ undersurface disruption $(\Delta z_2=z_2-\xi(T_2-0)$, $z_2=\xi(T_2+0))$ takes place again etc. Hence $\xi(T)$ is a piecewise function in this case.

In each time interval $[T_n, T_{n+1}]$ (n=0,1,2,...) the function $Q(z,T)$ $(z>z_n)$ is determined from the equation (1). The following integral equation for surface wear rate in interval $[T_n, T_{n+1}]$ is derived based on (1) and (2)

$$\frac{dT}{d\xi}=-\frac{1}{q(0,P)}\left[\int_{z_n}^{\xi}\frac{\partial q(\xi-\zeta,P)}{\partial\xi}\frac{dT}{d\zeta}d\zeta+\frac{dQ_n(\xi)}{d\xi}\right] \qquad (3)$$

Equation (3) for P(t)=const is Volterra integral equation of the second kind which can be solved using Laplace-Karson transformation.

Thus if we know function $q(\zeta,P)$, $Q(z)$ it is possible to describe the kinetics of the surface failure.

FAILURE PROCESS IN CASE $q(\zeta,P)=K\tau_{max}^N$, P(t)=const

In case of complex stress field it is common practice to determine a rate of damage accumulation in terms of equivalent stresses (for example, maximum shear or maximum normal stress) which play the main part in the given kind of failure (Collins 1981).

For the sake of definiteness an amplitude value of the maximum shear stress τ_{max} will be used below as a criterion of damage accumulation, power law $q(\zeta,P)$ and τ_{max} relationship is postulated:

$$q(\zeta,P)=K\tau_{max}^N(\zeta,P) \qquad (4)$$

Parameters K and N can be determined in special frictional fatigue tests. There are data which demonstrate for several materials quantitative coincidence of parameters of surface and bulk fatigue failure (Kragelsky et al.) for this materials parameters K and N can be determined in standard fatigue tests.

We consider such oscillating stress field in an elastic half-space for which the function τ_{max} coincides with the one occurring along the vertical axis z of spherical indenter (R is its radius) contacting with an elastic half-space. It is approximatly valid for identic spherical indenters sliding without friction along the surface of half-space. Using an equation for τ_{max} (Hamilton and Goodman, 1966) and equation (4) we can write:

$$q(\zeta,P)=K\left[0.5p_0\psi\left(\frac{\zeta}{a}\right)\right]^N, \qquad a=\left[\frac{3PR}{4E^*}\right]^{1/3}, \quad p_0=\frac{3P}{2\pi a^2}, \qquad (5)$$

$$\psi(t)=1+\nu-1.5(1+t^2)^{-1}-(1+\nu)t\ \mathrm{arcctg}(t)$$

Specific features of the function $q(\zeta,P)$ which determine the disruption process are its nonmonotone character (presence of maximum) and the fact that $\lim_{\zeta\to\infty} q(\zeta,P)=0$.

In this case it is possible to consider damage Q(z,P) of material at each instant as to be the same for all points at the specified depth z. Thus disruption of the half-space has delamination character and the shape of the contact recovers every time.

Consider fracture process in case N=5, $Q_o(z)=0$. If P=const in dimensionless coordinates

$$z'=\frac{z}{a(P)}, \quad \theta=\frac{T}{T_m}\left[\frac{P}{P_m}\right]^N$$

the function $Q(z',\theta)$ does not depend on the load. Consequently an influence of the load shows itself only in modification of time and distance scales (in accordance with coordinate transformation).

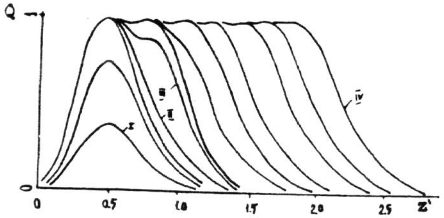


Fig.1 Damage accumulation function Q in failure process ($P(t)=P_m$, N=5).

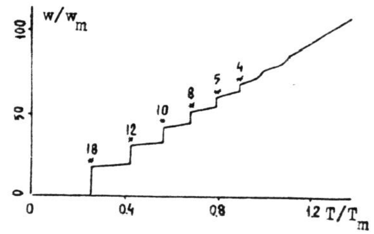


Fig.2 Wear process w(t) for $P(t)=P_m$, N=5.

The kinetics of the process was studied numerically. The function $Q(z',\theta)$ is depicted in Fig.1. Before the first act of disruption there is a typical shape (I) of $Q(z',\theta)$. After the first disruption $Q(z',\theta)$ is the monotone function (II) which has its maximum on the surface. In the process of damage accumulation a bending point appears at some depth

(III). When undersurface maximum value is equal to 1 the next act of disruption occurs etc. After six acts the undersurface disruption ends and the surface wear rate approaches to a constant. Then $Q(z',\theta)$ takes a shape which is characteristic for the surface wear (IV).

The wear process for this case is illustrated in Fig.2. The instants of undersurface disruption are marked with stars, numbers near the stars show the depth of disruption.

Calculations reveal the influence of the exponent N on the process. In case N=3 only one act of undersurface disruption occurs, if N=5.5 then 28 acts take place. However in case P=const there are common features of the fracture process. They are monotone diminishing of separating layers thickness, cessation of undersurface disruption, transit to the steady-state surface wear.

INFLUENCE OF P(t) ON THE FAILURE PROCESS

In real contacts P(t) has typical features as a result of the discrete contact area, waviness, periodic character of failure etc. We simulate it in a simple manner by periodic function P(t) and study its influence on the failure process.

Consider P(t) as the piecewise constant function with the period T_o:

$$\frac{P(t)}{P_m}=\begin{cases}1+\delta, & kT_o\le t<(k+0.5)T_o\\ 1-\delta, & (k+0.5)T_o\le t<(k+1)T_o\end{cases} \qquad k\in N$$

We have analyzed an influence of δ on the process. The results of calculation are depicted in Fig. 3(a,b,c) for $\delta=0.2, \delta=0.5, \delta=0.8$ ($T/T_m=2/5$). For the small δ the process is similar to that for the case $P(t)=$const, i.e. after several acts of undersurface desruption there is only surface wear (with its rate changing periodically) .When δ increases there is no more cessation of undersurface disruption. It is possible to divide the process in two stages : initial one when appearance of undersurface disruptions not the direct consequence of load change and the second one (steady-state stage) when the undersurface disruption occurs periodically with some delay after an instant of load increasing. A number of acts of disruption per each period depends on δ and increases with its growth.

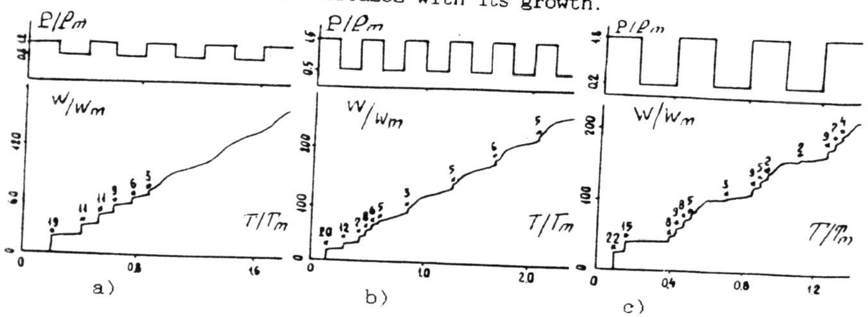
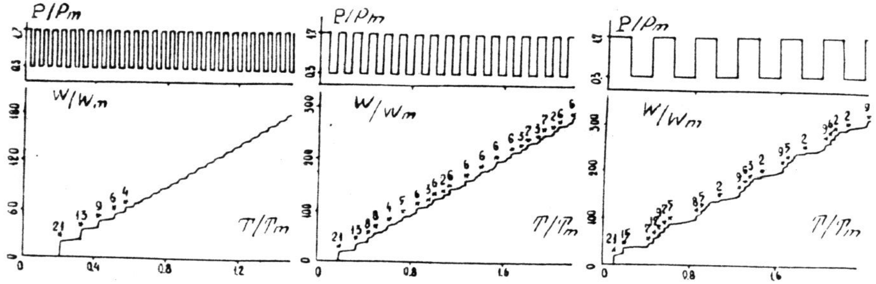


Fig.3 Wear process for different amplitudes of load variation

In Fig.4(a,b,c) you can see the failure process as a function of time in cases $T_o/T_m=1/20$, $T_o/T_m=2/15$, $T_o/T_m=2/5$ ($\delta=0.7$).

In spite of the fact the average value of $P^N(t)$ is the same for the three processes , there is qualitative difference between them. If the period is small the system "does not feel" the changes of P(t) and undersurface disruption ends (Fig.4a). When the period increases undersurface disruption does not end (Fig.4b). For greater periods it is periodic in accordance with function P(t).

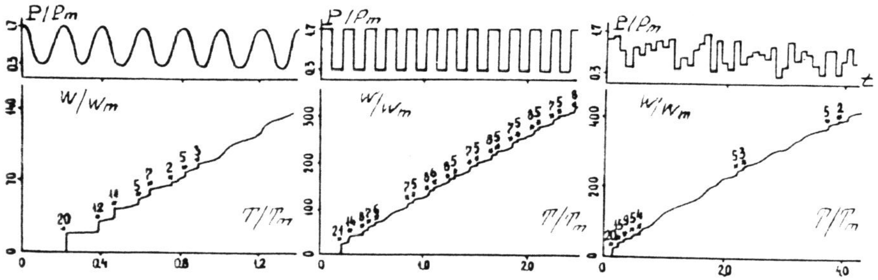
We have studied also the wear process when limits of changes and specific time intervals were the same ($\delta=0.7$, $T_o/T_m=1/5$) but the functions P(t) were different.



a) b) c)

Fig.4 Wear process for different periods of load variation

The results of calculations show that for smooth function $P(t)=1+\delta\cos(2\pi t/T_o)$ undersurface disruption ends (Fig.5a). In case of piecewise continuous function P(t) in the steady-state stage we have steady-state undersurface disruption (Fig.5b). Fig.5c shows the dependence of the wear versus time in case P(t) is a piecewise constant random function with uniform distribution on the interval [0.3,1.7]. In this case undersurface disruption does not end, the instants of its arising are correlated with the instants of significant jumps of P(t).



a) b) c)

Fig.5 Wear process for various types of function P(t)

STEADY-STATE STAGE CHARACTERISTICS

Determine total damage accumulated by material at an instant T as,

$$\Omega(T)=\int_{\xi(T)}^{\infty} Q(z,T)dz,$$

where $\xi(T)$ is the coordinate of surface which changes during the wear. $\Omega(T)$ decreases as a result of material separation $\Delta\Omega=-\Delta z\bar{Q}$ where $\bar{Q}$ is an average (over the time interval Δt) damage of separating material, Δz is its thickness. On the other hand $\Omega(T)$ increases in process of damage accumulation:

$$\Delta\Omega=\int_{T}^{T+\Delta t}\int_{0}^{\infty} q(z,P(t))dzdt$$

If P(T) is a periodic function with the period T_o, then on the steady-state stage of the wear process

$$\Omega(T+T_o)-\Omega(T)=-\Delta z\bar{Q}+\int_{T}^{T+T_o}\int_{0}^{\infty} q(z,P(t))dzdt=0.$$

Hence

$$\frac{\Delta z}{T_o}=\frac{1}{\bar{Q}}\int_{0}^{\infty}\bar{q}(z)\,dz, \qquad (6)$$

where $\bar{q}(z)=\frac{1}{T_o}\int_{T}^{T+T_o} q(z,P(t))dt$.

If there is only surface wear (Q=1) the average wear rate $\Delta z/T_o$ determined from the equation (6) has the minimum value. If an undersurface wear takes place also then Q<1,and the average wear rate is higher .As we can see from Fig.1 there is no great difference between $\bar{Q}$ and 1. Hence in a steady-state stage total wear rate in presence and absence of undersurface disruption will not greatly differ.

In the table 1 data are summarized on average rate of surface, undersurface and total wear for processes similar to ones shown in Fig.4 (a,b,c). The data confirm analytical predictions.

Table 1
Wear rate for piecewise-constant function P(t) $\delta=0.7$

T_o/T_m	1	0.4	0.2	0.13	0.04
surface wear rate	100	92.5	70	76	128
undersurface w.r.	30	45	62.5	59	0
total wear rate	130	137.5	132.5	135	128

CONCLUSIONS

The conclusions of the work can be formulated as follows.

1. Two types of failure process may take place depending on the character of function $q(\zeta,P)$: continuous surface wear (when failure condition is correct only on the surface), continuous surface wear which is accompanied by separation of layers of some thickness in discrete instants (delamination).

Within the framework of the model it is possible to determine numerically such characteristics of the process as rate of surface, undersurface and total wear, instants of delamination, thickness of separating layers.

2. Wear process correlates with function $P(t)$ (Fig.2-5). It is shown for the periodic function $P(t)$ (in particular for $P(t)$=const) that the process of wear may be divided in two stages: initial one and the steady-state one.

For the steady-state stage of wear an integral evaluation of total wear rate may be obtained analytically if $q(\zeta,P)$ is known.

The conclusions related to discontinuos nature of the fatigue failure of surfaces is qualitatively confirmed by the results (Cooper,Dowson, Fisher,1991) on testing the polymer material, used for production of joint prothesis (Ultra-high molecular weight polyethylene) in contact with steel pin(pin-disk tests) and by experimental results (Kragelsky, Nepomnyachy, 1965) on the frictional fatigue for a set of different rubbers (scheme of the experiment was analogous).

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