DISCUSSION OF THE TWO-DIMENSIONAL FRACTURE SURFACE MARKS OBTAINED BY THE MODEL OF A PRIMARY CRACK PROPAGATING CIRCULARLY

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ABSTRACT

The two-dimensional fracture surface marks of a both-ends closed mark (BECM) and a one-end opened mark (OEOM) observed in brittle fracture of the plastics specimens are expressed mathematically by an interference model between a primary crack propagating circularly and a secondary crack which is activated and initiated at its nucleus in front of the primary crack. This equation excels the previous equation of the marks based on a primary crack propagating linearly in that a mark at a specified primary crack propagation distance can be expressed and be related to the experimental primary crack velocity at the point.
The variation of K_{D} and $\mathscr{G}_{D}$ with respect to the crack propagation distance are calculated from the experimental primary crack velocity. The numbers and the dimensions of BECMs and OEOMs observed in a specimen are related to those values and the physical meanings of the mark is discussed.

KEYWORDS

Fracture Surface Mark, Brittle Fracture, Plastics, Equation of Marks, Model of a Crack Propagating Circularly

INTRODUCTION

Fracture surface marks of an ellipse (OEOM) and a parabola (BECM) are known to be products of an interference between a primary and a secondary propagating cracks [1], [2] and the model of generating the marks are qualitatively given [3], [4]. We have presented a quantative model of forming the marks by a primary crack propagating linearly and a secondary crack propagating circularly [5], which gave the various shapes of marks having the dimensions of e (interference distance), ℓ (chordal length at its nucleus), d (critical

distance for the primary crack to activate the secondary crack nucleus) and so on. The model expressed the shapes and physics of the individual experimental fracture surface marks nicely.
Here, we present a model of generating the marks by an interference between a primary and a secondary cracks proceeding circularly from their initiation points. Analytical relations by this model are given in detail and the comparison between this model and the previous model is presented. Experimental data of the fracture surface marks of a center-notched tensile sheet specimen of unsaturated polyester resins (UP) are given. The frequency distribution of velocity ratio $\beta_c' = V_{2max}/V_1$ is calculated by the equation of the marks based on this model using the experimental values of e and ℓ. The dynamic elastic-strain energy release rate and the dynamic fracture toughness related to generate the marks are estimated.

A THEORETICAL MODEL AND AN EXPERIMENT

We take the fracture of a sheet specimen of thickness t, having a center crack length of $2C_{01}$ under tension. A primary crack initiates at the point O_1 of the crack tip in Fig.1. The crack propagates circularly a distance of p^*-d and it will activate a secondary crack nucleus at O_2. Then, the secondary crack initiates and propagates circularly. The primary and secondary crack fronts first interfere at x=-e and the locus of the succeeding interference points shows

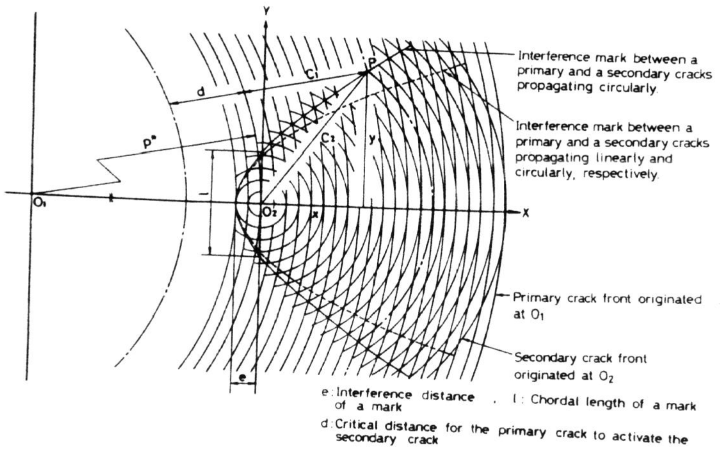


Fig.1 A model of the interference marks between a primary and a secondary crack fronts. The symbols e, ℓ and d of a solid-line parabola are defined in the figure.

a contour of a parabola (the solid-line parabola in Fig.1). Here, we define a critical distance d , an interference distance of the mark, e and a chordal length at the nucleus of the mark, ℓ. These values give the physical meanings of a mark. Mathematics of this model is given below. The primary crack is assumed to propagate with a velocity given by Berry [6]. The velocity for the primary crack to propagate the very short distance $\sqrt{(p^*+x)^2+y^2}-p^*+d$ ($\leq 200\mu m$) is calculated to increase very little and it may be set to be nearly constant. Then, the time (t_1) needed for the primary crack to propagate the above distance is,

$$t_1=\{\sqrt{(p^*+x)^2+y^2}-p^*+d\}/V_1 \quad (1)$$

Similarly, the secondary crack is assumed to propagate with velocity [6] and the time (t_2) needed for the secondary crack to propagate the length C_2 is given by

$$t_2=(C_{02}/V_{2max})[\sqrt{C_2/C_{02}-1}\sqrt{C_2/C_{02}-(n_2-1)} + n_2\ln\{\sqrt{C_2/C_{02}-1}+\sqrt{C_2/C_{02}-(n_2-1)}\} - (n_2/2)\ln(2-n_2)] \quad (2)$$

where C_2 : the secondary crack propagation distance.
C_{02}: a half of the secondary crack nucleus ($C_{02}\leq 1\mu m$).
V_{2max}, n_2: material's constants.

Setting $t_1=t_2$, we obtain the equation of the interference marks as,

$$\sqrt{C_2/C_{02}-1}\sqrt{C_2/C_{02}-(n_2-1)} + n_2\ln\{\sqrt{C_2/C_{02}-1}+\sqrt{C_2/C_{02}-(n_2-1)}\} - (n_2/2)\ln(2-n_2) = \beta_c'\{\sqrt{(p^*+x)^2+y^2}-p^*+d\}/C_{02} \quad (3)$$

where $\beta_c' = V_{2max}/V_1$: the velocity ratio.

The parabola, shown by the dotted line in Fig.1 is the one by the model of the primary crack propagating linearly.
The numbers of the experimental fracture surface marks and their dimensions are measured under the microscope on a sheet specimen of UP fractured under tension. The dimensions of the specimen was 260mm length ×240mm width ×4mm thickness having a center crack of $2C_{01}$=9.76mm (its crack tip radius $\leq 3\sim 5\mu m$).

RESULTS AND DISCUSSION

<u>Theoretical marks and relationships</u> The various interference marks by Eq.(3), are presented in Fig.2, which are very close to the experimental fracture surface marks. Here, p^*/C_{02}=500 , β_c'=1.0, n_2=1.6 are assumed. The marks change from a small ellipse to a parabola as d/C_{02} increases. The d/C_{02} value generally increases as the primary crack velocity increases. The chordal length ℓ at a

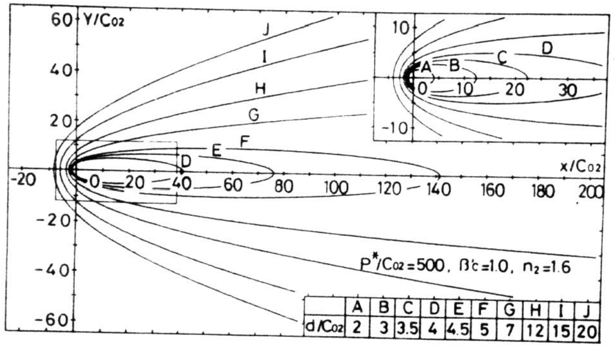


Fig.2 Example of various interference marks given by Eq.(3).

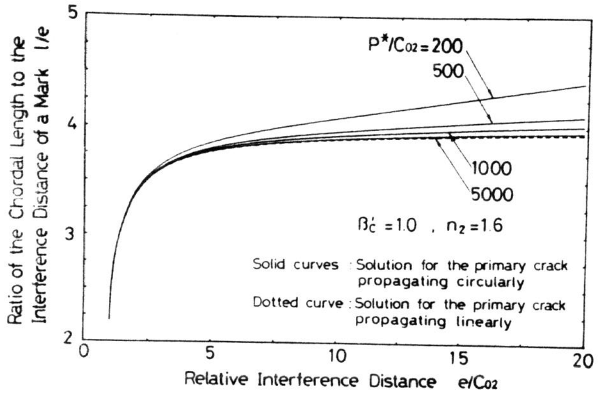


Fig.3 Theoretical relations between ℓ/e and e/C_{o2} of a mark. Effect of the propagation distance p^*/C_{o2} is shown.

nucleus of a mark increases with the interference distance e. Theoretical relation between ℓ/e and e/C_{o2} by Eq.(3) is given in Fig.3 for $\beta_c'=1.0$, $n_2=1.6$. The ℓ/e value approaches 4.0 as e/C_{o2} and p^*/C_{o2} increase. Here, we show only a theoretical relationship which is qualitatively clear in Fig.2. Eq.(3) gives the other useful theoretical relationships of the marks by changing the value of a specified term and setting the rest of terms to be constant.

Experimental fracture surface marks The fracture surface of the tensile sheet specimen of UP presented a clear change of the fracture marks from a small

ellipse to a parabola and/or a nearly hyperbola as the primary crack propagated to fracture. The numbers of each appeared mark changed with both the propagation distance and the velocity. Fig.4 shows the variation of numbers of the BECMs and the OEOMs which have the secondary crack nuclei (a nucleus is not always observed in every mark) as a function of the relative propagation distance α_1

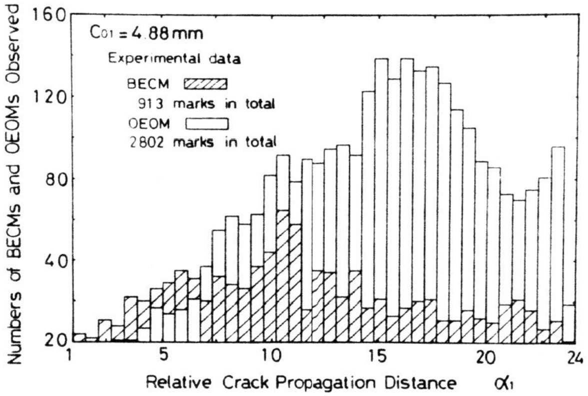


Fig.4 Observed numbers of the one-end opened marks (OEOM) and the both-ends closed marks (BECM) are shown as a function of the relative crack propagation distance $\alpha_1=C_1/C_{o1}$.

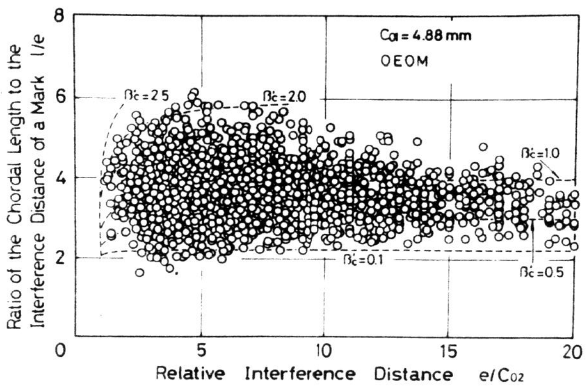


Fig.5 Experimental data of ℓ/e and e/C_{o2} of the OEOMs (the marks in Fig.4) and theoretical relations(the dotted curves) by Eq.(3).

$=C_1/C_{01}$. The number of the BECMs increased to a peak value and it decreased as α_1 increased. The OEOMs appeared at a little later α_1 and it reached a nearly constant number as α_1 increased to fracture point (here, many marks without nuclei were also observed). The experimental ℓ/e and e/C_{02} values of OEOMs only marks which have clear nuclei) are measured under microscope (by the magnification of 200×~1500×) and their relationship is plotted in Fig.5. The ℓ/e value shows the smallest one of nearly 2 and the largest one of nearly 6. This value is controlled not only by e/C_{02} and p^*/C_{02} as shown in fig.3, but also by β_c'. We know that the ℓ/e value increases as β_c' increases and as p^*/C_{02} decreases by the equation of the marks, Eq(3). The larger ℓ/e values with the smaller e/C_{02} values in this figure are seemed to be due to the larger β_c' values in the process of generating the marks.

Calculated frequency distribution of β_c' When e/C_{02}, ℓ/e and p^*/C_{02} of an experimental mark are measured, we can calculate β_c' of the mark by Eq.(3), setting n_2 to be constant. Fig.6 gives calculated frequency distribution of β_c', f_β, of the experimental marks by the measured e/C_{02} and ℓ/e values in Fig.5, setting p^*/C_{02} to be a value (as a simple example). Two different distributions are for the marks appeared in the narrow areas of fracture surface, i.e., 5.0 (C_1=24.4mm)$<\alpha_1<$7.5 (C_1=36.6mm) and 17.5 (C_1=85.4mm)$<\alpha_1<$20.0 (C_1=97.6mm). The calculated f_β for the marks in the band of $5.0<\alpha_1<7.5$ are on the left-hand abscissa of the figure and these small values by Eq.(3) give more or less the BECMs. On the other hand, the values for marks in the band of $17.5<\alpha_1<20.0$, distribute on all over the abscissa in the figure. The larger β_c' value in Eq.(3) gives the OEOMs.

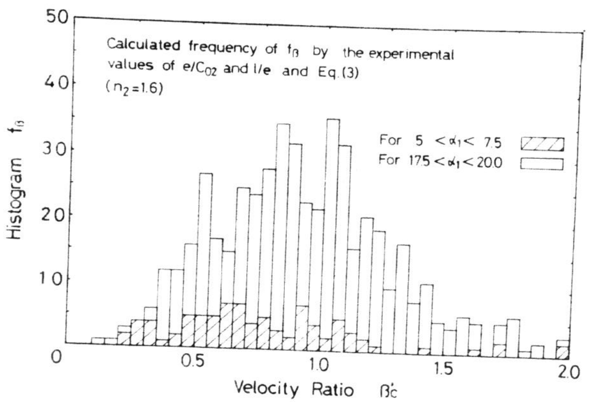


Fig.6 Frequency distribution of velocity ratio β_c', f_β is calculated by Eq.(3) and the experimental e/C_{02} and ℓ/C_{02} in the range of α_1 indicated.

Crack velocity and dynamic elastic-energy release rate We have measured the experimental crack velocities of about 20 sheet specimens of up having center-cracks under tension, and we know that the crack velocity of the specimens having the shorter crack length than $2C_{01}\fallingdotseq$15mm (the specimen width =240mm) is nicely expressed by the theoretical solution by Berry [4] under constant load. We

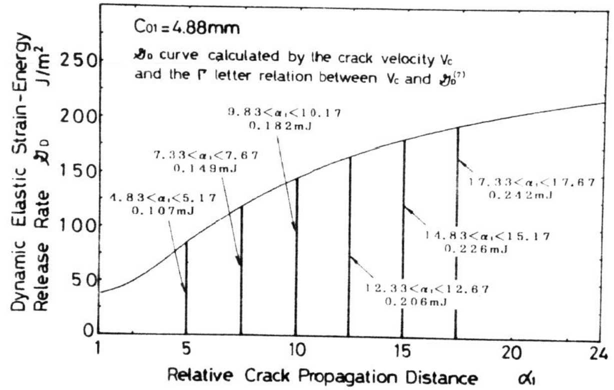
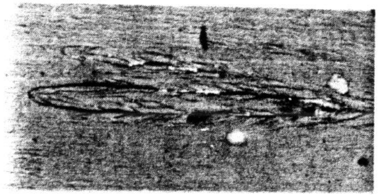


Fig.7 Variation of $\mathscr{G}_D$ with respect to α_1. The calculated energy, released in the indicated narrow areas at each $\Delta\alpha_1$, is given.

(a)

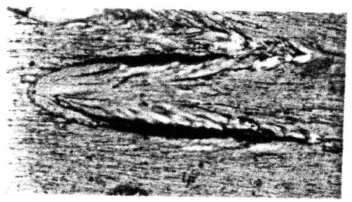


α_1 = 5. 4

0. 79 μJ

20 μm

(b)



α_1 = 1 0. 6

3. 3 6 μJ

20 μm

Fig.8 Experimental marks at the indicated α_1 and the energies supposedly related to these.

calculate the K_D variation with respect to the crack propagation distance α_1 using the Γ-letter relation between the velocity and K_D (7), (8) and $\mathcal{G}_D$ is calculated by the equation (9), and the solid curve in Fig.7 is that. We calculate the dynamic elastic energy, released in the very narrow area (crack propagation distance ΔC_1 × specimen thickness t) indicated, in the figure. The energy corresponding to each narrow area is supposed to be released to create fracture surfaces of the plane surface (without the BECMs and the OEOMs), the surface covered by the BECMs and the one by the OEOMs. We estimate the energies supposedly related with a BECM and a OEOM at a specified α_1 from the energy released and the surface areas measured. The examples are given in Fig.8.

CONCLUSION

The equation of the interference marks is derived by a model of a primary and a secondary crack propagating circularly. It gives various useful theoretical relations of the marks. The numbers and dimensions of the experimental fracture surface marks are measured on a sheet specimen of UP under tension. The theoretical relations by the equation are shown in qualitative agreement with the experimental results. The crack velocity is measured and the energies supposedly related to generate a BECM and a OEOM are estimated.

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