RUPTURE KINETICS AND FATIGUE LIFETIME OF CRACKED PLATES CONSIDERING TWO-STAGE FRACTURE

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ABSTRACT

The problem of rupture kinetics and lifetime computation of thin elastic plates with internal cracks under isothermal high cyclic loading is considered. The united theory considering the crack initiation and crack propagation stages is made up within the framework of Continuum Damage Mechanics principles. Fatigue lifetime is defined through the moment when structure is broken into some parts. Some fatigue problems for thin plates with internal cracks are solved.

KEYWORDS

Continuum Damage Mechanics, rupture kinetics, fatigue lifetime, plane problem, internal crack.

INTRODUCTION

Investigations of the fatigue crack growth are known among quite early researches in Fracture Mechanics. During the last twenty years many of experiments have been done and nowadays, two approaches of principle difference have been developed in strength theory for the fatigue life prediction of structural components.

The first engineering one has been based on stabilised stress-strain state analysis using parametric relations containing number of cycles up to destruction [Serensen et al., 1975]. As a matter of fact, the lifetime computation in this case is carried out on the crack initiation time in the most stressed point. The crack propagation stage is out of consideration and results therefore are always appear to be below the real value [Golub and Panteleyev, 1992].

Another approach has been based on modelling of the fatigue rupture kinetics. The linear and nonlinear Crack Mechanics methods and Continuum Damage Mechanics concepts have been spread mostly in that approach.

The linear Fatigue Crack Mechanics is virtually generalization of the Griffits-Irwin approach. Fatigue crack growth rate is obtained from empirical and semiempirical formulas received due to compact specimen tests covering the rate dependence from stress intensity coefficient range [Cherepanov, 1974; Panasjuk et al., 1977]. To account the crack nucleation stage a characteristic function determined from additional tests is taken into consideration [Andrejkiv, 1982].

In the nonlinear Fatigue Crack Mechanics the COD [Kaminskiy, 1990] and J-integral [Vardar, 1982] are in use as "moving force". In this case the crack nucleation stage can not be also predicted without additional hypothes. The crack nucleation conditions and crack propagation up to the length when Crack Mechanics principles could be used are out of consideration.

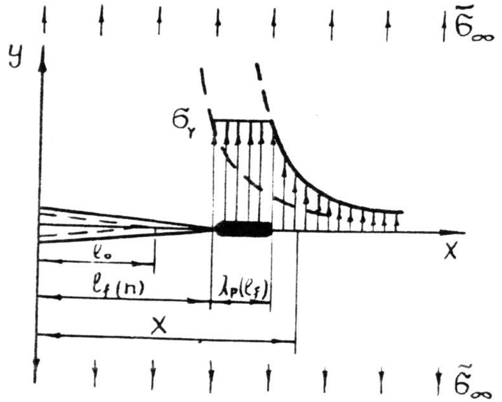

Figure I.

An approach based on the Continuum Damage Mechanics conception [Kachanov, 1974] seems to be more suitable in solving such kind of problems. According to this approach the source of crack growth are microdefects distributed over whole volume and their density is maximal near the crack tips. Some problems have been solved within the framework of the Continuum Damage Mechanics [Astafjev, 1973; Murakami et al., 1988; Bolotin, 1990].

The subject of this paper is to construct the theoretical model of fatigue rupture considering two-stage character of this process.

FATIGUE RUPTURE MODEL

Let us consider the rupture problem of thin isotropic plate (Fig.1) subjected to uniaxial cyclic load σ_∞ at infinity. The crack surfaces are supposed to be unloaded. Neglecting inertia forces, the stress tensor components σ_{ij} at any point of the plate with radius vector $\vec{x}$ may be represented as

$$\sigma^{\infty}_{ij}(\vec{x},n)= \sigma^{m}_{ij}(\vec{x})+ \sigma^{a}_{ij}(\vec{x})\sin(2\pi n)$$
$$\sigma_{eqv}(n)= F[J_k(\sigma^{m}_{ij})]+ F[J_k(\sigma^{a}_{ij})]\sin(2\pi n) < \sigma_y, \forall\vec{x}, \forall n \qquad (1)$$

Here σ_{eqv} is an equivalent stress; σ^{m}_{ij}, σ^{a}_{ij} are mean and amplitude components of a cyclic stress tensor; $F[\cdot]$ is a function of stress tensor invariants which is dependent on the plasticity condition; σ_y is the yield point of the material; n is the cycle number of load change ($n=ft$).

Let us assume that the extremal values of the maximal stress tensor components $\sigma^{max}_1 = \sigma^{m}_1 + \sigma^{a}_1$ and $\sigma^{min}_1 = \sigma^{m}_1 - \sigma^{a}_1$ follow each other quite often ($f>10Hz$). In this case the most part of the plate material is deformed linear-elastically and fatigue rupture is brittle and multicyclic ($n_R \geq 10^5$).

We consider the crack as a sharp tipped slot. Under external load action the plastic zone appears near the crack tip. Write $2l_0$ for initial crack length, label current crack length by $2l_f$, and write λ for plasticity zone length. For the λ we have

$$\lambda= \left(\frac{\sigma_\infty}{\sigma_y}\right)^2 \frac{l_f}{2}. \qquad (2)$$

Stress distribution near the crack tip let us define in accordance with model of Irwin of small plasticity zone in which all nonlinear effects are concentrated. In this crack tip area the stress $\sigma_{yy}(x,0)$ is supposed to be constant and equal to σ_y. If (x,o) point is located outside of this zone, dependence of σ_{yy} on x coordinate appropriates to the stress distribution rule for the normal opening crack in linear elastic material. In this case we can write

$$\sigma_{yy}(x,0)= \sigma_y\left[H(x-l_f)- H(x-l_f- \lambda(l_f)) \right] + \frac{\sigma_\infty}{\sqrt{2}} \sqrt{\frac{l_f}{x-l_f}}\, H\left[x-l_f- \lambda(l_f) \right]. \qquad (3)$$

Here $H[\cdot]$ is Heaviside step function. Fig.1 shows the graphic interpretation of equation (3).

As a "moving force" of crack propagation let us consider the damage accumulation process. The ability of solving dynamic problems of Crack Mechanics was principally formulated in [Kachanov, 1958]. Proceeding from the automodel assumption, for the damage measure ω_f we have the differential equation

$$\frac{\partial \omega_f(\vec{X},n)}{\partial n} = C_f \left[\frac{\sigma_{eqv}(\vec{X},n)\, H[\,\sigma_{eqv}(\vec{X},n)\,]}{1-\omega_f(\vec{X},n)} \right]^k \qquad (4)$$

Here C_f, k are the coefficients which reflect fatigue properties of the material.

Let us consider the fatigue rupture phenomenon as a two stage process consisting of nucleation stage and crack propagation stage. The crack propagation stage we will model as a motion of certain rupture edge. The moving condition of the rupture edge we assume as

$$\omega_f\left[\vec{X}_R,\, n\right] = 1 \qquad (5)$$

or taking equation (4) into consideration as

$$C_f(1+k) \int_0^n G^k\left[\sigma_{eqv}(\vec{X}_R,\tau)\right] d\tau = 1 \qquad (6)$$

Here $G[f(n)] = f(n)\, H[f(n)]$; $\vec{X}_R$ - radius vector of the moving crack tip.

CONSTITUTIVE EQUATIONS OF FATIGUE RUPTURE KINETICS

General relation for half length l_f of moving fatigue crack can be written as

$$l_f(n) = \mathop{F}_{\tau=0}^{\tau=n}\left[\sigma_{eqv}(\vec{X}_R,\tau),\ \omega_f(\vec{X}_R,\tau)\right], \qquad (7)$$

Here $F[\cdot]$ is functional of loading history and damage accumulation process.

To write relation (4) in case of crack with arbitrary orientation, equations (4)-(6) should be solved with equilibrium and compatibility equations. For the straight crack shown on Fig.1 and its moving direction coinciding OX axle ($\vec{X}_R = (x=l_f, 0)$), the resolving equations may be reduced to

$$\begin{cases} \sigma_{eqv}(x,n) = \sigma_{yy}\left[\,x,\ l_f(n),\ n\,\right] \\ \dfrac{\partial \omega_f(x,n)}{\partial n} = C_f \left[\dfrac{\sigma_{eqv}\, H(\sigma_{eqv})}{1-\omega_f(x,n)} \right]^k \\ \omega_f(x,0) = 0, \quad \forall x \\ \omega_f(x,n) = 1, \quad x = l_f(n) \end{cases} \qquad (8)$$

where $\omega_f(x,n)$ is changed from some value to 1 for any point of OX axle.

Substituting (3) into (8) and integrating differential equation in (8) we obtain relation

$$1-\left[1-\omega_f(x,n)\right]^{1+k} = (1+k)\, C_f \left(\frac{\sigma^a}{\sqrt{2}}\right)^k \times$$

$$\times \int_{n_*}^{n} \left[\frac{l_f(\tau)}{x+\lambda[\,l_f(\tau)\,]-l_f(\tau)} \right]^{\frac{k}{2}} G^k\left[\,\sin(2\pi n)\,\right] d\tau, \qquad (9)$$

which ties the stress state near the crack tip and damage value at the point with coordinates $(x,0)$ at any time moment n. Here assumed that the plastic zone is concentrated in the crack tip, and $\sigma_m = 0$. From equation (9) considering (5) we obtain relation for fatigue crack length l_f depending on cycle numbers n

$$1-(1+k)C_f\left(\frac{\sigma_a}{\sqrt{2}}\right)^k \int_0^{n_*} \left[\frac{l_0}{l_f(n)+\lambda(l_0)-l_0}\right]^{\frac{k}{2}} G^k\left[\sin(2\pi n)\right] d\tau =$$

$$= (1+k)C_f\left(\frac{\sigma_a}{\sqrt{2}}\right)^k \int_{n_*}^{n} \left[\frac{l_f(\tau)}{l_f(n)+\lambda[\,l_f(\tau)\,]-l_f(\tau)}\right]^{\frac{k}{2}} G^k\left[\sin(2\pi n)\right] d\tau \qquad (10)$$

Here n_* is the time when initial crack starts its moving.

To obtain analytical solution of the equation (10) we use approach suggested in [Astafjev, 1979]. For fatigue crack growth rate we finally have

$$\frac{dl_f}{dn} = \left(1+\frac{1}{k}\right) 2^{\frac{k}{2}}\, C_f \left[\frac{\sigma_a}{\sigma_y}\right]^{2-k} (\sigma_a)^k\, l_f\, G^k\left[\,\sin(2\pi n)\,\right] \qquad (11)$$

Here from follows that fatigue crack growth rate is the power function of stress amplitude σ_a, depends on current crack length l_f and on yield stress σ_y. The structure of equation (11) obtained theoretically is not in conflict with well known experimental data.

THE MAIN MECHANICAL EFFECTS

In accordance with equation (11) and rupture edge moving criterion (5), the influence of damage distribution in front of crack tip (Fig.1) on fatigue crack growth rate is taken into account. In particular from (5) we have

$$\frac{dl_f}{dn} = -\frac{\partial\omega_f/\partial n}{\partial\omega_f/\partial x} \qquad (12)$$

Herefrom follows, when $\omega_f \to 1$ then fatigue crack growth rate converges to infinity and influence of $\omega_f(x)$ is decreased.

Special feature of equation (11) is also that fatigue crack growth rate is a function of range of stress intensity coefficient ΔK. In this case we have

$$\frac{dl_f}{dn} = \left(1+\frac{1}{k}\right) C_f \frac{(\Delta K)^k}{2\pi^k \left[\lambda(l_f)\right]^{\frac{k}{2}-1}} G^k\left[\sin(2\pi n)\right] \qquad (13)$$

where assumed $\Delta K = 2\sigma_a\sqrt{\pi l_f}$. Equation (13) is the extension of known empirical relations establishing dependence of crack growth rate on the range of stress intensity coefficient. In particular, if $\lambda(l_f)=\lambda(l_0)$, from (13) considering (2) for brittle materials we can obtain

$$\frac{dl_f}{dn} = \left(1+\frac{1}{k}\right) C_f \left(\frac{\Delta K}{2\sqrt{\pi}}\right)^k \left[\frac{\sigma_a}{\sigma_y}\right]^{2-k} \left(l_0\right)^{1-\frac{k}{2}} G^k\left[\sin(2\pi n)\right] \qquad (14)$$

and for plastic materials when λ depends on the current l_f value

$$\frac{dl_f}{dn} = \left(1+\frac{1}{k}\right) C_f \, \Delta K^2 \left(\frac{\sigma_y}{4\pi}\right)^{k-2} G^k\left[\sin(2\pi n)\right] \qquad (15)$$

here $k>2$. Increase of index k of the ΔK base while material brittling is also confirmed with experimental data [Parton, 1990].

RUPTURE PROCESS OF CRACKED PLATES

To solve the problem of fatigue crack growth rate calculation in thin plates on the basis of equation (11) it is necessary to have material parameters C_f and k which characterise the material resistance to fatigue rupture. They may be obtained due to fatigue tests of smooth cylindrical specimens. Every points of such kind of specimen are of equal strength and incubation stage finish coincides with the moment when specimen is breaking into parts. So that for $n_R=n_*$ from (10) we have

$$n_R = 1/\left[\,(1+k)\, C_f\, (\sigma_y)^k \int_0^{0.5} \sin^k(2\pi n)\, dn\,\right] \qquad (16)$$

Here integrating is in effect only for tension half cycles. In logarithmic coordinate system, line (16) approximates rightly the experimental data. Parameters C_f and k just define the line location.

Dependence of crack growth rate and crack length on cycle number is shown on Fig.2 as an example. Results obtained in

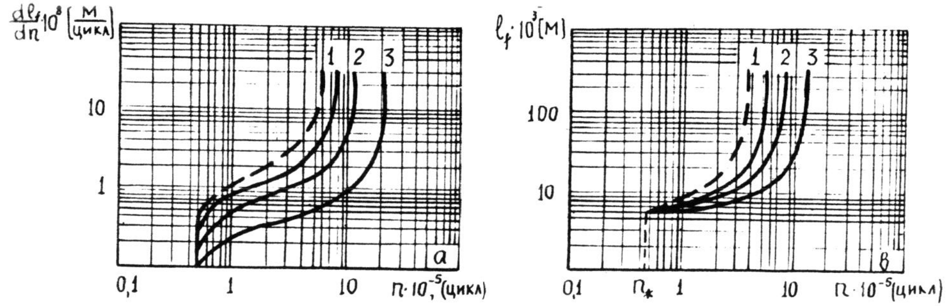

Figure 2.

accordance with (14) and (15) relations for the plate of EP718 alloy at 20°C with initial crack halflength $l_0 = 5X10^{-3} m$, in σ_a=500 (1), 400 (2) and 300 (3) MPa. Line obtained in accordance with (14) is dashed, with (15)-solid. Consideration of dependence on plastic zone length comes to decrease of fatigue crack growth rate and lifetime increase. If plastic zone length converges to 0 crack growth rate increases up to infinity. In this case the moment when crack starts its moving coincides with total destruction moment like in case of homogeneous stress state.

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