DYNAMIC FRACTURE IN SOLIDS UNDER PULSE DEFORMATION

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ABSTRACT

This paper presents the authors' investigations on numerical modelling for motion of two-dimensional compressive, unloading and tensile waves in solids with spalling effects taken into account. Tensile waves resulting in spalling fracture are formed with the output of a compressive pulse on the contact boundaries with less rigid media, including free surface. To solve the problems in question a series of software has been developed on the basis of a continual model of a damaged medium taking into account the possibility of closing and healing as well as generation and opening of macrocracks under alternate loading.

KEYWORDS

Dynamic loading, unloading waves, damage accumulation, spalling fracture, two-dimensional model, damage tensor, numerical experiment.

INTRODUCTION

The process of dynamic fracture is known to be characteristic for formation of space-oriented microdamages (MDs), and their growth and coalescence into spalling macrocracks. These MDs influence the properties of a damaged medium (DM) and, thus, it should be necessary to introduce a damage degree when describing the DM behaviour, and take it into account in the defining model equations (Akhmadeyev and Nigmatulin, 1982). Contrary to Akhmadeyev and Nigmatulin (1982) let us link the MDs to the tensor $\hat{\Xi}$, that being a symmetric second-rank one (Ilyushin, 1967). Now, let us represent the tensor $\hat{\Xi}$ as a sum of the dyads $\hat{\Xi}(\nu)$:

$$\hat{\Xi} = \sum_{\nu} \hat{\Xi}(\nu) = \sum_{\nu} \xi(\nu)\vec{n}(\nu)\otimes\vec{n}(\nu), \quad \xi(\nu) = V_{\xi}(\nu)/V. \qquad (1)$$

Here, $\xi(\nu)$ is a volime fraction (density) of the ν th MDs, $V_\xi(\nu)$ and V are volumes of the ν th MDs and DM, $\vec{n}(\nu)$ is a normal unit vector to the MDs section plane, cutting its maximum diameter, for example. It follows out of (I) that

$$\xi^{ij} = \sum_\nu \xi(\nu) n^i(\nu) n^j(\nu), \quad J_1^\xi = \xi^{ii} = \xi = V_\xi / V, \qquad (2)$$
$$(V = V_m + V_\xi, \quad V_\xi = \sum_\nu V_\xi(\nu)).$$

where J_1^ξ is the first $\hat{\xi}$ invariant equal to the MDs volume density ξ, and its limiting value ξ_* can be assumed as a microdamage criterion; V_m and V_ξ are volumes of DM and MDs matrices.

BASIC EQUATIONS

The laws of conservation and completing relations for the DM have the form:

$$\dot\rho + \rho \dot\varepsilon^{ij} \delta^{ij} = 0, \quad \rho \dot v^j = \nabla^i \sigma^{ij}, \quad \rho \dot e = \sigma^{ij} \dot\varepsilon^{ij},$$
$$\sigma^{ij} = 1/2 (\varphi^{im} \sigma^{omj} + \sigma^{oim} \varphi^{mj}), \quad \varphi^{ij} = \delta^{ij} - \alpha^{ij},$$
$$\alpha^{ij} = \gamma^{ijk\ell} f^{k\ell}, \quad \gamma^{ijk\ell} = a\delta^{ik}\delta^{j\ell} + b\delta^{ij}\delta^{k\ell}, \quad f^{k\ell} = f^{k\ell}(\xi^{k\ell}),$$
$$\rho = \rho^\circ(1 - \xi), \quad \sigma^{oij} = -p^\circ \delta^{ij} + \tau^{oij}, \quad p^\circ = p^\circ(\rho^\circ, T), \quad e^\circ = e^\circ(\rho^\circ, T),$$
$$\dot\varepsilon^{ij} = \dot\varepsilon^{oij} + \dot\Delta^{ij}, \quad \dot\Delta^{ij} = \dot\varepsilon^{ij} \dot\theta^{-1} \dot\xi / (1 - \xi), \quad \dot\theta = \dot\theta^\circ + \dot\xi / (1 - \xi), \qquad (3)$$
$$\dot\xi^{ij} = \dot\xi^{ij}(\sigma^{ok\ell}, \xi, \eta), \quad \dot\xi = \dot\xi^{ii}, \quad (i, j, k, \ell, m = 1,2,3).$$

In (3) the tensor components of deformation velocities defined through the MDs dynamics are denoted with Δ^{ij} and obtained by the assumption that $\dot\varepsilon^{ij}/\dot\varepsilon^{oij} = \dot\theta/\dot\theta^\circ$ (Kanel and Shcherban, 1980); θ is a velocity of volume deformation; φ^{ij} are components of the weakening tensor (Leckie and Hayhurst, 1977), and α^{ij} are components of the MDs surface density tensor $\hat{\mathcal{D}}$; f^{ij} is the tensor function, corresponds to structural parameters in the expression for the MDs kinetics; other notations are conventional (Akhmadeyev and Nigmatulin, 1982). The Hooke's law with the effective values of shearing modulus and yield point – $\mu = \mu^\circ(1 - R(\xi))$ and $\tau_s = \tau_s^\circ(1 - R(\xi))$, $R(\xi)$ being the weakening parameter, is applied in the elastic flow domain for the deviator τ^{oij} (the values ρ°, σ^{oij}, etc. which characterize the DM matrix properties bear the upper zero index). Here, as in Murakami and Ohno (1981), the material isotropy is assumed along with the MDs anisotropy. τ^{oij} are conserved on the yield surface while passing into the plastic yield domain.

NUMERICAL RESULTS

Two problems have been solved numerically in two-dimensional plane statement with the DM model applied: problem A – on the collision of an aluminium impactor 2x50 and iron target 10x50 (the dimensions are given in mm both here and below) with the velocity 600 m/s; problem B – on the collision of two aluminium plates 3x50 and 5x50 with the velocity 390 m/s. Numerical computations are done with the Lagrange approach applied (Swegle, 1980) under experimental conditions (Tarasov, 1974) and (Wilkins, 1964).

Problem A. Fig.I shows the position of the damaged target after the impact at the time moment $t = 4.0$ mks according to the Eiler's coordinate system xoy.

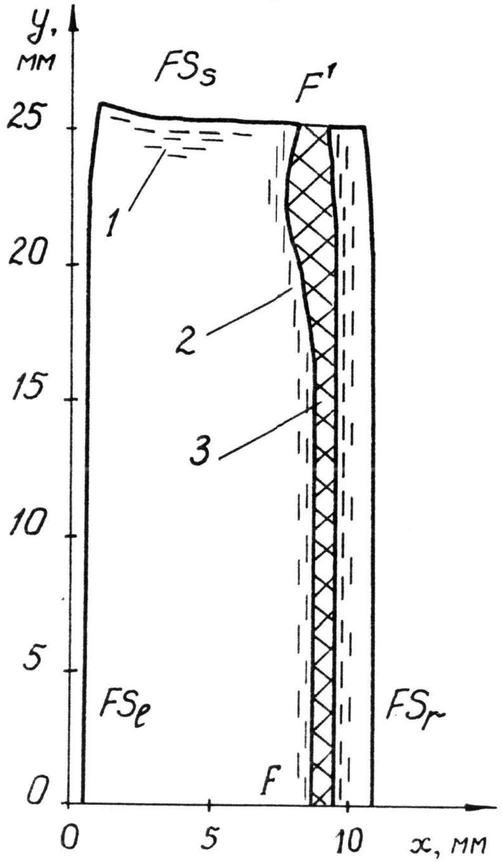

Fig.I. Iron target state at the time moment t = 4.0 mks

The target loading is performed along the Ox axis of symmetry. The MDs kinetics in the form of disc-like cracks in iron is defined in the principal axes of the stress tensor ($i, j = x^*, y^*$) while accounting the MDs effect on σ^{ij} - in those of the damage tensor ($i, j = x' y'$). Kinetics is set with the following relations (has no summing):

$$\dot\xi^{ij} = (1+\xi)(\sigma^{oi} - \sigma_L^i)/(\tau_\kappa \sigma_L^{oi}) \quad \text{at} \quad \sigma^{oi} > \sigma_L^i \quad \text{and} \quad \xi < \xi_*,$$
$$\dot\xi^{ij} = 0 \quad \text{for} \quad i \neq j, \quad \sigma_L = \sigma_L^o(1 - R(\xi)). \qquad (4)$$

In (3) and (4) the following parameters are used: $f^i = (\xi^i/\xi_*)^{1/2}$, $d^i = d - h(\xi - \xi_*)(1-d)$, $d = af^i + bJ_1^f$, $J_1^f = \sum f^i$, $a + b = 1$, b = 0.41; $R(\xi) = (\xi/\xi_*)^{1/2}$, σ_L^o = 1.8 GPa, τ_κ = 0.07 mks, ξ_* = 0.075, $h(z)$ is the Heaviside's function. In fig.1 the damage zones are marked with dashed lines: zones 1 and 2 correspond to local spalls, and zone 3 - to complete one. In the process of the target unloading damage zone 1 is the first to be formed from the time moment 1.6 mks as a result of interacted rarefaction waves moved out of the front FS_ℓ (after the recoil of the impactor) and lateral FS_s surfaces; the MDs are oriented parallel to the lateral surface in zone 1. Zones 2 and 3 are formed later on due to the target unloading at the back free FS_r surface. The interaction of head-on rarefaction waves from FS_ℓ and FS_r surfaces causes the formation of a spalling macrocrack FF', which is practically parallel to FS_r. At $y > 15$ some increase in the crack width and a slight distortion near the surface FS_s can be observed, that being explained by the influence of lateral unloading. Computational data are tested by the experimental evidence along the velocity section of the surface FS_r.

Problem B. Fig.2 and 3 show the development of a wave pattern in the plates by the diagrams in the Lagrange coordinate system XOY (OX is the axis of symmetry, OY is the line of collision) at different time moments (in mks). It is noted in Wilkins (1964) that the value of the impact threshold velocity at which spalling is formed and, thus, the velocity of the MDs growth depend on rolling orientation (in the experiments the impacts were performed along and across rolling). These data suggest to determine the velocities of the MDs growth as spherical voids in the numerical experiments. Kinetics of the MDs growth are considered in the concomitant system of the coordinates $x'oy'$, that being the principal one for $\hat\Xi$, and the axis ox' is chosen along the rolling direction. At ' = 0 the systems of the coordinates XOY and $x'oy'$ coincide. The damage tensor is represented as a sum

$$\hat\Xi = \sum_i \hat\Xi^i, \quad \hat\Xi^i = \sum_\gamma \xi^i(\gamma) n^i(\gamma) n^i(\gamma),$$

and the MDs kinetics has the form (i has no summing):

$$\dot\xi^i = (1+\xi)(\sigma^{oii} - \sigma_L^{ii})/(\tau_\kappa^{ii}\sigma_L^{oii}) \ \text{at} \ \sigma^{oii} > \sigma_L^{ii} = \sigma_L^{oii}(1 - R(\xi)) \ \text{and} \ \xi < \xi_*. \ (5)$$

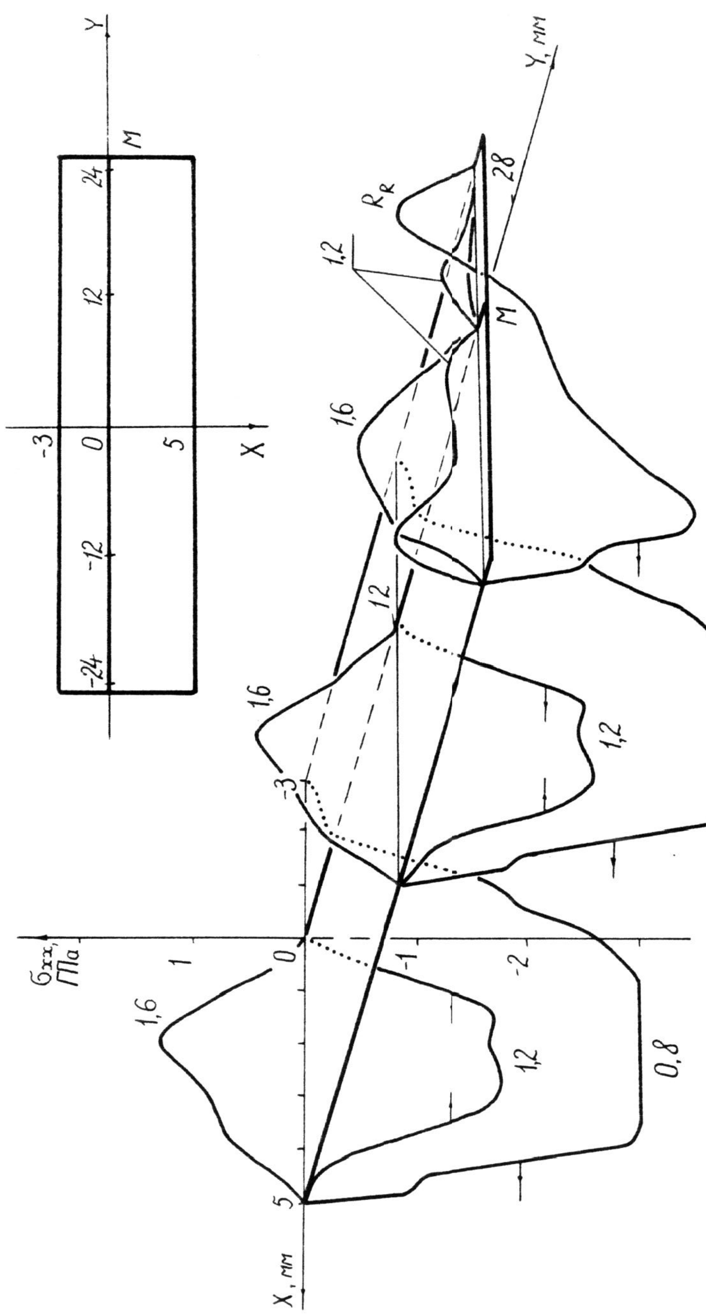

Fig.2a. Stress diagrams σ^{xx} at the time moments (in μs) 0.8, 1.2, 1.6

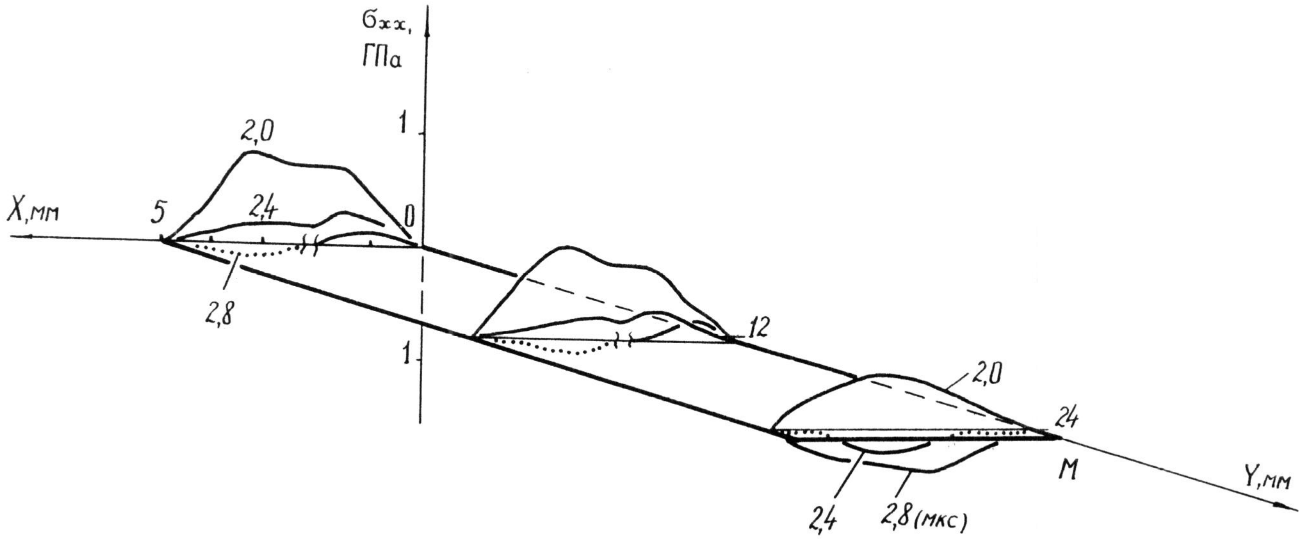

Fig.2b. Stress diagrams σ^{xx} at the time moments (in μs) 2.0, 2.4, 2.8

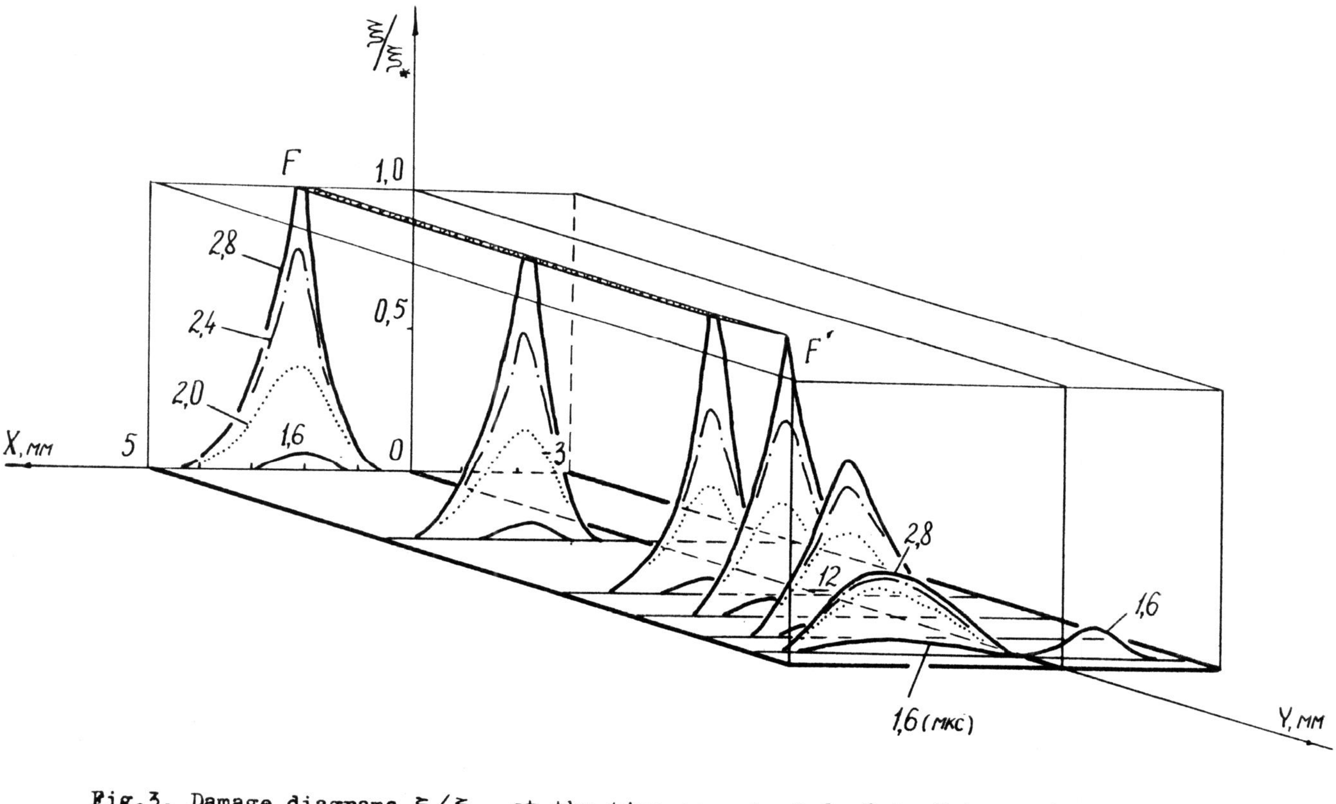

Fig.3. Damage diagrams ξ/ξ_* at the time ~~moments~~ I.6, 2.0, 2.4 and 2.8 μs in the sections of the impactor/target system at Y = 0, 8.5, I6.5, I9.0, 2I.5 and 24.0 (in mm)

Fig.2 shows the diagrams σ^{xx} in the sections $Y = 0$, 12.5 and 24. Fig.2a represents a completely formed compressive pulse, the latter reflecting from the back surface of the target ($X = 5$) unloads with a tensile pulse originated. Unloading when produced from the point M (0; 25) at the impact moment changes the flow pattern considerably near the lateral surface ($Y = 25$) and causes the earlier tension here and, as a result, the MDs growth. Fig.2b represents a final stage of the tension phase. As the MDs grow (see the MDs diagrams in fig.3) a practically complete relaxation of tension takes place in the target not only where $\xi = \xi_*$, but along the whole length of the target in the internal domain as well. It should be noted that the ratio between the components ξ^{xx} and ξ^{yy} is 3/2. The domain of complete fracture (see fig.3) presents a spalling crack 0.2 c wide appr. with its apix at the point F' (2; 21.5) lying within the compressive stress field at $t = 2.8$ mks (see fig.2b).

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