MATHEMATICAL MODELLING OF DYNAMIC PROCESSES OF DEFORMATION AND MICROFRACTURE OF THERMO-ELASTO-PLASTIC MEDIA

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ABSTRACT

Three new models of deformable solids are being presented, which take into account the accumulation of micro-structural damage in the material, in the process of dynamic deformation, the influence of damage to a stress-strain state and the temperature effects. Two models (the damaged and porous thermoelastoplastic media) serve as a description of deformation and the ductile failure of metals and explosives. These models belong to a class of models with internal variables. The third model explains the brittle fracture of materials, which can be interpreted as the process of the formation of a large number of micro-cracks in different directions, as a result of which the isotropic material acquires anisotropy. The entropy criteria of macro-failure are proposed. The testing of the models and the numerical solutions of some dynamic problems are discussed.

KEYWORDS

Thermoelastoplasticity, dynamic continuum fracture, damage, numerical modelling.

INTRODUCTION

The evaluation of the life of materials under intensive short-term stress is one of the basic problems of the mechanics of solids. Dynamic fracture is a complicated multistage process, including the appearance, development and confluence of microdefects and the formation of embryonic micro-cracks, their growth right up to the break-up of the bodies with their division into separate parts. Three basic types of dynamic fracture can be singled out: ductile, brittle and adiabatic shear failure. Ductile fracture, observed under normal conditions in metals, solid rocket fuels and explosives, are characterized by the nucleation and growth of dispersed spherical micropores under plastic deformation. A large number of orientared, coin-type micro-cracks, capable of growing in the process of deformation are formed in the brittle fracture of the material. Fracture of this type can be observed in berilium, concrete, mineral rock and certain types of steel. The mechanism of shear failure is observed under high speeds of deformation, for example, when a "plug" is forced out of the target. In this case the resulting tear is concentrated in thin layers with a thickness of up to several tens of a micron, are positioned along surfaces with maximum tangent stresses. This leads to the development of intensive plastic flow.

Below, three new models are discussed: two models, describe the initial stages of ductile fracture (formation and growth of microdefects) and the third models for brittle fracture. These models are geared towards modern usage of the modelling of nonstationary non-homogenous processes of deformation and fracture of bodies in a complex stress-strain state.

MODELS OF CONTINIOUS DUCTILE FRACTURE

Model of the damageable thermoelastoplastic medium. This model belongs to the class of model media with internal variables, in which additional scalar or tensor variables of state, that characterise damages are introduced (Coleman and Gurtin, 1967; Kondaurov and Nikitin, 1990 et al.). An understanding of the measure of damage to material was first introduced in the works of Ilyushin, 1967; Kachanov, 1958; Rabotnov, 1966. The scalar internal variables of state ω is used for models of the damageable media (Kiselev and Yumashev, 1990 b). This describes the appearance and growth of the damaged material in the deformation process(ω varies from 0 in an undamaged material to 1 in a complete fracture). Let it be assumed that the full deformation ε_{ij} can be expessed in the form of the sum $\varepsilon_{ij}=\varepsilon^e_{ij}+\varepsilon^p_{ij}$, where ε^e_{ij}- elastic deformations and ε^p_{ij} plastic deformations, while: $\varepsilon^p_{kk}=0$.

Returning to the heat equation and the second law of thermodynamics, expressed in the form of the Clausius-Duhame inequality we get

$$\dot{\eta}T = \left(\frac{1}{\rho}\sigma_{ij} - \frac{\partial F}{\partial \varepsilon^p_{ij}}\right)\dot{\varepsilon}^p_{ij} - \frac{\partial F}{\partial \omega}\dot{\omega} - \frac{1}{\rho}\mathrm{div}\vec{q}, \qquad (1)$$

$$d = d_M + d_F + d_T \geqslant 0, \qquad d_M = \left(\sigma_{ij} - \rho\frac{\partial F}{\partial \varepsilon^p_{ij}}\right)\cdot\dot{\varepsilon}^p_{ij},$$

$$d_F = -\rho\cdot\frac{\partial F}{\partial \omega}\cdot\dot{\omega}, \quad d_T = -\frac{\vec{q}\cdot \mathrm{grad}T}{T}, \quad \sigma_{ij} = \rho\frac{\partial F}{\partial \varepsilon^e_{ij}}, \quad \eta = -\frac{\partial F}{\partial T},$$

where U - specific internal energy, ρ -density,, σ_{ij} - components of the stressed tensor, $\vec{q}$ - heat flow, η - specific entropy, T - absolute temperature, d_M- mechanical dissipation, d_F- dissipation of continuum fracture , d_T- thermal dissipation, $(\sigma_{ij}- \rho\frac{\partial F}{\partial \varepsilon^p_{ij}})$ - tensor of "active" stress.

In the frameworks of the linear thermodynamics, with assumptions of small elastic deformation and the nonnegativity of each of the components of functions of dissipation; introducing a specific heat capacity under constant stress c_σ, accepting that module K and μ depend on the variables of damage ω in the following way:

$$K = K_0(1-\omega), \qquad \mu = \mu_0(1-\omega), \qquad (2)$$

where K_0, μ_0 - are the modulii of the undamaged material, assuming that the behavior of the material can be described by a flow equations with Mises' criterion of plasticity, and that the variable of damage ω is expressed by a kinetic equation of the Tooler-Butcher type, finally we end up with the following system of constitutive equations:

$$\begin{aligned} &\sigma' = K_0\ \varepsilon_{kk} - \alpha_v(T-T_0) + \frac{\Lambda}{3}\int_0^\omega \frac{\partial\dot{\omega}}{\partial\sigma}\,d\omega, \\ &(\tau'_{ij})^\nabla + \lambda\tau'_{ij} = 2\mu_0(\dot{\varepsilon}_{ij} - \tfrac{1}{3}\dot{\varepsilon}_{kk}\delta_{ij}), \quad \tau'_{ij}\tau'_{ij} \leqslant \tfrac{2}{3}Y^2, \\ &\rho c_\sigma\dot{T} + \alpha_v\dot{\sigma}T = \tau_{ij}\varepsilon^p_{ij} + \Lambda\dot{\omega}^2 + \mathrm{div}\vec{q}, \qquad (3) \\ &\dot{\omega} = B(\sigma' - \sigma_*)^m H(\sigma' - \sigma_*), \quad \tau'_{ij} = \tau_{ij}/(1-\omega), \quad \sigma' = \sigma/(1-\omega), \end{aligned}$$

where $\sigma=\sigma_{kk}/3$, H(x) - function of Heaviside, B, m, σ_*- constants of the material. The symbol ∇ designates Jauman' time derivation. Here,the yield strength Y and the shear modulus μ depend on temperature, pressure and other variables of state (Wilkins, 1984).

Model (3) generalizes the Prandtler-Reuss model of elastoplastic flow and takes into account the anisotropy of plastic deformation (in the case where $\Gamma\neq 0$), the accumulation of damage in area of intense tension, the effects of the processes of the deformation and accumulation of micro-structurial damage, and termal effects. Model (3) is used to express the behavior of metals.

Model of porous thermoelastoplastic medium. The model of porous thermoelastoplastic medium (Kiselev and Yumashev, 1992 b) is suggested to explain the dynamic behavior of solid rocket fuel and explosives, which even in their initial condition have scattered micropores.The system of constitutive equations of the porous medium turns out to be analogous to that of the model of the damaged medium (3), if instead of the variable of damage ω the variable of porosity α ($0\leqslant\alpha\leqslant 1$) is inserted - volumetric total contents of micropores (the voids in the materials). As a kinetic equation for variable α the equation of the ductile growth of pores is used, taking into account the influence of its gases (Kiselev and Yumashev, 1992 a) :

$$\dot{\alpha}/\alpha = \frac{(\sigma-\sigma^+)}{4\eta} H(\sigma-\sigma^+) + \frac{\sigma-\sigma^-}{4\eta} H(\sigma^- - \sigma), \qquad (4)$$

$$\sigma^+ = -\tfrac{2}{3}Y\cdot\ln\alpha - p_o(\alpha_o/\alpha)^k, \quad \sigma^- = \tfrac{2}{3}Y\cdot\ln\alpha - p_o(\alpha_o/\alpha)^k .$$

Here η is the dynamic ductility of the material, α_o- initial porosity, p_o- initial pressure of the gas in a pore, k - index of the adiabatic constant of the gas. The first term in (4) explains the process of the expansion of micropores, the second it's plastic swelling.

Criteria of macrofracture (the origin of cracks - the new free surface in the material) is the condition for the achievement of the specific dissipation of maximum meaning D_* (Kiselev and Yumashev, 1990 a,b):

$$D = \int_0^{t_*} \frac{1}{\rho}(d_M+d_F+d_T)dt = D_*.$$

Here t_*- time of fracture, D_*- the constant of the material, experimentally defined.

Concerning the parameters of the models. The material characteristics are selected from the experimental data from the flat collision of two plates with results of numerical modelling (Kiselev and Yumashev, 1990 a,b). The deformation anisotropic parameter Γ can be defined from the experiments of tension-pressure or normal shear (Bykovtsev and Lavrova, 1989). In future calculations let Γ=0 given the absence of necessary experimental data for the investigation of material. In particular experiments (Kanel' et al., 1987) for the break-off fracture of a 10 mm titanium alloy targets which impact from 2 mm aluminum plates with a wide range of velocities. Numerical investigation was carried out with an adiabatic approach. The significance of the limit of the strength $D_* = 75$ kJ/kg was defined for titanium alloys.

For the definition of porous, thermoelastoplastic model parameters, the problem of the impact-compression of micropores (Kiselev and Yumashev, 1992 a) is used. This is used to define the interactive parameter Λ of deformation and micropore evolution (Λ is replaced by A in the equation for the model of porous media). Firstly the problem of the adiabatic compression of individual micropore with initial inner-radius a_o and external radius b_o, with or without gas was solved. External pressure was defined as the following with τ representing the duration of the process: $P(t)=P_oH(\tau-t)$. From the average temperature of pores when t=τ can be defined: T_{av} in the case where $a_o \neq 0$ and $\overset{o}{T}_{av}$ in the case $a_o = 0$. The increment of temperature $\Delta T_{av} = T_{av} - \overset{o}{T}_{av}$ was attained due to the porosity of

the individual cell. The problem of dynamics of micropore can be solved with a one-dimensional approach. The gas in the pores is considered ideal. As constitutive equation for the material of pores the equation for thermoelastoplasticity is used (Perzyna, 1963) with the same coefficient of dynamic ductility η, which is found in equation (4). Then the problem of plate collision with velocity V_o is numerically solved. However in the plate of the investigated material initial porosity $\alpha = (a_o/b_o)^3$ is introduced. The velocity of collision V_o and the thickness of the striker h are selected so that the surface pressure equals p_o and the duration τ. The selection of parameter A must make sure that the increment of surface temperature equals ΔT_{av}. Fof VRA-fuel we managed to attain the value A: A=5 kPa·s. The results of the calculations correspond to experimental data and the calculations of other investigations.

MODEL OF BRITTLE FRACTURE

In the case of brittle fracture, a thick net of arbitrary directed microcracks are formed in the material. As each crack has its own direction their accumulation leads to anisotropic condition of materials.The basis of the given model is the idea that represents the breaking-up material as a variable, elastic module (Zelensky, 1985).

Originally the continuous medium is represented as one with increasing crackability. This is modelled on the increase of the coefficients of the malleability matrix. For a quantitative description of malleability matrix changes a subsidiary problem is concerned: in the continuous medium with malleability matrix A_{ij} (here the usual definitions for plane deformation are used: $e_1=e_{11}$, $e_2=e_{22}$, $e_6=2e_{12}$, $\sigma_1=\sigma_{11}$, $\sigma_2=\sigma_{22}$, $\sigma_6=\sigma_{12}$; Hook's law is written down thus: $e_i=A_{ij}\sigma_j$, i,j=1,2,6) the system of N cracks with length of 2a and parallel to the x_1 axis is introduced. "Effective" elastic module have to be found. Here effective modulii are called quantities $\bar{A}_{ij}$, connecting average deformations e_i and average stresses σ_i of cracked media: $e_i = \bar{A}_{ij}\sigma_j$. The average deformation of a material element consists of elastic deformation and the deformation due to cracks:

$$e_{ij}=e^e_{ij}+ \frac{1}{2}\int_S(n_i[U_j]+n_j[U_i])dS,$$

where n_i- components of the vector normal to the crack; $[U_i]$ - components of displacement leap under transformation through the crack. Therefore, malleability of cracked media $\bar{A}_{ij}$ consists of malleability of initial media A_{ij} and additional malleability Π_{ij}, which is represented by means of displacement leaps. These leaps are defined based on the work of Paris and Si, 1968. Consequently we can conclude for each individual crack, length 2a, parallel to the x_1-axis:

$$[U_1] = 2\sqrt{a^2-x_1^2}\cdot A_{11}[\sigma_2 Im(\mu_1\mu_2) + \sigma_6 Im(\mu_1+\mu_2)],$$
$$[U_2] = -2\sqrt{a^2-x_1^2}\cdot A_{22}[\sigma_2 Im(1/\mu_1+1/\mu_2) + \sigma_6 Im(1/\mu_1\mu_2)].$$

Hear σ_i - active stresses; μ_i - roots of characteristic equation for this anisotropic media. In conclusion we can express additional malleability in the following way:

$$\Pi_{22}=-\pi A_{22} Im(1/\mu_1+1/\mu_2)\Omega,\quad \Pi_{26}=-\pi A_{22} Im(1/\mu_1\mu_2)\Omega,\quad \Omega=a^2N,$$
$$\Pi_{62}=\pi A_{11} Im(\mu_1\mu_2)\Omega,\quad \Pi_{66}=\pi A_{11} Im(\mu_1+\mu_2)\Omega,\quad \Pi_{11}=\Pi_{12}=\Pi_{21}=\Pi_{16}=\Pi_{61}=0.$$

For the self-consistency of this model we have to add the kinetic equation for Ω. The constants of this equation are defined from experimental data of one-dimensional tension.

In extreme cases (model of ideal brittle media) this model fully corresponds to the results of the work of (Mainchen and Sack, 1967). The model of brittle fracture was realised in the problems of interaction of elastic waves with hole.

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