

# FATIGUE ANALYSIS METHOD FOR WING LOWER-SURFACE ELEMENTS OF TRANSPORT AIRPLANE AT RANDOM LOADING

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## ABSTRACT

A method based on precise equivalent stresses of a random load fatigue test program for transport aircraft wings is used to improve the fatigue strength prediction accuracy. A modified linear equation for damage cumulation was put into the basis of the method.

## KEYWORDS

Fatigue life, fatigue analyses, fatigue tests, random load program, equivalent stresses.

## INTRODUCTION

Since the introduction of computers into experimentation practice, a number of random load programs representing most features of in-service aircraft structural member loadings were developed and implemented. For the stretched longitudinal elements of transport aircraft wings such programs are, for example, the "TWIST" standardized program (De Jonge et al., 1973), wing fatigue testing programs for Fokker 27, Fokker 28, Boeing 757, 767, Airbus A310, A320, "PUSK" and "PIROUETTE" programs (TsAGI), wing fatigue testing programs for IL-96-300 and Tupolev Tu-204 airplanes. The use of these and other similar programs allowed the specialists to make experimental fatigue life predictions substantially more precise; however, it also led to further complication of fatigue analyses.

At present there is a great number of fatigue damage theories. The most widely used is the Palmgren-Miner theory of linear damage cumulation. In accordance with this theory, the total damage (D) is determined as

$$D = \sum n_i/N_i, \quad (1)$$

where  $n_i$  is the number of cycles at the  $i$ th loading level, and

$N_i$  is the number of cycles to failure at the same level.

At the moment of damage,  $D=1$ . As is known, this theory poorly agrees with experimental results in many cases, first of all, when the programs with notable nonstationarity (including random programs) are used. In such cases methods of calculation using pre-estimated sums  $D \neq 1$  are often applied. This approach is called "relative linear cumulation". The accuracy is higher, but manhours increase noticeably, because the value of  $D$  is closely related to a specific element, program, loading level, etc. and its precise, usually experimental, determination requires much effort.

#### PROPOSED METHOD

The present study suggests a method that indirectly accounts for the realities of damage cumulation in precise values of equivalent stresses, for a case of random fatigue loading of the lower surfaces of transport airplane wings, with the aim to reach the acceptable accuracy of calculations and reduce manhours (in comparison with the "relative linear cumulation" analyses). Speaking in more details, the essence of the method suggested may be presented as follows:

1. Physically, the method is based on the modified Palmgren-Miner theory, i.e. the principle of linear cumulation of damage of random loading programs flights:

$$\bar{D} = \sum \bar{n}_i / \bar{N}_i, \quad (2)$$

where  $\bar{n}_i$  is the number of flights of the  $i$ th type of the random program load spectrum;  $\bar{N}_i$  is the number of flights, same type, to failure. It is assumed that  $\bar{D} = 1$  at the moment of failure.

2. The S-N curve of the structural element under regular repeating-stress cycles is used for analyses:

$$N S_o^{m/c} = 10^{co} \quad (3)$$

Equation (3) is also assumed to be presented in the following forms:

$$\bar{N}_i S_{eq,i}^{m/c} = 10^{co} \quad (4)$$

and

$$\bar{N} \bar{S}_{eq}^{m/c} = 10^{co}, \quad (5)$$

where  $S_{eq,i}$  is the equivalent stress of the  $i$ th type of flights in the block (i.e. the maximum stress of the repeating-stress cycle which is equivalent to the flight of the  $i$ th type as for damage introduced),  $\bar{S}_{eq}$  is the equivalent stress of the average flight in the block;  $\bar{N}$  is the number of average flights to failure. It is evident that, when  $\bar{N}$  is well high, we may assume that

$$\bar{N} = \sum \bar{N}_i.$$

3. On the basis of the theory of linear damage cumulation (1) and equation (3), the equivalent stress of any program (including random load programs) is determined as:

$$S_{eq} = \left( \sum n_i S_{o,i}^{m/c} \right)^{c/m}, \quad (6)$$

where  $S_{o,i}$  is the maximum stress of the repeating-stress cycle that is equivalent, in respect of damage, to the  $i$ th load cycle of the program;  $n_i$  is the number of repeating-stress cycles with the maximum stress equal to  $S_{o,i}$ .

As stated above, such approach does not provide satisfactory correlation with experimental results in many cases. This is why the present study suggests to estimate the equivalent stresses of random loading programs for transport airplane's wing lower-surface elements by using the system of equations:

$$\begin{aligned} \bar{N} \bar{S}_{eq}^{m/c} &= 10^{co} \\ \bar{N} S_{mf}^{m/c} &= 10^c \end{aligned} \quad (7)$$

where  $\bar{N} S_{mf}^{m/c} = 10^c$  is the equation describing typical S-N curve of transport airplane's wing lower-surface elements at random loads ( $S_{mf}$  is the mean stress of the load spectrum defined by the level of nominal stresses in the element tested under operational load). The equation

$$\bar{N} S_{mf}^{m/c} = 10^c$$

is based on the assumption of linearity of  $\log \bar{N}$  vs.  $\log S_{mf}$  and is confirmed by results obtained by Schutz and Lowak (1975), Schijve et al. (1974), Schijve (1985).

Solving the system (7) with respect to  $\bar{S}_{eq}$ , the following formula is obtained:

$$\bar{S}_{eq} = S_{mf}^{m/mo} 10^{(co-c)/mo} \quad (8)$$

#### DISCUSSION

It is evident that the accuracy of  $\bar{S}_{eq}$  calculated according to (8), as well as the accuracy of the whole fatigue analyses would depend on the accuracy of parameters  $m$  and  $c$  of the S-N curve

$$\bar{N} S_{mf}^{m/c} = 10^c$$

( $m_o$  and  $c_o$  for the S-N curve (3) are assumed to be known to sufficient accuracy). As shown by the results of above mentioned studies, the values of  $m$  and  $c$  depend on the type of a test element, as well as on the type of random loads, and may be either determined by special-purpose tests or found in data bases on airframe fatigue resistance.

In case of special-purpose tests there are all reasons to believe that the amount of tests necessary to determine two

parameters (m and c) of the linear log  $\bar{N}$ -log Smf function will be substantially less than at necessary to find out the sums  $D \neq 1$  according to the "relative linear cumulation", because up to the present time there are no simple and reliable functions of D vs random load parameters.

### SOME RESULTS

Advantages of the suggested method can be illustrated using the example of design estimation of IL - 96-300 aircraft wing lower panel endurance for the area of open holes (Kt=2.5) under loads of the PUSK-96-300 program at three values of Smf: 85, 102 and 119 MPa.

Initial data:

1. The PUSK-96-300 flight load block is shown in Table 1.

Table 1. PUSK-96-300 flight load block.

Flight type	Number of flights in one block	Amplitude level Sa/Smf (No. and value)					Total number of cycles per flight
		I	II	III	IV	V	
		0.725	0.66	0.52	0.39	0.25	
A	1	1	3	16	62	116	198
B	9		1	8	28	65	102
C	65			1	18	40	59
D	435				1	30	31
E	690					15	15
Total number of cycles in one block		1	12	153	1919	26701	
Cumulative number of cycles in one block		1	13	166	2085	28786	

2. Equations of S-N curves for rolled-strips of 1163T7 - alloy with open holes ( Kt=2.5) are derived on the basis of Kulyna's experimental data (TsAGI):

$$\bar{N} S_o = 10^{4.36} \cdot 14.849 \quad \text{for regular repeating- stress cycles,}$$

$$\bar{N} S_{mf} = 10^{4.04} \cdot 12.313 \quad \text{for PUSK-96-300 random load program.}$$

3. The results of an estimation of accumulated damage sums D for PUSK-96-300 program as applied to the above described strips

are shown in Table 2 (Kulyna's data).

The estimation of fatigue endurance was carried out by three methods:

a) using linear damage cumulation theory:

$$\sum n_i/N_i = D=1, \quad \bar{S}_{eq} = (\sum n_i S_{o,i}^{m_0})^{1/m_0}, \quad \bar{N} = 10^{c_0} \bar{S}_{eq}^{-m_0};$$

b) using the theory of "relative linear cumulation":

$$\sum n_i/N_i = D \neq 1, \quad \bar{S}_{eq} = (\sum n_i S_{o,i}^{m_0})^{1/m_0}, \quad \bar{N} = 10^{c_0 + \log D} \bar{S}_{eq}^{-m_0};$$

c) according to the method suggested in this study:

$$\sum \bar{n}_i/\bar{N}_i = \bar{D}=1, \quad \bar{S}_{eq} = S_{mf} \cdot 10^{m_0/m_0} (c_0 - c)/m_0, \quad \bar{N} = 10^{c_0} \bar{S}_{eq}^{-m_0}.$$

Calculation error was evaluated as:

$$Er = \frac{\bar{N} - \bar{N}_{exp}}{\bar{N}_{exp}} \cdot 100\%$$

where  $\bar{N}_{exp}$  is the experimentally determined fatigue endurance of the specimens (Kulyna's data). Table 2 represents the results of the evaluation. The Er values clearly demonstrate advantages of the suggested method.

Table 2. Estimation results and error of IL - 96-300 wing lower panel endurance analyses. Area of open holes (Kt=2.5)

Fatigue analysis method	Smf, MPa	$\bar{S}_{eq}$ , MPa	D ( $\bar{D}$ )	$\bar{N}$ , flights	$\bar{N}_{exp}$ , flights	Er, %
$\sum n_i/N_i = D=1$	85	172	1.0	126500	33003	283
	102	206	1.0	57615	15031	283
	119	241	1.0	29070	8471	243
$\sum n_i/N_i = D \neq 1$	85	172	0.23	29095	33003	11.8
	102	206	0.21	12100	15031	19.5
	119	241	0.21	6100	8471	27.9
$\sum \bar{n}_i/\bar{N}_i = \bar{D}=1$	85	235	1.0	33270	33003	0.8
	102	278	1.0	15930	15031	6.0
	119	321	1.0	8550	8471	0.9

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