THE NUMERICAL ANALYSIS OF COMPOSITE MATERIALS DYNAMIC FRACTURE

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ABSTRACT

The special manner of finite difference solving technique for some dynamic loading behaviour and fracture problems of anysotrophic materials and constructions is proposed. The governing equation system in terms of displacements, which containes the second order derivatives, by the special replacement is transformed to the first order system of hyperbolic type with unknown displacements and their first derivatives with respect to the time and coordinates. The numerical example concerns the thin-walled composite constructions dynamic loading. The additional speciality of the developed method for the hyperbolic Timoshenko type equations is the separation of the contribution of the right side terms with large factors. The bearing ability of cracked composite shells can be determined according K_{IC} fracture criterion.

KEYWORDS

Composites, dynamic behaviour, non-linear fracture, thin-walled structures, grid-characteristic technique.

THE NONLINEAR DYNAMIC FRACTURE EQUATIONS

In brittle fracture of composites, crack can propogates at velocity large enough, so that the near-tip fields of stress and deformation are significantly influenced by elastodynamic effects.
Above this, the composites are such materials, which reveal the considerable nonlinearity under the off-axis loading and

if the crack rapidly propogates in some orthotropic composite material at some angle to the direction of the large elastic modulus then the dynamic and nonlinear effects are coupled.

The nonlinear behaviour is accounted by the model, based on the complementary energy density (Hahn and Tsai, 1973). Using this approach to the laminated composite the nonlinear stiffness matrice may be written as

$$[Q^{*k}_{ij}] = [Q^k_{ij}] + f(\varepsilon_{mn})[T(\varphi^k)] , \tag{1}$$

$$(i,j = 1,2,6; \quad n,m = x,y; \quad k = 1,\dots,N)$$

where $[Q^k_{ij}]$ - elastic stiffness matrice, $[T(\varphi^k)]$ - transformation matrice, taking into account the angle φ^k of fiber orientation in k-th laminae (N - the number of layers in laminate), and $f(\varphi_{mn})$ - the real root of some cubic equation.

Thus, consider the plane crack, which propogates in an orthotropic composite material at some angle to the direction of material axis, whith the velocity c(t) and, furthemore let $c(t) < C_R$, where C_R is the Rayleigh waves velocity. For the case of the in-plane fracture the system of displacement equations of motion is as follows (Zaytsev et al., 1991):

$$\begin{aligned} &Q^{*}_{11}\frac{\partial^2 u}{\partial x^2} + 2Q^{*}_{16}\frac{\partial^2 u}{\partial x\partial y} + Q^{*}_{66}\frac{\partial^2 u}{\partial y^2} + \\ &+ Q^{*}_{16}\frac{\partial^2 v}{\partial x^2} + (Q^{*}_{12} + Q^{*}_{66})\frac{\partial^2 v}{\partial x\partial y} + Q^{*}_{26}\frac{\partial^2 v}{\partial y^2} = \rho\frac{\partial^2 u}{\partial t^2} , \\ &Q^{*}_{16}\frac{\partial^2 u}{\partial x^2} + (Q^{*}_{12} + Q^{*}_{66})\frac{\partial^2 u}{\partial x\partial y} + Q^{*}_{26}\frac{\partial^2 u}{\partial y^2} + \\ &+ Q^{*}_{66}\frac{\partial^2 v}{\partial x^2} + 2Q^{*}_{26}\frac{\partial^2 v}{\partial x\partial y} + Q^{*}_{22}\frac{\partial^2 v}{\partial y^2} = \rho\frac{\partial^2 v}{\partial t^2} , \end{aligned} \tag{2}$$

where ρ - density; u, v - displacements along axes x and y, respectively. The origin of motion Cartesian coodinate system is placed at the crack tip and x-axis is directed along the failure trajectory. The complete formulation of the problem would be accomplished by appropriate initial and boundary conditions.

Thus, so called mathematical physics initial-boundary problem for nonlinear system of second order partial time and space coordinates derivatives differencial equations is achieved.

TRANSFORMATION OF THE GOVERNING EQUATIONS

Let's transform the governing equations system (2) to the first order hyperbolic system. Introduce the new vector-column of unknown variables

$$U = \left[\frac{\partial u}{\partial t}, \frac{\partial v}{\partial t}, \frac{\partial u}{\partial x}, \frac{\partial v}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial y}, u, v\right]^T . \tag{3}$$

The components of vector U are the displacements u, v and their first partial time t and space x and y coordinates derivatives. It must be emphased that after calculating U in given point at some time moment it is easy to obtain all the necessary deformation and stress parameters.

The substitution of vector (3) into system (2) leads to governing system in displacements:

$$\frac{\partial U}{\partial t} + A(U)\frac{\partial U}{\partial x} + B(U)\frac{\partial U}{\partial y} = F(t,x,y,U) \tag{4}$$

where A(U), B(U) are the square matrices of 8x8 elements, F(t,x,y,U) is the vector-column. There are coupled derivatives terms in system (2) and various forms of A and B exist, e.g.

$$A = \begin{bmatrix} 0 & 0 & -Q^{*}_{11} & -Q^{*}_{16} & -2Q^{*}_{16} & 0 & 0 & 0 \\ 0 & 0 & -Q^{*}_{16} & -Q^{*}_{66} & -(Q^{*}_{12}+Q^{*}_{66}) & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{5}$$

and

$$B = \begin{bmatrix} 0 & 0 & 0 & -(Q^{*}_{12}+Q^{*}_{66}) & -Q^{*}_{66} & -Q^{*}_{26} & 0 & 0 \\ 0 & 0 & 0 & -2Q^{*}_{26} & -Q^{*}_{26} & -Q^{*}_{22} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{6}$$

(Obviously there is some kind of symmetry and coupling among matrices A and B in the formulas (5) and (6)). The right side vector-column can be written as

$$F = \left[0,\ 0,\ 0,\ 0,\ 0\ ,\ 0,\ \frac{\partial u}{\partial t},\ \frac{\partial v}{\partial t}\right]^T .$$

The system (4) with matrices (5), (6) is a hyperbolic one because there are 8 real eigenvalues of matrice A and 8 real eigenvalues of matrice B, which can be found from the characteristic equations:

$$\det[A - \lambda^A E] = 0\ , \qquad \det[B - \lambda^B E] = 0$$

and also, there are the complete systems of the linear independent left eigenvectors-strings for A and B matrices, which can be obtained from the vector relationships:

$$\omega^i_A A = \lambda^A_i \omega^i_A\ , \qquad \omega^i_A B = \lambda^A_i \omega^i_A\ .$$

In other words, the characteristic decomposition of A and B matrices are:

$$A = \Omega^{-1}_A \Lambda_A \Omega_A\ , \qquad B = \Omega^{-1}_B \Lambda_B \Omega_B\ ,$$

where Λ_A , Λ_B - diagonal matrices from A and B eigenvalues; Ω_A , Ω_B - matrices, which have the appropriate eigenvectors as it's strings (Ω^{-1}_A, Ω^{-1}_B - inverse matrices to Ω_A , Ω_B ; E - unit matrice of 8x8 elements).

After the representation of problem as initial-boundary for the first order hyperbolic system (4) it may be utilized one of finite difference techniques with good numerical properties, e.g. that one, based on charachteristic properties of the governing system (Courant et al., 1952; Magomedov and Holodov, 1988, etc.)

FRACTURE CRITERION

The complex structure of composites and special conditions of their technology and exploitation cause the presence of various defects in material. The most dangerous among these defects is the crack. In order to predict the moment of crack starting under dynamic loading, it is possible to use the fracture criterion based on dynamic stress intensity factor. According

to this criterion the crack begins to propogate when the stress intensity factor at the crack tip K_I caused by external dynamic loads becomes more then some critical value K_{IC}. For the crack length 2l and tensile stress σ acting perpendicular to the crack direction the expression for K_I is

$$K_I = \sigma Y(l)^{1/2}, \tag{7}$$

where Y - the crack geometry depending function. The critical value K_{IC} for given composite material is determined from the tests - impact tension - and may be found from relationship suggested by the authors

$$K_{IC} = \tfrac{1}{2}\sigma(\pi)^{1/4}(Y)^{1/2}\Big(\delta_c + 2l(\beta_1 + \beta_2)\Big)^{1/2} .$$

Here δ_c - critical crack opening near the tip; β_1 , β_2 - imaginary parts of the roots of the characteristic equation for given composite.

THIN-WALLED COMPOSITE STRUCTURE

The development of the dynamic behaviour model of the thin-walled composite shell and it's numerical testing is based on the above described approach and governing equations introduced by E.V.Morozov (see Evseev and Morozov, 1992). The cracked shell bearing ability can be determined according K_{IC} fracture criterion.

One of the practical difficulties in the numerical analysis of Tymoshenko type systems by explicit methods is the presence on the right side of terms with large factors. There are strongly oscillating and temporary weakly damped harmonic components in the solution. For decreasing these negative additions the ideas of splitting method which makes it possible to separate out strongly oscillating components in a special manner so that stability of explicit schemes is guaranteed for the steps, determined by Courant conditions are used (for detailes see: Evseev and Semenov, 1990).

The Timoshenko type governing equations in dimensionless form for the stiff bounded semicircle toroidal ring under the out of plane deformation can be written from the above mentioned article (Evseev and Morozov, 1992) as follows

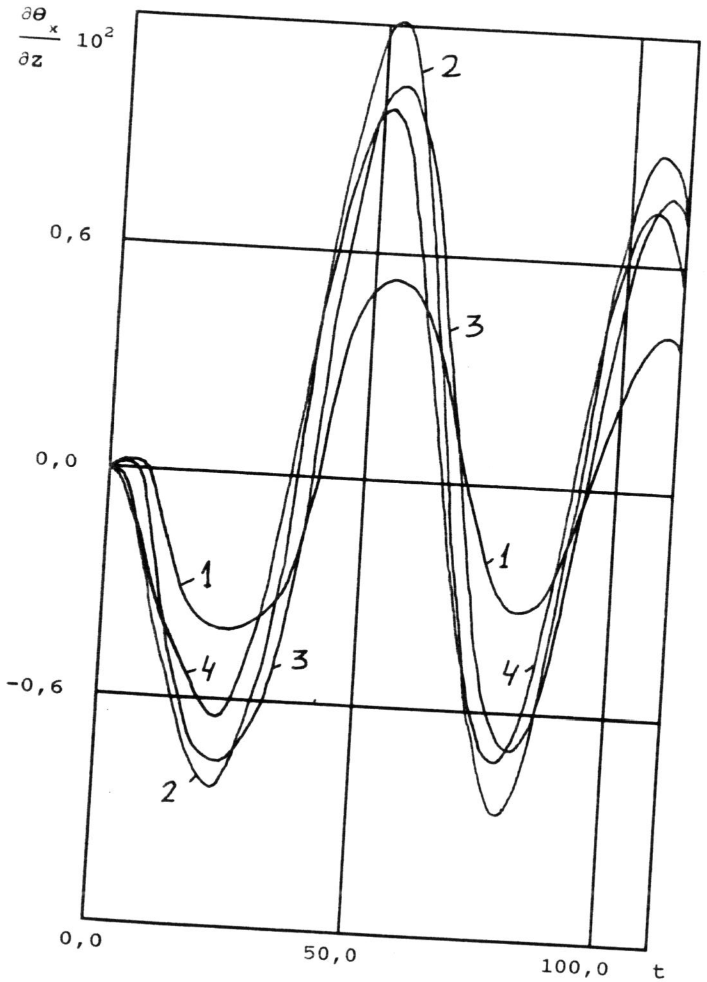

Figure. The time dependance of normal stress linear proportional dimensionless parameter.

$$\frac{\partial^2 v}{\partial t^2} - \xi^2\zeta^2\frac{\partial^2 v}{\partial z^2} = \xi^2\zeta^2\frac{\partial\theta_x}{\partial z} + q(t,z) \ ,$$

$$\frac{\partial^2 \theta_z}{\partial t^2} - \xi^2\frac{\partial^2 \theta_z}{\partial z^2} = \frac{1}{4}\eta^2\frac{\partial\theta_x}{\partial z} \ ,$$

$$\frac{\partial^2 \theta_x}{\partial t^2} - \eta^2\frac{\partial^2 \theta_x}{\partial z^2} = -2\xi^2\zeta^2\frac{R^2}{r^2}\frac{\partial v}{\partial z} + 2\xi^2\frac{\partial\theta_z}{\partial z} - 2\xi^2\zeta^2\frac{R^2}{r^2}\theta_x \ .$$

Here v and θ_x, θ_z - cross section centre displacement and it's rotations; q(t,z) - external load (z - angle coordinate which origin is located in symmetry plane); coeffitients $\xi^2 = G_{12}(1-\nu_{12}\nu_{21})/E_2$, $\eta^2 = 2(1-\nu_{12}\nu_{21})$, $\zeta^2 = 1/2\pi$; R and r - shell axes radius and cross section radius; G_{12} , ν_{12} , ν_{21} and E_2 - elastic constants.

Let the uniformly distributed external load q = 0,017 is instanteously applied to the shell with r/R = 0,002, ν_{12} = 0,3, ν_{21} = 0,15, E_2/G_{12} = 10,0. The calculated time dependent normal stresses linear proportional dimensionless parameter is depicted at Figure. The curves 1, 2, 3 and 4 are for the angles z = π/8, π/4, 3π/8 and 19π/40 respectively.

The bearing ability of the structure under consideration may be treated as a critical crack length by supposing that a through crack is placed at the most dangerous cross section with maximal normal stresses. The location of this section and stress values may be determined from the numerical results and according formula (7) the stress intensity factor K_I can be obtained as a crack length l function. By plotting the experimental value K_{IC} on graphic "K_I-l" the critical crack size may be determined.

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