CRITERION OF DYNAMIC FRACTURE OF BRITTLE SOLIDS

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ABSTRACT

The problem of modeling a dynamic fracture under the high rate loading is discussed. New structure-time criterion that reflects a "quantum" nature of dynamic failure process is presented. Approach suggested in this paper associates the dynamic strength properties of the brittle media with the special material parameter dimension of time. In case of crack growth initiation problem the introduced structure time may be interpreted as an incubation time in well-known minimum time conception. The problem of crack instability under the short pulse loads is analyzed. The results of well-known experiments in terms of an assumed structure-time criterion are discussed.

KEYWORDS

Fracture criterion, dynamic loading, structure time, critical stress intensity, fracture toughness, short pulse load, crack instability.

INTRODUCTION

In many practical cases materials and structures experience loading rates many orders of magnitude greater than in the quasistatical conditions. Therefore it is very important to work out an effective handy method of exploration and testing the dynamic strength properties of the materials that operate at high loading rates. There are some principle effects of dynamic fracture of brittle solids that can not be explained on the basis of traditional models of failure (Nikiforovsky and Shemiakin, 1979; Knauss, 1984; Kalthoff, 1986). Even typically brittle failure is remarkable of some dynamic effects which can not be analyzed by Griffith and Irwin criteria. One of the most essential problem is modeling the process of crack growth initiation and crack instability in dynamic conditions The difficulties in determining the critical parameters of dynamic fracture of brittle media and specific behaviour of the dynamic strength and fracture toughness at high loading rates generate the different opinions about possible ways of description of dynamic failure of materials. Morozov and Petrov (1990) supposed that well-known rate dependencies of dynamic fracture toughness of brittle media may be not interpreted as the material functions and showed the principal way to the analysis of the effects of rate of loading within the framework of the system of fixed material parameters.

Approach suggested in the present paper associates the dynamic strength peculiarities of the brittle media with the existence of the special physical constant, dimension of time, that must supplement the quasistatical fracture toughness and strength of the material forming, jointly with them, the united system of the fixed material parameters. In special case the introduced structure time may be interpreted as an incubation time in well-known minimum time criterion proposed and explored by Kalthoff and Shockey (1977), Homma et al. (1983), Shockey et al. (1986). Brittle fracture theory suggested in the present paper allows to calculate the fracture toughness rate dependencies on the basis of a system of the fixed material parameters.

CRITICAL STRESS INTENSITY IN CONTINUAL MECHANICS

We shall demonstrate that the conventional continual mechanics principle of critical intensity of the stress field is not consistent with the fundamental low of conservation (change) of momentum in dynamic conditions.

Lets consider the one-dimensional problem of spalling and suppose that rupture of the material is caused by the triangular pulse of stress of duration T. We determine the threshold, i.e. minimal for given T, value of momentum providing the failure $U=U_c(T)$. Using classical critical stress criterion $\sigma \leq \sigma_c$ we obtain $U_c=1/2\sigma_c T$. The appropriate threshold is shown in Fig.1 by dotted line. We can see that in accordance with that criterion even the infinitely small pulses (momenta) can produce a macrofracture.

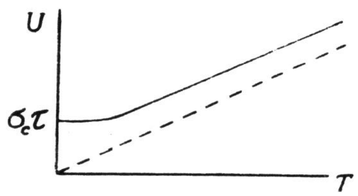

Fig.1. Limiting values of momenta by spalling

Let the semi-infinite straight crack is situated in the unbounded plate and loaded on its faces $x \leq 0$, $y=0$ by the uniform rectangular stress pulse of amplitude P and duration T: $\sigma_y=-P[H(t)-H(t-T)]$, $\sigma_{xy}=0$, where $H(t)$ is the Heaviside function. The corresponding expression for the maximal normal stress on the prospective crack line ($\vartheta=0$, $r=x$) is given by

$$\sigma_y = K_I(t)/\sqrt{2\pi r} + O(1), \quad r \longrightarrow 0; \tag{1}$$

$$K_I(t) = P\Phi(c_1,c_2)f(t), \quad f(t) = [\sqrt{t}\,H(t) - \sqrt{t-T}\,H(t-T)],$$

where $\Phi(c_1,c_2)=4c_2\sqrt{c_1^2-c_2^2}/(c_1\sqrt{\pi c_1})$, c_1, c_2 are velocities of longitudinal and shear waves. Using classical critical stress intensity criterion $K_I \leq K_{Ic}$ we receive $U_c = K_{Ic}\sqrt{T}/\Phi(c_1,c_2)$. Therefore at $T \longrightarrow 0$ the threshold momenta become infinitely small.

These conclusions contradict to common sense and the well-known experiments on fast fracture show various effects that are not consistent with the conventional approaches.

It is assumed in classical criteria that in dynamic failure processes the energy and momentum used to form the new surfaces and damaged domains in material are always consumed by the continuous way. We want to show in the next section that the elementary taking into account a discrete nature of dynamic fracture process allows to avoid some contradictions of the traditional "continual" models.

DISCRETE NATURE OF DYNAMIC FAILURE PROCESS

The principal parameter of linear fracture mechanics is a certain structure size d that describes the elementary cell of failure. The classical approaches by Griffith and Irwin contain this characteristic as a latent quality. In case of plane stress state and brittle fracture it is convenient to take (Morozov, 1984): $d=K_{Ic}^2/(\pi\sigma_c^2)$. The elementary cell of fracture has not got a simple physical interpretation. It may be interpreted by various ways depending on class of problems.

Lets put in consideration the elementary portion ("quantum") of momentum required to make a failure of one structure cell: $U_1=\sigma_c\tau$; here τ is structure time of fracture postulated as a given parameter for the material and appropriate class of problem.

Being in conditions of spalling we assume that a threshold momentum of given duration T was produced in medium and some of the structure elements were destroyed. To produce a failure of m structure cells there are required the pulses:

$$U_m = \sigma_c \tau\, m, \qquad m = 1,2,3,\ldots \tag{2}$$

We regard the parent distribution:

$$P_m = C \exp(-U_m/\alpha T), \tag{3}$$

where P_m – the probability of failure of m structure elements, α is parameter dependent on the profile of the applied stress pulse and determinate by the condition of coincidence the threshold critical momenta with the corresponding quasistatic values in case of long-duration pulses. C is normalizing factor defined by the correlation $\sum_m P_m = 1$. An average value of the threshold momentum of failure can be determined from the formula:

$$U = \sum_m P_m U_m \tag{4}$$

and so for the above mentioned triangular pulse of stress from (2),(3) and (4) becomes:

$$U = \sigma_c \tau/(1 - \exp(-2\tau/T)) \tag{5}$$

The appropriate threshold is shown in Fig.1 by solid curve. We can see that now the finite limiting values of critical momenta correspond to the short-duration loading pulses. For the long-duration loads $\tau \ll T$ and so the threshold characteristics may be calculated on the basis of critical stress criterion: $U \simeq 1/2 \sigma_c T$.

STRUCTURE-TIME FRACTURE CRITERION

We define failure as a breakage of at any rate one structure element (Novozhilov, 1969). Then the corresponding criterion may be written in the form:

$$\int_{t-\tau}^{t} \sigma(t')\, dt' \leq \sigma_c \tau . \tag{6}$$

In accordance to (6) dynamic strength of the "defectless" materials can be considered as a calculated characteristic. Criterion of this type allows to construct analytically the time-strength dependence in conditions of spalling (Morozov, Petrov and Utkin, 1990)

In case of crack-containing solids we consider the average, within the limits of structure size d, values of the local tension stresses. Therefore the rupture of material at the defect's tip (r=0) can be explored by means of criterion:

$$J(t) \leq J_c, \quad J(t) = \int_{t-\tau}^{t} dt' \int_0^d \sigma(t',r)\, dr, \quad J_c = \sigma_c \tau\, d . \tag{7}$$

Time of fracture t_* must be determinate by equation $J(t_*) = J_c$. By search for the parameters of load corresponding to the minimum of the expenditures to failure it is important to have the threshold condition:

$$J(t_*) = \max_t J(t) = J_c, \tag{8}$$

which may be considered as a condition of instability.

In special case of slow process the quasistatical criterion by Neuber and Novozhilov (Neuber, 1937; Novozhilov, 1969) from (7) comes out.

SHORT-PULSE CRACK INSTABILITY

We want to demonstrate that the effects of cracks instability and specific bahaviour of dynamic fracture toughness of brittle materials can be obtained and analyzed within the framework of the simplest theoretical model and suggested criterion.

We are interested in critical pulse amplitude for crack instability. It may be determined from (1) and (8) and expressed by simple formula:

$$P_1 = \frac{\tau K_{Ic}}{\Phi(c_1, c_2) \max_t \int_{t-\tau}^{t} f(s)\, ds} \tag{9}$$

On the other hand from traditional criterion of maximum stress intensity $\max_t K_I(t) = K_{Ic}$ (Sih, 1968) becomes

$$P_2 = \frac{K_{Ic}}{\Phi(c_1, c_2) \max_t f(t)} \tag{10}$$

In our case we find that

$$\max_{t} \frac{1}{\tau} \int_{t-\tau}^{t} f(s)\, ds \leqslant \max_{t} f(t), \qquad (11)$$

and so it follows from (9),(10) and (11) that

$$P_1 > P_2. \qquad (12)$$

This result conforms with the experiments on short pulse crack instability (Shockey et al., 1986), in which was shown that for short-duration pulses (so, that the stress intensity history were independent of crack length) the critical values of the stress amplitudes were greater than the same values calculated from maximum stress intensity criterion and could not be explained by classical fracture mechanics.

STRUCTURE TIME AND INITIATION TOUGHNESS

The dependence of stress intensity at the crack tip on time is shown in Fig.2. The calculations show that in accordance to structure-time criterion (7) the instable growth of the crack does occur with the delay, i.e. in stage of diminution of the local stress intensity near the crack tip. At the moment of fracture t_* integral $\int_{t-\tau}^{t} f(s)ds$ reaches its maximum value, therefore $f(t_* - \tau) = f(t_*)$. Due to monotony properties of function $f(t)$ we can conclude that $K_I(t)$ exceeds the value $K_{Id} = K(t_*)$ for the period τ.

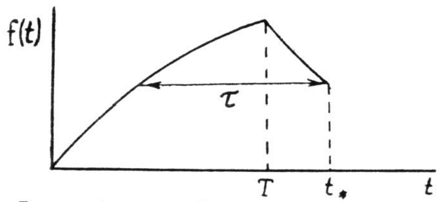

Fig.2. Dependence of stress intensity at the crack tip on current time.

Thus theoretical analysis of the crack instability shows that structure parameter τ has the features of incubation time t_{inc} introduced by Kalthoff and Shockey (1977), and so

for the problems of crack growth initiation it may be taken

$$\tau = t_{inc} \qquad (13)$$

It is noteworthy that dynamic fracture toughness may here be regarded as a calculated characteristic. The values of the dynamic fracture toughness become less than the appropriate quasistatical value

$$K_{Id} = P_1 \Phi(c_1, c_2) f(t_*) = \frac{\tau K_{Ic} f(t_*)}{\int_{t_*-\tau}^{t_*} f(s)\, ds} < K_{Ic}, \qquad (14)$$

that was observed in the above mentioned crack instability experiments.

Taking, for example, the characteristics of 4340 steel $c_1 =$ 6mm/μsec, $\nu=0.3$, $K_{Ic} = 47$ MPa, $\sigma_c = 1490$ MPa, $t_{inc} = 7$ μsec and duration of pulse T=18 μsec we receive from (9),(13) the value of critical stress amplitude $P_1 =$ 155 MPa that well corroborates with the experiments by Homma et al.(1983), where was obtained the same value of critical stress pulse amplitude providing the jump of instable crack for the distance $d = 2K_{Ic}^2/(\pi\sigma_c^2) \simeq 0.6$mm.

CONCLUSIONS

In the present paper the new structure-time approach was considered that allowed to get rid of the principal shortcoming of the conventional "continual" models. This approach gives the finite limiting values of the critical momenta for the infinitely short durations of the loading pulses. The connection between critical stress intensity factor of crack growth initiation and history of loading becomes within the framework of brittle fracture model described by set of fixed material parameters.

The principal parameter that is responsible for dynamic reaction by brittle failure is the certain structure time τ, which supplements the quasistatical fracture characteristics system σ_c and K_{Ic} (or d). Analysis of instability of cracks under short pulse loads has shown that introduced structure time could be interpreted as an incubation time in well-known minimum time criterion by Kalthoff and Shockey (1977).

Specific behaviour of dynamic fracture toughness and critical stress amplitudes that corresponds to the instable growth of macrocrack can be theoretically predicted and calculated. The results indicate good corroboration between theoretical predictions and well-known experiments.

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REFERENCES

Homma, H, Shockey, D.A., and Murayama, Y. (1983). Response of Cracks in Structural Materials to Short Pulse Loads, *J. Mech. Phys. Solids*, Vol.31, 261-279.

Kalthoff, J.F. (1986). Fracture Behavior Under High Rates of Loading, *Engineering Fracture Mechanics*, Vol.23, 289-298.

Kalthoff, J.F., and Shockey, D.A. (1977). Instability of Cracks Under Impulse Loads, *Journal of Applied Physics*, Vol.48, 986-993.

Knauss, W.G (1984). Fundamental Problems in Dynamic Fracture. In: *Advances in Fracture Research. Proceedings of the ICF-6* (S.R.Vallury et al. eds.), Vol.1, Pergamon Press, Oxford-New York, 625-652.

Morozov, N.F. (1984). *Mathematical Questions of Crack Theory*, Nauka, Moscow (in Russian).

Morozov, N.F., and Petrov, Yu.V. (1990). Dynamic Fracture Toughness in Crack Growth Initiation Problems, *Izv.AN SSSR. MTT (Solid Mechanics)*, No.6, 108-111 (in Russian).

Morozov, N.F., Petrov, Yu.V., and Utkin A.A. (1990). On the Analysis of Spalling within the Framework of Structural Fracture Mechanics, *Dokl. AN SSSR*, Vol.313, 276-279 (in Russian).

Neuber, H. (1937). *Kerbspannungslehre (Notch Stresses)*, Verlag von Julius Springer, Berlin.

Nikiforovsky,V.S., and Shemiakin E.I. (1979). *Dynamic Fracture of Solids*, Nauka, Moscow (in Russian).

Novozhilov, V.V. (1969). Necessary and Sufficient Criterion of Brittle Strength, *Prikl. Matematica i Mechanika*, Vol.33, 212-222 (in Russian).

Shockey, D.A.,and Erlich, D.C., Kalthoff, J.F. and Homma, H. (1986). Short-Pulse Fracture Mechanics, *Engineering Fracture Mechanics*, Vol.23, pp.311-319.

Sih, G.C. (1968). Some elastodynamic problems of cracks, *Int. J. Fract. Mech.*, Vol.4, pp.51-68.