SHORT CRACKS FRACTURE MECHANICS

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ABSTRACT

On the basis of an energy analysis a generalized fracture criterion is derived for a body with crack-like defects of various size. Using the criterion an expression for the fatigue crack growth rate is proposed. Observed and predicted results of the physically short fatigue crack behavior are discussed.

KEYWORDS

Energy analysis, fracture criterion, short fatigue cracks.

INTRODUCTION

Until recently the decrease of plain strain fracture toughness and the increase of fatigue crack growth rate (for ΔK=*const*) caused by the crack length decreasing was usually explained by means of the influence of the plastic zone at the crack tip and/or microstructural features.

However today we have the problem of physically short fatigue cracks, which are "long" in terms of continuum mechanics (*e.g.* linear elastic fracture mechanics (LEFM)) analyses, which also show higher propagation rates than correspondingly long cracks under the same nominal driving force. This therefore represents a limitation in the criterions basis of LEFM. This paper examines this problem.

A Fracture Criterion

From the first it should be given the quantity determination of the free surface energy density of the body γ for the reversible process

$$dR = \gamma\, dS, \quad (1)$$

where dR is the work spending on the reversible total body surface derivation dS.
The relationship (1)(compare to the Griffith one) represents the condition of the new surface appearance for the reversible process. For a cracked body of homogeneous and isotropic elastic material under static loading the expression (1) can be written in the form (Cherepanov,1977)

$$-d \int^{V} (\mathbf{W} - \sigma_{ij}\varepsilon_{ij} + \mathbf{K}\mathbf{E}_0) dV = \gamma\, dS \ , \quad (2)$$

where $\mathbf{W} = \int \sigma_{ij} d\varepsilon_{ij}$ is strain energy density, $\mathbf{K}$ and $\mathbf{E}_0$ are temperature and entropy per unit volume, respectively, S is the total body surface.

Here $\mathbf{G}_0 = (\mathbf{W} - \sigma_{ij}\varepsilon_{ij} + \mathbf{K}\mathbf{E}_0)$ is the free Gibbs energy per unit volume.

Notes: (i) The expression (2 is not only for fracture,but for deformation instability as well. The second case was not considered in this paper.(ii) The expression (2) is valid for the arbitrary body region. (iii) For the body loading process the volume V as well as the total body surface S are variable.

Under the condition of surface traction $\mathbf{F}_i = const$ or surface displacement $\mathbf{u}_i = const$ expression (2) can be written in the form (see the note (iii))

$$- d \int_0^{\mathbf{V}(\mathbf{S})} \mathbf{G}_0(\mathbf{S},\mathbf{V}) dV = \gamma\, dS \ . \quad (3)$$

The left part of the expression (3) represents the differential of the integral with the variable parameters. Using the Leibnitz rule we can derive

$$- \int^{V} \frac{\partial \mathbf{G}_0}{\partial S} dV - \bar{\mathbf{G}}_0 \frac{\partial V}{\partial S} = \gamma \ . \quad (4)$$

In the more general case the surface energy densities of a "smooth" surface of the loaded body and of a crack surface are different. So we should write the criterion (4) in the form

$$- \frac{1}{\gamma_{cr}} \int^{V} \frac{\partial \mathbf{G}_0}{\partial S} dV - \frac{1}{\gamma_{sm}} \bar{\mathbf{G}}_0 \frac{\partial V}{\partial S} = 1 \ , \quad (5)$$

where γ_{cr} and γ_{sm} are the effective surface energy densities of a crack surface and of a "smooth" surface of the loaded body.

The validity of the expression (5) will be shown below.

From experiments it is known that the entropy term is negligible for an elastic body. Taking this into account criterion (5) can be written in the form

$$\frac{1}{\gamma_{cr}} \int^{V} \frac{\partial \Omega}{\partial S} dV + \frac{1}{\gamma_{sm}} \bar{\Omega} \frac{\partial V}{\partial S} = 1 \ , \quad (6)$$

where $\Omega = \sigma_{ij}\varepsilon_{ij} - \mathbf{W}$.

For the two-dimensional case criterion (6) indicates the path dependence of the J-integral.

For the condition of unstable surface "extension",i.e. for $\mathbf{u}_i = const$ ("fixed-grips") the criterion expressed by (6) can be written in the form

$$- \frac{1}{\gamma_{cr}} \int^{V} \frac{\partial \mathbf{W}}{\partial S_{cr}} dV - \frac{1}{\gamma_{sm}} \bar{\mathbf{W}} \frac{\partial V}{\partial S_{sm}} = 1 \ , \quad (7)$$

In the criteria (4)-(7) $\bar{\mathbf{G}}, \bar{\Omega}$ and $\bar{\mathbf{W}}$ are the mean values per V.

Let us examine two extreme cases of criterion (7).
The first case: the surface of the body varies given by the crack surface variation ∂S_{cr} (i.e. dV=0).In this case we have the classical energy fracture mechanics criterion

$$- 2 \int^{V} \frac{\partial \mathbf{W}}{\partial S_{cr}} dV = 2\, \gamma_{cr} = \tilde{J}_c \ , \quad (8)$$

where $\tilde{J}_c$ is the limit critical value of the J-integral for the infinite crack.
The second case: the body has no crack. In this case we have the limit energy condition for a smooth body.
Hence

$$\mathbf{G}_{0f} = \gamma_{sm} / (- \frac{\partial V}{\partial S_{sm}}) \quad \text{from eq.(5)}$$

or

$$\mathbf{W}_f = \gamma_{sm} / (\frac{\partial V}{\partial S_{sm}}) \quad \text{from eq.(7)} \ , \quad (9)$$

where $\mathbf{G}_{0f}$ and $\mathbf{W}_f$ are the critical values of the $\mathbf{G}_0$ and $\mathbf{W}$ for the material of the smooth body.

Note that in the more general case we should write equation (8) from (5) in the form

$$2 \int^{V} \frac{\partial \Omega}{\partial S_{cr}} dv = 2\, (\gamma_{cr} \pm K \int^{V} \frac{\partial \mathbf{E}_0}{\partial S_{cr}} dv) = 2\gamma_{eff} \ ,$$

which is similar to the Orowan-Irwin representation of γ.

The damage parameter $\partial V/\partial S$ has the dimension of a length and represents the size effect too, i.e. the greater the parameter $\partial V/\partial S$ (i.e. the character body size) the greater contribution of the second term in the left hand parts of the criteria (4) to (7).

Using expressions (7), (8) and (9) we can suggest an engineering fracture criterion for a body with crack-like defects of various size

$$\frac{J_c}{\tilde{J}_c} + \frac{W_c}{W_f} = 1 , \qquad (10)$$

where J_c and W_c are respectively the critical values of the J-integral (calculated by the traditional methods) and the critical nominal strain-energy density for the body with a crack of finite size.

The criterion given by expression (10) enables one to describe some effects don't comming out the traditional fracture mechanics approach (Vasyutin,1988), e.g.:
(i)the crack size effect on the fracture toughness parameters;
(ii)the relation of COD to J_c;
(iii)the type of failure (e.g."leak before failure" condition).

Fatigue Crack Growth

Let us introduce the generalized dimensionless load for the cracked body

$$L = \frac{J}{\tilde{J}_c} + \frac{W}{W_f} . \qquad (11)$$

The expression for the fatigue crack growth rate may be proposed as follows (Vasyutin,1988)

$$da/dN = B\,(\Delta L_{eff})^{p} , \qquad (12)$$

where $\Delta L_{eff} = (\tilde{J}_{max} - \tilde{J}_{min} - \Delta J_{th})/\tilde{J}_c$, $\quad (\Delta J_{th} = \Delta K^2_{th(R=0)}/E')$

$$\frac{\tilde{J}_{max}}{\tilde{J}_c} = \frac{J_{max}}{\tilde{J}_c} + \frac{W_{max}}{W_f} ,$$

$$\frac{\tilde{J}_{min}}{\tilde{J}_c} = \frac{J_{min}}{\tilde{J}_c} + sgn(R)\frac{W_{min}}{W_c} , \quad (\text{for } R \leqslant 0, \; J_{min} = 0)$$

Let us examine two particular cases.
The first case: $\bar{\sigma} \leqslant 1$, $a \Rightarrow \infty$

$$\frac{da}{dN} = \frac{B}{\tilde{K}_c^{2p}} (K^2_{max} - K^2_{min} - \Delta K^2_{th(R=0)})^{p} , \qquad (13)$$

(for $R \leqslant 0$ $\quad K_{min} = 0$).

The second case: $\bar{\sigma} \leqslant 1$, $a \Rightarrow 0$

$$\frac{da}{dN} = B \left[\frac{\sigma^2_{max} - sgn(R)\sigma^2_{min}}{2\,E'\,W_f} - \frac{\Delta K^2_{th(R=0)}}{\tilde{K}^2_c} \right]^{p} \qquad (14)$$

From the first case the Paris relationship parameters can be derived as $C_{(R=0)} = B/\tilde{K}_c^{2p}$, $m_{(R=0)} = 2p$.

From expression (12) for $\sigma/\sigma_y \leqslant 1$ and the condition $da/dN = 0$ we can derive an expression for the Kitagawa-Takahashi diagram

$$\frac{\sigma^2_{max} - sgn(R)\sigma^2_{min}}{2E'W_f} + \frac{K^2_{max} - K^2_{min}}{\tilde{K}^2_c} = \frac{\Delta K^2_{th(R=0)}}{\tilde{K}^2_c} , \qquad (15)$$

or,in particular for R=-1 and using the relationship $K=\sigma\sqrt{a}\,Y$

$$\sigma_{-1d} = \sigma_{-1} (E'W_f Y^2 a / \tilde{K}^2_c + 1)^{-0.5} \qquad (16)$$

where σ_{-1d} is the fatigue limit of the elements with a crack size a, σ_{-1} is fatigue limit for R=-1, and Y is a dimensionless geometry function for the crack. Furthermore, the curves in coordinates da/dN versus $\sqrt{\Delta \tilde{J} E}$ are coinsidered, i.e. the derived relationship automatically allows for a stress ratio effect and the level of the fatigue crack growth rate, ΔK_{th} and the fatigue limit (Vasyutin,1991,1992).

Conclusions

(i) In this paper the author tries to present a new interpretation for the fracture mechanics energy approach which takes account of the total surface of a loaded cracked body;

(ii) This approach removes the general contradiction of the Griffith theory of an infinite nominal critical stress at the nil crack size situation;

(iii) This approach leads to a two-parameter fracture criteria for a cracked body. In particular, we may derive an engineering criterion given by equation (10).

(iv) The infinite body situation should be insensitive to a crack of limited size;

(v) The J-integral should be dependent on the value of the area S bounded by the path of integration Γ, i.e.
$\tilde{J} = J + \Omega\, \partial S/\partial\Gamma$.

The derived relationship for the fatigue crack growth rate allows one to derive the following

(i) a finite crack growth rate at zero crack length (i.e. at $\Delta K=0$), not using a fictional crack length;

(ii) using the condition $da/dN=0$ the fatigue limit of components with cracks of various size (the analytical expression for the Kitagawa-Takahashi diagram);

(iii) the effect of stress ratio (in particular the effect of compressive part of a cycle) and stress level on short fatigue crack kinetics and the fatigue limit.

References

Cherepanov G.P.(1977) Mechanics of brittle fructure. McGraw-Hill.

Vasyutin A.N.(1988) Strength criteria for a material containing short cracks. *Soviet materials science*, V.24,3, pp.273-279.

Vasyutin A.N.(1991) Fracture mechanics of physically short cracks. In:The Proceedings of the 10th Congress on Material Testing(Editor:Czoboly E.),Vol.1,pp.186-191,Budapest.

Vasyutin A.N.(1992) Fracture mechanics of physically short cracks. *Fatigue Fract.Engng Mater.Struct*, Vol.15,2,pp.203-212.