PRINCIPLE OF MAXIMUM ENERGY DISSIPATION RATE IN CRACK DYNAMICS

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ABSTRACT

The principle of maximum energy dissipation rate - maximum excess of the energy flux into a propagating crack tip - is introduced as an energy criterion for crack dynamics. As a result, the upper limits of the crack velocity in perfectly elastic and elastic-plastic bodies are obtained. It is found that the theoretical maximum crack velocity in an isotropic elastic body (in the first mode of crack propagation) is approximately equal to half the shear wave velocity.

The comparison of these theoretical results with some experimental data shows that under ordinary conditions a crack propagation looks like the "maximum dissipation rate" process.

It is shown also (using some physical considerations and variational technique) that the principle of maximum energy dissapation rate is possible to consider as a consequence of the Hamilton's principle.

KEYWORDS

crack velocity limit, elastic body, elastic-plastic body

INTRODUCTION

The energy criterion for fracture consists of the comparison of two quantities: the macroscopic energy release G caused by a crack propagation (in elastostatic - the energy release under a crack position variation) - with the surface energy by GRIFFITH (1920) or an effective surface energy by IRWIN (1948) and OROWAN (1955).

A dynamic problem of the crack propagation can be solved if the effective surface energy is constant or, in any case, is a function of the crack velocity. In this case the use of the energy criterion allows to obtain, in principle, the crack's velocity as a function of time. However the experimental data show that the effective surface energy is not a constant and what is more there is no connection between the effective surface energy and the crack velocity.

The numerous experimental results which concern the crack velocity limits in brittle materials are well known. The survey of brittle crack velocity are represented by RAVI-CHANDAR AND KNAUSS (1984). This survey is very important for us and it is shown in Table 1 (the ratios of crack velocities to other two characteristic velocities is not shown).

Table 1. Survey of brittle crack velocities by RAVI-CHANDAR AND KNAUSS (1984)

Material	Author	ν	v/c_2	v/c_R
Glass	Bowden	0.22	0.42	0.51
	Edgerton	0.22	0.43	0.47
	Schardin	0.22	0.47	0.52
	Anthony	0.22	0.60	0.66
Plexiglas	Cotterell	0.35	0.54	0.58
	Paxson	0.35	0.58	0.62
	Dulaney	0.35	0.58	0.62
Homalite-100	Beebe	0.31	0.31	0.33
	Kobayashi	0.345	0.35	0.38
	Dally	0.31	0.35	0.38
	Smith	0.31	0.38	0.41

Here ν is Poisson's ratio, c_2 is the shear wave velocity and c_R is the Rayleigh wave velocity. The references of the original papers can be found in the paper by RAVI-CHANDAR AND KNAUSS (1984).

The results show that the crack velocity (under ordinary conditions) do not achieve the Rayleigh wave velocity c_R. These results contradict the theoretical result obtained by FREUND (1972) assuming the effective surface energy to be constant. The experimental values of the crack velocity limits equal only about half this velocity. One can see also from the experimental results by RAVI-CHANDAR AND KNAUSS (1984) that the crack velocity is constant under any variable stress intensity factor. *The process looks independent on the energy flux into the propagating crack tip.*

Another result should be in a "weakly bonded plane" in which the energy absorption is strongly limited. It was pointed out by RAVI-CHANDAR AND KNAUSS (1984) and investigated experimentally by LEE AND KNAUSS (1989). In this case the crack velocity achieved the Rayleigh wave velocity.

The interesting results have been obtained recently by FINEBERG, GROSS, MARDER AND SWINNEY (1991), (1992). They discovered an almost regular roughness structure of the crack surfaces and the high frequency oscillation of the crack velocity in polymethylmethacrylate. This phenomena (the non-regular roughness was observed earlier repeatedly) arises when the crack velocity is high enough, especially when the crack velocity achieves its limit. The energy release - energy flux into the propagating crack tip per unit area is almost constant when the crack velocity increases. The energy release increases when the "average" crack velocity is constant (when it is equal to the crack velocity limit: about 0.5 c_R). The fact that the energy radiation as an effect of the crack velocity occilation was pointed out by RICE (1978). A periodical varibility of the sizes of the fracture process zone as a result of the crack velocity oscillations was pointed out by BOTSIS AND CHUDVOVSKY (1987).

Another reason of the effective surface energy increase is the structure of the medium influence. Recent developments in crack propagation in elastic periodically structured media such as chains, lattices, composite materials and rock joints by SLEPYAN (1981), KULACHMETOVA, SARAIKIN and SLEPYAN (1984), MICHAILOV and SLEPYAN (1986), SLEPYAN (1990) - demonstrate a number of effects which cannot be discovered using the classical model of non-structured solids. The energy radiation from the front of fracture is the most important phenomenon. It can be heat transfer, sound emission or high frequency seismic oscillations depend upon the scale of the structure. This energy outflow non-monotonically depends on the crack velocity and increases boundlessly if the crack velocity tends to the critical velocity.

The experimental and theoretical results show that the effective surface energy is formed in a brittle material under influences of some "micro" factors such as the structure of the medium, the roughness of the crack surfaces and the crack velocity oscillation. All these factors cause a radiation - high frequency waves which carry energy from the crack. The roughness increases the crack surface area, and this phenomenon also increases the effective surface energy. The roughness and the crack velocity oscillation, in their turn, depend on the structure of the medium and on macrolevel factors such as the energy release (G) and the "average" smooth crack velocity (v). Thus, we have here the coupled problem of macro - micro processes interaction.

THE CRITERION FORMULATION

Consider an elastic body under given external forces. A dynamic problem is completely defined if the boundary of the body, the boundary and initial conditions are given. However, in a fracture the boundary is not known in advance: an additional part of the boundary is formed by the crack propagation, and this process is out of the elasticity framework. If we want to consider the problem on the macrolevel we need a criterion to obtain the velocity and the trajectory of the crack.

Let G be the energy release per unit area of a dynamic crack, let N be the energy flux into the propagating crack tip: $N = Gv$, and let N_* be the excess of the energy flux: $N_* = (G - 2\gamma)v$, where γ is the effective surface energy for a quasistatic growth of the crack, v is the crack velocity. Taking into account the above mentioned fracture dynamic phenomena assume that at each given moment the crack velocity as vector corresponds to the maximum energy dissipation rate - the maximum excess of the energy flux into the propagating crack tip per unit time. We also use the Griffith's (or Irwin - Orowan's) criterion but only as the lower boundary of the energy release per unit area. So, we assume that the crack velocity vector $\mathbf{v}$ is defined by the requirement

$$N_*(\mathbf{v}) \equiv N - 2\gamma v = N_{*max}, \qquad \frac{dN}{dv} = 2\gamma \tag{1}$$

if $G = N/v \geq 2\gamma$ ($v = |\mathbf{v}|$). In the opposite case $G < 2\gamma$ the crack does not propagate. Let the criterion (1) be satisfied by the equality $v = v_0$, and the equation

$$\frac{dN}{dv} = 0 \tag{2}$$

is satisfied if $v = v_*$. We assume that the derivative dN/dv is a monotonous decreasing function (that is what it is under ordinary conditions, see the book by SLEPYAN (1990)). In this case $v_0 < v_*$. However, $v_0 \to v_*$ if the energy flux N increases. Really, the energy flux N can be represented (see Section 3) in the form $N = f(v)g(t)$, and the equation (2) follows from criterion (1) if $g(t) \to \infty$. So, we see that v_* is the crack velocity limit,

and to obtain this limit it is possible to use the equation (2) for all energy flux into propagating crack tip.
The extremal dissipation principle was used earlier by NIKOLAEVSKY (1987) for the investigation of some aspects of crack growth in a visco-elastic material but without any variation of the crack velocity.
We also consider the crack propagation in an elastic-plastic body. In this case the rate of the plastic strain energy is required to be maximum. The same is required of the *maximum elastic energy release rate.*

THE CRACK VELOCITY LIMIT IN AN ELASTIC BODY

Consider the generalized plane problem for a semi-infinite straight running crack in an unbounded elastic body (the initial conditions are zero). The velocity limit v is obtained as a value which satisfies the equation: The results of calculations of the ratio of crack velocity limit to shear wave velocity for the fracture modes I and II are represented in Table 2.

Table 2. The Crack Velocity Limit

Poisson's ratio	0	0.1	0.2	0.3	0.4	0.5
c_R/c_2	0.874	0.893	0.911	0.927	0.942	0.955
v/c_2 (fracture mode I)	0.476	0.492	0.507	0.517	0.520	0.482
v/c_2 (fracture mode II)	0.539	0.568	0.601	0.638	0.674	0.711

The theoretical results for fracture mode I are very close to the experimental results (see Table 1): the theoretical ratio for glass ($\nu = 0.22$) : $v/c_2 = 0.51$, the average experimental ratio $v/c_2 = 0.48$. The same ratios for plexiglass ($\nu = 0.35$) are 0.52 and 0.57 respectively. The experimental results for Homalite-100 differ from the theoretical ones somewhat more (0.52 and 0.35 respectively). Perhpas, the decrease of the velocity limit is the effect of the viscosity and plasticity influence which is not taken into account in the theoretical consideration (the plasticity influence is estimated below). Note here that a small change of the crack velocity (in comparison with the velocity limit) leads to a smaller (second order) change of the energy dissipation rate, because the limit corresponds to maximum rate.
The theoretical results for fracture modes II and III predict higher values of the limit crack velocity and shear cracks are expected to be faster. This result is confirmed, to some extent, indirectly from data for the shear crack velocity. Some results are pointed out by BEROZA AND SPUDICH (1988) using two methods of the natural data estimation. It turns out that the shear crack velocities $v = 0.7c_2$ - from one method and $v = 0.8c_2$ - from another method are the most suitable for the natural data description (Application to the 1984 Morgan Hill, California, earthquake). See also HEATON(1990).
Consider for example a transient problem for the fracture mode I assuming the surface crack loading is a constant:

$$\sigma_- = const H(t)H(l(t) - x)$$

where H is Heaviside's function, and $l(t)$ is the crack tip coordinate. Initial conditions are zero, $l(0) = 0$. If the energy dissapation rate criterion is in force the motion of the crack as follows. At some time $t = t_0 > 0$ the crack motion begins, its velocity is somewhat

smaller than the velocity which satisfies the Griffith's criterion, then the velocity tends to its limit (Fig.1).

ELASTIC-PLASTIC BODY

Consider an elastic-plastic problem for fracture mode III. We assume for simplicity that the plastic region is a narrow zone in front of the crack - we use the well known foundation by BARENBLATT (1959), DUGDALE (1960), LEONOV (1961) and PANASYUK (1968). We consider the self-similar problem for a semi-infinite straight crack in the unbounded elastic body (the plasticity appears only in boundary conditions). We have the boundary conditions ($y = 0$) for the half-plane x, y ($-\infty < x < \infty,\ y \geq 0$)

$$\sigma_{yz} = \sigma_- = [-pH(vt - x) + kH(x - vt)H(at - x)]H(t)\ (x < at) \qquad (3)$$
$$u_z = u = 0 \qquad (x > at)$$

where z is the third axis, $-p = const < 0$ is a shear stress which acts on the crack surfaces ($x < vt$), $k = const > 0$ is the same stress but in plastic zone, a is the velocity of the plastic zone front, and v is the velocity of its internal boundary - the crack velocity. The initial conditions are at zero. We also have an additional condition: there is no energy flux into the moving point $x = at$.
The results of calculations of v as a function of λ are represented in Fig. 2.

CONCLUDING REMARKS. THE PRINCIPLE OF MAXIMUM ENERGY DISSIPATION RATE AND HAMILTON'S PRINCIPLE

Consider, at last, a possible connection of the principle of maximum energy dissapation rate with the Hamilton's principle. We have the equality (see FREUND (1990))

$$\frac{\partial L}{\partial l} = G \qquad (4)$$

where L is a lagrangian of the crack dynamic problem, and l is a crack length. We obtain from (4) omiting a possible item which is independent on l.

$$L = \int_0^l G dl = \int_0^t N dt \qquad (5)$$

The lagrangian L is independent on the instantaneous crack velocity $v(t)$, and it is impossible to obtain this velocity by the Hamilton's principle using the lagrangian L in the form (17). To overcome this difficulty we take into account the fact that the macroproblem foundation is possible to consider only as a long wave approximation. Some small time-interval τ exists which we cannot divide on the macrolevel. (Otherwise we have to include the discription of the microphenomena into the lagrangian.) Therefore the introduction of an averaged lagrangian is justified. The averaged lagrangian:

$$\bar{L} = \frac{1}{2}(L(t) + L(t-\tau)) = \int_0^l G dl - \frac{1}{2}\int_{t-\tau}^t G v dt \sim$$
$$\sim \int_0^l G dl - \frac{1}{2}\tau N(t, v),\ v = \frac{dl}{dt} \qquad (6)$$

This averaged lagrangian depends on instantaneous crack velocity v. The variation of the action integral gives us:

$$\delta \int_{t_1}^{t_2} \bar{L} dt = \int_{t_1}^{t_2} (G + \frac{\tau}{2}\frac{d}{dt}\frac{\partial N}{\partial v})\delta l dt \qquad (7)$$

Let this variation leads to the same result as above.

$$\delta \int_{t_1}^{t_2} \bar{L}dt = \delta \int_{t_1}^{t_2} Ldt = \int_{t_1}^{t_2} G\delta l dt \tag{8}$$

So, we require the energy release to be independent on the (small) time-interval on which the average is obtained. We have to believe it using the averaged lagrangian because the energy release is exactly defined on the macro-level.
We obtain from (7), (8):

$$\frac{d}{dt}\frac{\partial N}{\partial v} = 0, \qquad \frac{\partial N}{\partial v} = const \tag{9}$$

At the same time:

$$\frac{\partial N}{\partial v} = G + v\frac{\partial G}{\partial v} \to G \to 2\gamma \qquad (v \to 0) \tag{10}$$

and hence:

$$\frac{\partial N}{\partial v} = 2\gamma \tag{11}$$

We see now that the principle of maximum dissipation rate (1) is a consequence of the Hamilton's principle if the above physical considerations are taken into account.
Of course the value of the extremal crack velocity can be changed under an additional condition's influence, such as the above mentioned weak bound. In such cases the variational principle should be used to obtain a relatively extremum.

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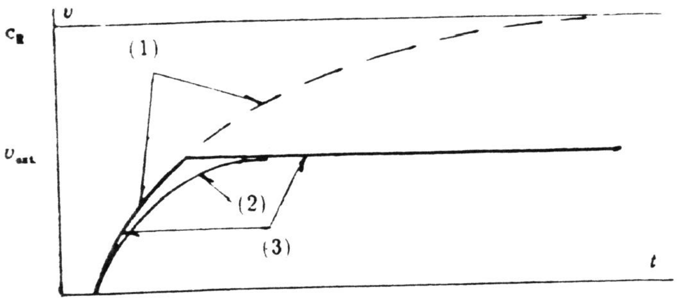

Fig. 1. Crack velocity based on Griffith's criterion (1), crack velocity based on criterion of maximum energy dissipation rate (2), and the crack velocity boundary (3).

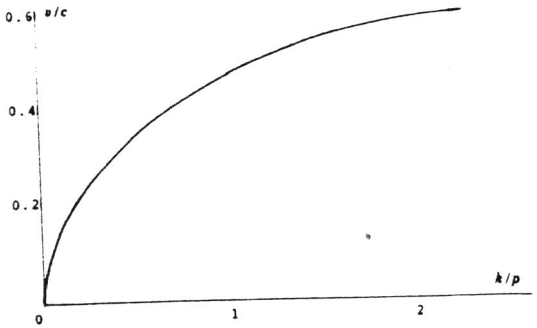

Fig. 2 Crack velocity limit in an elastic-plastic body.

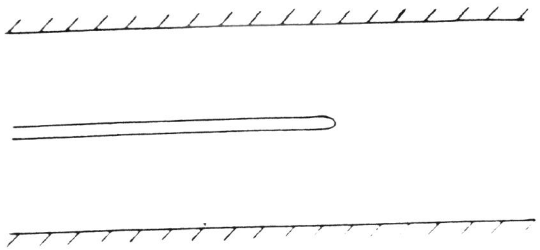

Fig. 3 Crack propagation in an initial stressed elastic strip