MODELLING STRAIN RATE EFFECT ON BEHAVIOUR OF THE PLASTIC ZONE NEAR A CRACK TIP IN MODE II

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ABSTRACT

This paper presents parametric study of the strain rate effect on the behaviour of stress fields near a crack tip in Mode II. Evolution of the plastic zone and boundaries of equal equivalent stress versus strain rate is modelled via corresponding quasi-static linear-elastic and elastic-plastic solutions which involve material plasticity parameters as functions of the strain rate. For a crack in mild steel, the plastic zone is evaluated at strain rates ranging from 10^{-4} s^{-1} to 10^3 s^{-1}.

KEYWORDS

Strain rate effect, Mode II crack, fracture toughness, plastic zone, crack initiation

INTRODUCTION

Analysis of crack initiation and determination of fracture toughness for metallic materials under various loading rates are the problems of prime interest in fracture mechanics. For Mode I and Mode III several investigations have been carried out to reveal the loading rate effect on fracture toughness. At the same time, systematic analyses have been performed for the majority of structural materials to study strain rate effect on the yield stress, ultimate stress, strain hardening exponent, etc. Using such data, some correlations between fracture toughness and parameters of stress-strain diagram at different strain rates have been derived (see Klepaczko, 1990).

For in-plane shear cracks (Mode II) such solutions are not available. Some features of crack initiation at high rates of shear loading have been studied experimentally by Kalthoff and Winkler (1987) and Mason et al. (1992). Important information on crack behaviour can be obtained from the analysis of the plastic zone geometry which is the most vivid characteristic of elastic-plastic fields near a crack tip. In the present paper some models of the plastic zone near a crack tip in Mode II are discussed. A parametric study of the strain rate effect on the plastic zone geometry is carried out through introduction of material plasticity parameters as functions of the strain rate into corresponding quasi-static solutions. This approximate analysis is applied to the evaluation of the plastic zone for a crack in mild steel at strain rates $\dot{\gamma}$ ranging from 10^{-4} s^{-1} to 10^3 s^{-1}.

APPROXIMATE MODELS OF THE PLASTIC ZONE

Several approaches have been introduced to evaluate the size and shape of the plastic zone within the small scale yielding (SSY) approximation (see, e.g., Broek, 1987).

Making use of Irwin's correction for plasticity, one can find the plastic zone size ahead of a crack tip in Mode II: $r_p = (1/\pi)(K_{II}/\tau_Y)^2$, where K_{II} is the stress intensity factor, τ_Y is the yield stress in shear. If asymptotic terms of the stress field near a crack tip are considered along with the Huber-Mises yield condition, an approximate solution for a plastic zone shape can be obtained (Broek, 1987):

$$r_p(\theta) = (1/6\pi)(K_{II}/\tau_Y)^2 \, [\, 3 + \sin^2(\theta/2) - (9/4)\sin^2\theta - 4\nu(1-\nu)\sin^2(\theta/2)\,] \quad (1)$$

Here ν is Poisson's ratio, θ is the polar angle, $\theta = \pm\pi$ on crack surfaces. For materials with low strain hardening, the use of Irwin's correction for plasticity to evaluate the plastic zone size ahead of a crack tip seems to be reasonable. This results in the value of $r_p(\theta=0)$ twice larger than that predicted by eq. (1), while the values of $r_p(\theta=\pi)$ and plastic zone hight are not changed.

Since the stress intensity factor is proportional to the applied stress, τ_{appl}, the plastic zone size in eq. (1) is proportional to $(\tau_{appl}/\tau_Y)^2$. Consequently, decreasing the stress level (due to increase of τ_Y with τ_{appl} = const or decrease of τ_{appl} with τ_Y = const) leads to diminishing the plastic zone proportionally to $(\tau_{appl}/\tau_Y)^2$, with no changes in its shape. This conclusion is in contradiction to the finite-element results by Banks-Sills and Sherman (1990) which demonstrate changes in the shape of plastic zone with increasing load level; namely, a significant decrease in the ratio of $r_p(\theta=\pi)$ to $r_p(\theta=0)$ with increase of the load level has been observed.

Influence of Non-Singular Stress Terms on the Plastic Zone Geometry. More realistic assessment of the plastic zone can be obtained by including non-singular terms of the crack tip stress field into consideration. In Mode I, the effect of the first non-singular stress term (the so called T-stress) has been studied by Larsson and Carlsson (1973) and Rice (1974) for various configurations of cracked specimens. An important piece of information which follows from Rice's (1974) analysis is that, when T-effect is taken into account, the solution predicts a more rapid increase in the plastic zone size with stress level than that with T-effect being neglected.

To study the effect of non-singular stress terms on the plastic zone geometry in Mode II, let us represent the crack tip stress components as follows (Williams, 1957):

$$\sigma_{ij}(r,\theta) = K_{II}\,(2\pi r)^{-1/2}\,\Phi_{ij}(\theta) + T_{ij}(\theta) + \tau_{appl}(r/H)^{1/2} A\,\Psi_{ij}(\theta) + \ldots \quad (2)$$

where r is the polar radius, H is a characteristic dimension, Φ_{ij}, T_{ij} and Ψ_{ij} are functions resulting from a solution of corresponding eigenvalue problem; to specify Φ_{ij}, see, e.g., Broek (1987). When solving the boundary value problem for a crack in Mode II, one can conclude that the T-stress terms vanish identically. The Ψ_{ij} functions are found to be

$$\Psi_{rr} = \sin(\theta/2)\,[\,2 + 5\cos(\theta/2)\cos(3\theta/2)\,]\,, \quad \Psi_{\theta\theta} = -5\sin(\theta/2)\cos(\theta/2)\cos(3\theta/2)\,,$$

$$\Psi_{r\theta} = (5/2)\cos\theta\cos(3\theta/2) - (3/2)\cos(\theta/2) \quad (3)$$

For certain crack geometry and loading conditions the amplitude A of Ψ-terms is to be determined from results of numerical treatment of a boundary value problem. In case of simple geometry an approximate assessment of the A value can be derived as follows. Consider a plate of width W containing an edge crack of length a; let H = W − a be a ligament size. The plate is subjected to shear stresses acting parallel with the crack line. Then eqs (2), (3) along with the equilibrium condition

$$\int_0^H \tau_{r\theta}(r,\theta=0)\,dr = \tau_{appl}\,W$$

yield

$$A = (3/2)\,(\,W/H - Y_{II}\sqrt{2a/H}\,) \quad (4)$$

where Y_{II} is the geometrical factor defined by $K_{II} = \tau_{appl}\sqrt{\pi a}\;Y_{II}$.

As an example, consider a semi-infinite crack in an infinite plate. Substituting a=H=W/2 and $Y_{II} = 2\sqrt{2}/\pi$ into eq. (4), we find A=1.09.

For the above crack configuration variation of the plastic zone size and shape versus stress level was calculated based on eqs (2), (3) and the Huber-Mises yield condition for plane strain with $\nu = 0.33$. Moreover, Irwin's correction for plasticity was employed for points ahead of a crack tip. The results are presented in Fig.1a in terms of a dimensionless radius of the plastic zone $R_p = r_p(\tau_Y/K_{II})^2$.

Fig.1a demonstrates a significant change in the shape of the plastic zone as compared to that predicted by eq. (1). The present two-parameter model predicts more rapid growth of the plastic zone ahead of a crack tip than that behind it, thus, with increasing the stress level, the most part of plastic zone area concentrates ahead of a crack tip. This feature of the plastic zone behaviour agrees with finite-element results by Banks-Sills and Sherman (1990). Quantitative comparison of the latter with our data (Fig.1b) justifies the validity of the derived approximate solution within the range of applicability of the SSY-approximation.

Application of the HRR-Solution. The plastic zone models considered above are based upon the linear-elastic solution for crack tip fields. The only strain rate sensitive plasticity parameter involved in those models is the yield stress. In reality, there exists material strain hardening which affects material behaviour in yielding and leads to redistribution of stress and strain fields. The dominant singularity of elastic-plastic crack tip fields has been studied by Hutchinson (1968) and Rice and Rosengren (1968) (the HRR-solution) for materials obeying the Ramberg-Osgood constitutive equation

$$\gamma/\gamma_Y = \tau/\tau_Y + \alpha\,(\tau/\tau_Y)^N$$

where $\gamma_Y = \tau_Y/\mu$ is the yield strain in shear, μ is the shear modulus, N is the strain hardening exponent ($N \geq 1$), α is material constant.

Including parameters τ_Y, α and N as functions of $\dot{\gamma}$ into the HRR-solution, an approximate study of strain rate effect on the behaviour of stress and strain fields near a crack tip can be performed. Since the HRR-solution yields the dominant singular stress and strain components, it is valid only at high stress levels or, equivalently, in a small vicinity of a crack tip with a radius being much smaller than the plastic zone size. Nevertheless, boundaries $r_\sigma(\theta)$ of the equal equivalent stress calculated by Rice and Rosengren (1968) for Mode I conditions at different values of strain hardening are in qualitative agreement with direct computations of the plastic zone size and shape (Shih, 1974). Thus, a qualitative analysis of the plastic zone behaviour at different strain rates seems to be possible employing the HRR-solution.

STRAIN RATE EFFECT ON EVOLUTION OF THE PLASTIC ZONE FOR A MODE II CRACK IN MILD STEEL

Rate Sensitivity of Material Properties. Strain rate effect on the behaviour of mild steel (0.12% C) in shear was studied by Campbell and Ferguson (1970). Their experimental

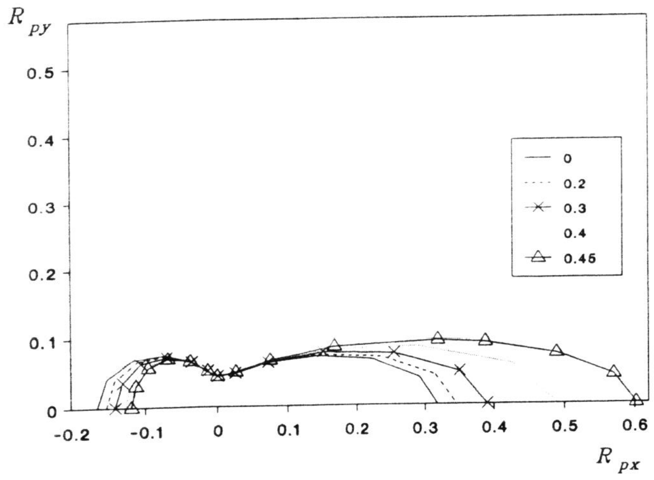

Fig. 1a. Plastic zone geometry at different stress levels (plane strain, $\nu = 0.33$).

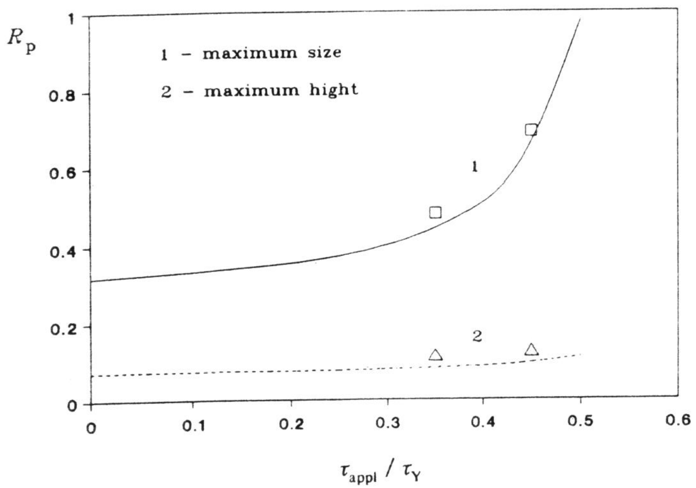

Fig. 1b. Comparison of the present approximate solution for plastic zone (curves) with the finite-element results by Banks-Sills and Sherman (1990) (points).

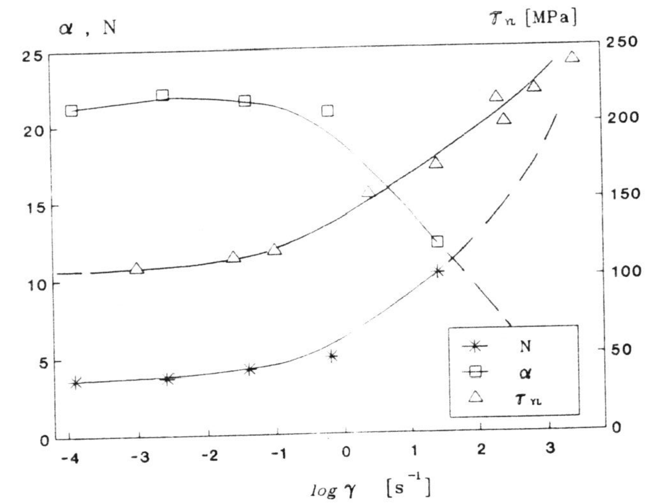

Fig. 2. Rate sensitivity of material constants for mild steel.

data on rate sensitivity of the lower yield stress, τ_{YL} , are shown in Fig.2. A series of stress-strain curves for technically pure iron in shear at strain rates ranging from $1.31 \cdot 10^{-4}\ s^{-1}$ to $55\ s^{-1}$ is available from the paper of Klepaczko (1969). Based on the latter results, an approximation of the stress-strain curves by the Ramberg-Osgood equation is derived, and variation of the constants α and N versus strain rate is presented in Fig.2. Solid lines in Fig.2 are averaged curves fitted to corresponding experimental data, while dashed lines represent appropriate extrapolation of the curves within the range of strain rate considered, i.e. $\dot{\gamma} = 10^{-4} \ldots 10^{3}\ s^{-1}$. One can conclude that both the lower yield stress and strain hardening exponent are rapidly increasing with $\log \dot{\gamma}$, while the α value is decreasing.

Variation of the Plastic Zone Geometry. Analysis of evolution of the plastic zone with strain rate is carried out based upon the two-parameter SSY model described above. Computations are performed for a semi-infinite crack subjected to constant value shear stress, i.e. $\tau_{appl}(\dot{\gamma}) = \mathrm{const}$, while the yield stress as a function of the strain rate is involved. Thus, the load level is characterized by the ratio of τ_{appl} to the lower yield stress of the material (τ_{YS}) in test with $\dot{\gamma} = 10^{-4}\ s^{-1}$, which is referred to as a quasi-static case. The ratio $\tau_{appl}/\tau_{YS} = 0.5$ is considered which is typical for fracture toughness tests.

Results on variation of the plastic zone size ahead and behind a crack tip versus strain rate are presented in Fig.3a in terms of the dimensionless radius $R_{ps} = r_p(\tau_{YS}/K_{II})^2$. One can find significant changes in the size and shape of the plastic zone. With an increase in strain rate from $10^{-4}\ s^{-1}$ to $10^{3}\ s^{-1}$, the plastic zone radius ahead of a crack tip is diminished 14.2 times, while that behind a crack tip is diminished 3.9 times. Asymmetry of the plastic zone shape, $r_p(\pi)/r_p(0)$, is changed from 0.12 to 0.42, respectively.

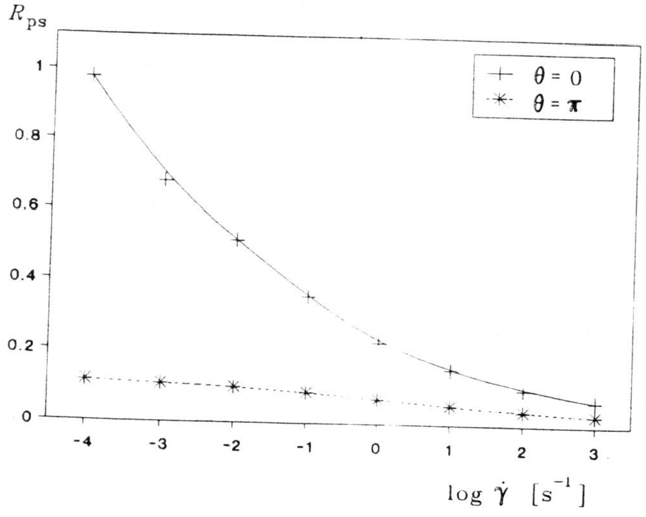

Fig. 3a. Variation of the plastic zone size versus strain rate. (Material - mild steel, plane strain, $\nu = 0.33$).

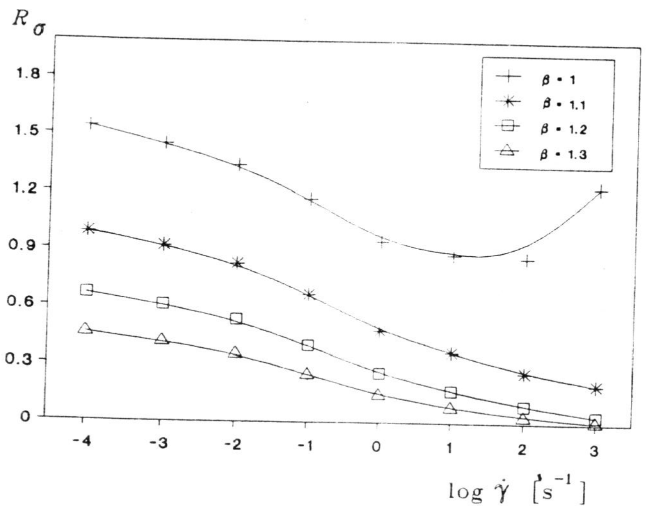

Fig. 3b. Variation of the value R_σ ($\theta = 0$) versus strain rate. (Material - mild steel, plane strain, $\nu = 0.33$).

Variation of r_σ-Boundaries. Making use of the HRR-solution and the Huber-Mises yield condition, the radius of the boundary of equal equivalent stress $\sigma_e = \beta \sigma_Y$ (with β = const) is evaluated. Fig. 3b presents variation of the dimensionless value $R_\sigma = r_\sigma(\tau_{YS}/K_{II})^2$ ahead of a crack tip ($\theta = 0$) for different values of β : 1, 1.1, 1.2 and 1.3. Calculations are performed with the use of data on strain rate sensitivity of material constants τ_{YL}, α and N for mild steel (Fig.2). Similarly to the results on plastic zone behaviour (Fig.3a), one can conclude from Fig.3b that the region of plastically deforming material near a crack tip reduces appreciably with increasing strain rate.

Crack Initiation Angle. For different materials, specimens and loading conditions, two alternative mechanisms of fracture in Mode II have been observed: (1) tear mode fracture with crack initiation in direction of about 70...80 deg. with respect to the initial crack line; (2) shear mode fracture with initiation of a crack in its original plane. One can suppose the first case to occur due to the action of maximum tensile stress (σ_1), while the second one due to the action of maximum shear stress (τ_{max}). Then conditions for crack initiation by tear or shear mechanisms are formulated respectively:

$$\sigma_1 > \sqrt{3}\,\tau_{max} \qquad \text{or} \qquad \sigma_1 < \sqrt{3}\,\tau_{max} \tag{5}$$

Fig.4 shows the ratio $\sigma_1/(\sqrt{3}\,\tau_{max})$ as a function of the strain hardening exponent calculated with the use of the HRR stress field. Based on conditions (5), one can assume transition of the fracture mode from shearing to tearing at $N \approx 8$. Note that for metallic materials the N value is normally increasing with strain rate, thus schematic illustration of crack initiation at different strain rates is shown in Fig.4. However, experimental results by Kalthoff and Winkler (1987) demonstrate reverse behaviour of a crack with increasing strain rate. This circumstance indicates limitations of the quasi-static approach

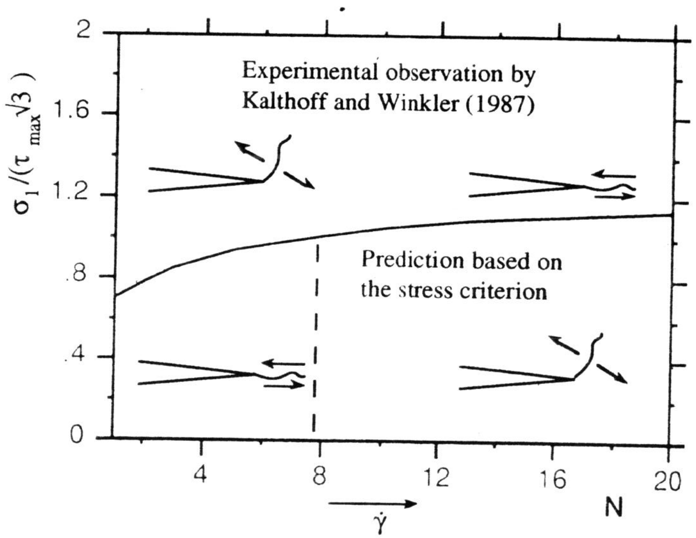

Fig. 4. Schematic representation of crack initiation angle.

and fracture criteria based on stress analysis when studying crack behaviour at high strain rates.

CONCLUSIONS

In the present study an approximate parametric analysis of the strain rate effect on the crack tip fields in Mode II is carried out. Evolution of the plastic zone and boundaries of equal equivalent stress at different strain rates is modelled via corresponding quasi-static solutions which take into account strain rate sensitivity of plasticity parameters (τ_Y , α , N). For materials with low strain hardening, the two-parameter model of the plastic zone derived provides an accurate prediction of the plastic zone versus stress level within the range of applicability of the SSY approximation; calculated values of the maximum size of the plastic zone are within 15% of the numerical solution by Banks-Sills and Sherman (1990).

Analyses of both plastic zone and r_σ-boundaries for a crack in mild steel reveal significant reduction (by an order of magnitude and more) of the zone of plastically deforming material with increasing strain rate from 10^{-4} s^{-1} to 10^3 s^{-1}. The present approximate approach is not valid for the evaluation of crack behaviour at higher strain rates; in this case rigorous analytical or numerical methods are to be used along with more general constitutive equations which take into account stress, strain, strain rate and temperature redistribution in the vicinity of the crack tip.

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