FATIGUE LIFE ESTIMATION UNDER NON-STATIONARY RANDOM LOADING

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ABSTRACT

As most operating processes possess a non-stationary character the paper aims at the computational assessment of influence of non-stationary process properties on fatigue life. Two kinds of non-stationary processes were used, viz. the most frequent Gaussian white noise with a time dependent mean and/or variance, and the Weibull white noise with time dependent parameters. These processes were simulated in real time, then analysed by the one- or two-parameter Rain Flow Method and the macroblocks obtained were used for the fatigue life estimation according to a few hypotheses. The results are mutually compared and some qualitative conclusions are formulated.

KEYWORDS

Fatigue life, rain flow method, Gaussian and Weibull non-stationary processes, random process simulation

INTRODUCTION

Permanently increasing requirements to higher performance and lower weight of machines force designers to consider factors of the true operation which were neglected before. One of them is the non-stationary behaviour of operating random processes manifesting itself in the majority of practical situations. Thus the aim of the present paper is to gain information on the computational influence of non-stationarities of some typical random processes on the fatigue life and to propose recommendations that could help in practical dimensioning against fatigue under operating loading.

NON-STATIONARY RANDOM PROCESSES

There are various definitions of non-stationarities of random processes (Cacko et al., 1988) based on various statistical moments. As a result some process parameters are time-dependent deterministic functions, practically, e.g., a *probability density function* $f(x,t)$ or *power spectral density* $S(f,t)$, or both of them. Considering, however, that data concerning this area of fatigue are very scarce and, moreover, depend on many other parameters, it is indispensable to limit the investigation to a certain class of non-stationarities. Following this idea, in this paper we shall be concerned with the fatigue life assessment under a non-stationary random process with a constant power spectral density $S(f,t) = const.$ and deterministic time-dependent probability density function (PDF) $f(x,t)$ (non-stationary white noise). This type of processes can be met, e.g., at vehicles operating outside the component resonant frequency ranges.

Despite this restriction there is still a variety of white noises with various PDFs. Here we shall present the results for two representatives described by the Gaussian PDF in the form

$$f_G(x,t) = \frac{1}{\sqrt{2\pi}} \exp\{-[x - \mu(t)]^2 / s^2(t)\} \tag{1}$$

and the three-parameter Weibull PDF in the form

$$f_W(x,t) = A(t)[x - a(t)]^{m(t)} \exp\{-A(t)[x - a(t)]^{m(t)+1} / [m(t) + 1]\} \tag{2}$$

where $\mu(t)$ and $s(t)$ are deterministic functions of time t, representing the mean value and standard deviation, resp., and $A(t)$, $m(t)$ and $a(t)$ are deterministic functions.

The choice of these two PDFs is not incidental but reflects their frequency of occurrence and importance in solutions of practical problems.

SIMULATION OF NON-STATIONARY RANDOM PROCESSES

Using the linear transformation

$$y(t) = [x - \mu(t)]/s(t) \tag{3}$$

for Eq. (1) and

$$y(t) = A(t)\ [x - a(t)]^{m(t)+1} / [m(t) + 1] \tag{4}$$

for Eq.(2) with the corresponding Jacobians

$J_G = s^{-1}(t)$ and $J_W = A(t)[x - a(t)]^{m(t)}$,

resp., we get the PDFs

$$f_G(y,t) = f_G(y) = \frac{1}{\sqrt{2\pi}} \exp(- y^2/ 2) \tag{5}$$

and

$$f_W(y,t) = f_W(x,t)/J_W = f_E(y) = \exp(-y). \tag{6}$$

It is obvious that the resulting processes are stationary with a normalized Gaussian PDF $f_G(y)$ and a normalized exponential PDF $f_E(y)$, resp.

The real time simulation algorithms are then obtained using the inverse transformation of Eqs. (3) and (4), which in the discrete form yields

$$x_{Gi} = s(t_i)\ \tau_i + \mu(t_i) \tag{7}$$

and

$$x_{Wi} = a(t_i) + [\frac{m(t_i) + 1}{A(t_i)} \lambda_i]^{[m(t_i)+1]^{-1}}, \tag{8}$$

where $s(t_i)$, $\mu(t_i)$, $a(t_i)$, $A(t_i)$ and $m(t_i)$ are the discrete values of the corresponding parameters in time t_i; τ_i, λ_i are random numbers with the normalized Gaussian and exponential distributions, resp.

FATIGUE LIFE ESTIMATION

As known, any fatigue life assessment using a fatigue damage accumulation hypothesis has its advantages and drawbacks. In order to avoid, therefore, unjustified conclusions based on one formula and at the same time to obtain a certain "scatter" of results, three types of hypotheses were adopted.

The simplest Palmgren-Miner (PM) linear damage rule for both strain and stress processes:

$$\sum_{(i)} N_i/ N_{fi} = D_M, \quad L = \sum D_M = 1, \tag{9}$$

the Serensen-Kogaev (SK) hypothesis also for both strain and stress processes:

$$\sum_{(i)} N_i/ N_{fi} = D_M, \quad L = \sum D_M = a, \tag{10}$$

and the Kliman (K1) hypothesis for strain processes:

$$\frac{1}{N_{fmin}} \sum_{(i)} N_i(\sigma_{ai}/ \sigma_{amax})^{(1+n)/n} = D_M, \quad L = \sum D_M = 1, \tag{11}$$

and the Kliman (K2) hypothesis for stress processes:

$$\frac{1}{N_{fmin}} \sum_{(i)} N_i(\varepsilon_{api}/\varepsilon_{apmax})^{n+1} = D_M, \quad L = \sum D_M = 1, \tag{12}$$

where N_i is the number of cycles at the *ith* stress σ_{ai} (strain ε_{ai}) level at which fracture occurs after N_{fi} cycles, σ_{amax} (ε_{amax}) is the maximum amplitude in the macroblock at which fracture occurs after N_{fmin}, *a* is the parameter depending both on the material properties and the macroblock form (Bily, 1989), D_M is damage caused by one macroblock, L is the total fatigue life (number of macroblock repetitions) to fracture, and *n* is the cyclic strain hardening exponent of the cyclic stress-strain curve

$$\varepsilon = \sigma/E + (\sigma/k)^{1/n}. \tag{13}$$

Because all these hypotheses are of a discrete type based on the macroblock representation of a random process (Bily, 1989), every simulated process was analysed by the Rain Flow Method (RFM) transforming it to a macroblock of sinusoidal cycles. Although most experimentally founded recommendations express the opinion that it is sufficient to use the one-parameter RFM (considering amplitudes only), they all are derived from stationary cases. This is why the results of the two-parameter RFM (amplitudes and their local mean levels) were also examined here.

EXPERIMENTS AND THEIR RESULTS

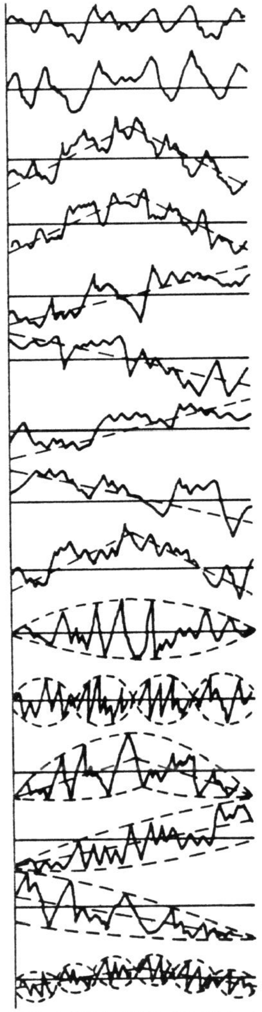
In accord with the aforementioned considerations Table 1 and Table 5 present variants of simulated processes with Gaussian and Weibull PDFs used in this investigation. In both cases the stationary processes (variants G1 and W1) were taken as references for the subsequent fatigue life estimation. All computations and experimental verifications were realized for low carbon steel with the following parameters: fatigue strength coefficient σ'_f = 1132 MPa, fatigue ductility coefficient ε'_f = 0.871, fatigue strength exponent b = -0.115, fatigue ductility exponent c = -0.579, cyclic strain hardening exponent n = 0.199, and cyclic strength coefficient k = 1164 MPa.

Various circumstances and questions motivated the choice of the subsequent variants of processes. In the following we shall present them in the form of the "question - answer".

GAUSSIAN PROCESSES

(a) what is the influence of time-dependent mean and time-dependent

Table 1. Variants of simulated processes with a Gaussian PDF



Variant	Type	Character	Non-stationary parameter	RFM applied
G1	stationary		---	2-parameter
G2	stationary		---	2-parameter
G3	non-stationary		mean $\mu(t)$	2-parameter
G4	non-stationary		mean $\mu(t)$	1-parameter
G5	non-stationary		mean $\mu(t)$	2-parameter
G6	non-stationary		mean $\mu(t)$	2-parameter
G7	non-stationary		mean $\mu(t)$	1-parameter
G8	non-stationary		mean $\mu(t)$	1-parameter
G9	non-stationary		mean $\mu(t)$	2-parameter segmented
G10	non-stationary		standard deviation s(t)	2-parameter
G11	non-stationary		standard deviation s(t)	2-parameter
G12	non-stationary		mean $\mu(t)$ and standard deviation s(t)	2-parameter
G13	non-stationary		mean $\mu(t)$ and standard deviation s(t)	2-parameter
G14	non-stationary		mean $\mu(t)$ and standard deviation s(t)	2-parameter
G15	non-stationary		mean $\mu(t)$ and standard deviation s(t)	2-parameter

variance on the fatigue life compared with the corresponding stationary process with the same parameters?

Answer: on comparing the fatigue curves for the stationary variant G1 with the non-stationary variants G3 (time-dependent mean), G10 or G11 (time-

Table 2. Numbers of mabroblock repetitions to fracture (in thousands) for some variants of strain processes from Table 1 and three fatigue hypotheses

Levels	Hypothesis	Variants G1	G3	G10/G11	G12/G15
Low	PM	10 320	10 510	2 360/2 380	8 960/10 300
$\varepsilon = 2.4\times10^{-4}$	SK	3 245	2 830	471/476	832/910
	K1	8 215	9 210	1 471/1 479	1 210/1 320
High	PM	11 200	13 100	6 300/5 990	3 900/3 310
$\varepsilon = 5.7\times10^{-4}$	SK	2 800	2 600	1 260/1 200	790/630
	K1	9 300	8 000	4 400/4 200	3 200/3 100

dependent variance) or G12 and G15 (both time-dependent mean and variance) one can deduce that all non-stationarities shift, in the average, the results to lower values but this shift does not seem to be significant. This is also clear from Table 2. No significant differences can be spotted even when various forms of standard deviations are applied: one large sinusoidal half cycle in variant G10, or more sinusoidal cycles in variant G11, or a linearly increasing-decreasing mean with one half cycle or more cycle sinusoidal standard deviation in variants G12 and G15, resp.

(b) what is the influence of the local amplitude mean levels considered or neglected in the one- or two-parameter RFM on the resulting fatigue life taking into account the non-stationary process behaviour?

Answer: on comparing the results for variants G3 and G4, G5 and G7, G6 and G8 from Table 1 one comes to the conclusion that the one-parameter RFM increases the fatigue lives for variants G7 and G8 but lowers them in case G4 for both low and high strain amplitudes. In either case it is, however, not too dramatic as obvious from Table 3. Because the two-parameter RFM results are closer to the experimentally determined fatigue lives than the one-parameter ones, the computational analysis of non-stationary random processes with time-varying means should always take into account the local mean levels and so the two-parameter RFM should be applied.

(c) considering that most measured operating processes represent strain but some recommendations still rely on stresses it is of interest to compare the fatigue lives obtained for both the strain process and its corresponding stress pair obtained from the strain process by means of the cyclic stress-strain curve (Eq. (13)). Is there any difference between these two approaches?

Answer: In this investigation the comparison was performed for the stationary variants G1 and G2 and the two-parameter RFM and the life results are practically identical. Similar observations are also related to other variants. Thus one can say that in the high cycle fatigue range there is practically no difference between the fatigue lives obtained for the strain process and the material Manson-Coffin curve, and those obtained for the stress process, recalculated from the strain process by means of the cyclic stress-strain curve (Eq. (13)), and the S/N curve.

(d) when the measured process is visually estimated to possess some non-stationary properties it is commonly recommended to split it into a few segments, evaluate their statistical properties and obtain the result as their average. Is this approach to the fatigue life estimation under non-

Table 3. Numbers of macroblock repetitions to fracture (in thousands) for some variants of strain processes from Table 1 and three fatigue hypotheses

Levels	Hypothesis	Variants G3/G4	G5/G7	G6/G8
Low	PM	617/444	856/1 174	868/1 216
$\varepsilon = 3.7 \times 10^{-4}$	SK	123/89	171/235	174/243
	K1	233/180	354/456	367/478
High	PM	13.1/11.1	130/160	170/203
$\varepsilon = 5.8 \times 10^{-4}$	SK	2.6/2.2	26/32	34/41
	K1	8.0/7.9	92/93	110/115

stationary processes to be recommended?

Answer: This approach was used for the non-stationary process with a time-dependent mean (variant G9) which was divided into ten segments; they were further analysed by the two-parameter RFM. The corresponding averaged fatigue lives are about 2 to 10 times higher compared with the lives obtained from the unsegmented process (variant G3). Qualitatively the same results were also obtained for non-stationary processes with time-dependent standard deviations. Thus the fatigue life estimation based on segmentation of a non-stationary process into short segments does not seem to be acceptable.

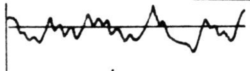
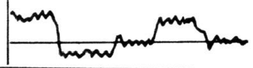
(e) when the measured process is in fact non-stationary but its time trends are not so pronounced that it is reckoned to be stationary after the visual "rough" appraisal then it may happen that the analysis is performed either on the "ascending" part (as in variants G5 or G13) or on the "descending" part (variants G6 or G14). Can these monotone trends influence the fatigue life estimations?

Answer: on comparing the fatigue lives obtained for variants G5,G6 and G13, G14 it is obvious (Table 4) that in all cases the results belong to the same set as they do not substantially differ. Moreover, they do not substantially differ from the results of variants G1 either (stationary process). Thus there is no obvious danger in the analysis of a measured non-stationary strain process as far as the choice of its analysed part is concerned, as the fatigue lives obtained for various (and sufficiently long) segments are approximately the same.

Table 4. Numbers of macroblock repetitions to fracture (in thousands) for some variants of strain processes from Table 1 and three fatigue hypotheses (standard deviation $s_{\varepsilon} = 3.7 \times 10^{-4}$)

Hypothesis	Variants G1	G5	G6	G13	G14
PM	613	856	868	508	199
SK	207	171	174	112	40
K1	424	354	367	256	144

Table 5. Variants of simulated processes with a Weibull PDF

Variant	Type	Character	Non-stationary parameter	RFM applied
W1	stationary		---	2-parameter
W2	stationary Rayleigh		A(t) = const in Eq. (15)	2-parameter
W3	stationary exponential		A(t) = const in Eq. (14)	2-parameter
W4	non-stationary Rayleigh		A(t) = 10t+1 in Eq. (15)	2-parameter
W5	non-stationary Rayleigh		A(t) = t + 1 in Eq. (15)	2-parameter
W6	non-stationary exponen.		A(t) = 1.2t+1 in Eq. (14)	2-parameter
W7	non-stationary		step-wise varying mean	2-parameter

WEIBULL PROCESS

The simulation algorithm for the random stationary or non-stationary process with the Weibull PDF given by Eq. (8) has many variants. First of all, if $a(t) = 0$, then for $m(t) = 0$ we get

$$x_{Ei} = A(t_i)^{-1} \lambda_i , \tag{14}$$

where λ_i are random numbers with an exponential distribution.

If for $a(t) = 0$ we take $m(t) = 1$ then we get

$$x_{Ri} = [2\, A(t_i)^{-1}]^{1/2} \sqrt{\lambda_i} . \tag{15}$$

In our investigations the simulation algorithms (14) and (15) were used and for the linear time-dependent ramp A(t) we got 6 variants shown in Table 5; variant W7 has a step-wise varying mean. In all cases the two-parameter RFM was applied.

Without a detailed comparison of fatigue lives obtained one can formulate approximately the same conclusions as for the Gaussian processes, i.e. neither the PDF shape (exponential, Rayleigh or Weibull) nor the time varying parameter A(t) have a practical influence on the resulting fatigue life. All these results are compared in Fig. 1 representing the Manson-Coffin curve for all Gaussian processes (line 1), Weibull processes (line 2) and for all processes taken together as one set (line 3) with a very narrow 50 % reliability band and 90 % scatter band. It is rather surprising but advantageous for practical analyses as the non-stationarities need not be taken into account.

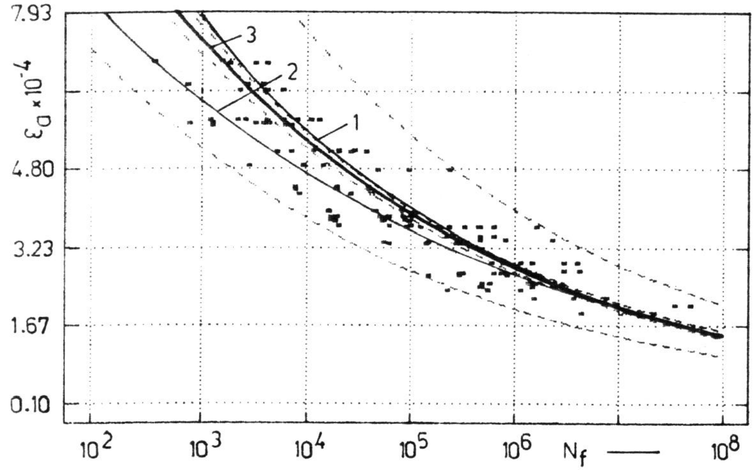


Figure 1. Manson-Coffin curves for Gaussian (1) and Weibull (2) processes from Tables 1 and 5; line 3 expresses a common set of fatigue lives

CONCLUSIONS

Computational estimation of the fatigue life under stationary and non-stationary white noise with the Gaussian, Weibull, exponential and Rayleigh probability density functions reveal the following facts:

- in the high cycle fatigue range there is no difference whether the fatigue life is estimated from a measured strain process or from a stress process, obtained by means of the cyclic stress-strain curve; one should be careful, however, because a possible discontinuity in the S/N curve can make this conclusion invalid (Bily, 1988);
- although no significant effect could be found after applying the one- or two-parameter Rain Flow Method, it seems to be more appropriate to analyze the non-stationary processes by the two-parameter method as it describes more fully non-stationary trends;
- variation of time-dependent means and variances of the Gaussian non-stationary processes lead only to minute changes in fatigue lives; the results fall into a relatively narrow scatter;
- variation of parameters of the Weibull probability density functions yielding stationary or non-stationary processes with the exponential and Rayleigh probability density functions has the same effect as above and the results also form one narrow scatter of fatigue lives;
- the Manson-Coffin curves for the Gaussian and Weibull stationary and non-stationary processes do not principally differ and both can be approximated by one common curve;
- segmentation of one long non-stationary process into short parts and computation of the total fatigue life as the average from the partial segment lives cannot be recommended because it yields a substantially higher estimation of the total fatigue life.

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