

## CRACK RESISTANCE OF THE STRUCTURAL ELEMENT WITH A SMALL CRACK

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### ABSTRACT

The method of determining the residual strength and the crack growth rate prediction for small cracks (from tenths of a millimeter to a few millimeters) in Al-alloys is presented. The residual strength determination method is based on the modification to the crack-resistance limit modification, introduced by Morozov E.M. (Parton and Morozov, 1974) for small cracks, propagating from stress concentrators, as well as on the replacement of the actual crack by the equivalent one. The approach of obtaining fatigue crack growth curves (FCGC) in the small crack range is proposed for small crack growth time analysis.

### KEYWORDS

Residual strength at small cracks, equivalent crack, small crack growth rate.

Small crack behaviour analysis at crack length from tenths parts of a millimeter to a few millimeters became the necessary part in the study of crack resistance and predict fatigue life. Some typical properties of small cracks, differing from the well-investigated long cracks properties can be noted: 1. Static failure of the structural element with a small crack can occur at the significant plastic deformations in the crack location. 2. Small crack size cannot often be defined by one parameter, by the length; crack front shape should be accounted that is not known beforehand.

Therefore, it is difficult to determine of structural element crack resistance without simplification and some assumptions. It is also important for the criteria for and both ranges to be based on a unified methodology the same material properties. This paper presents the method for determining crack resistance of flat elements with small cracks; use is made of stress intensity factors and material properties obtainable at standard crack resistance certification tests. Some aspects of this approach, as to the static and cyclic crack resistance are presen-

ted in the papers of Zverev et al. (1984) and Zverev et al. (1986).

In addition, the problems connected with modeling of the crack, with computation of stress intensity factors (SIF) at various stress concentration factor  $K_t$  and with approximation of FCGC over the entire reasonable range of the crack lengths are considered.

The small crack shape analysis performed on results of Al-alloy specimens with different geometric stress concentrators ( $K_t = 1 - 6$ ) shows that a stress concentrator with  $K_t < 2.5$  generates a corner crack covering a quarter of the circle. It should be noted that stress concentrators usually initiate one or two cracks. The number of fatigue crack initiation sites increases as  $K_t$  increases and finally these cracks can be considered a through-thickness crack at large enough  $K_t$  ( $K_t > 5$ ), (Fig. 1). As well, a multi-site damage appears under some types of random loading.

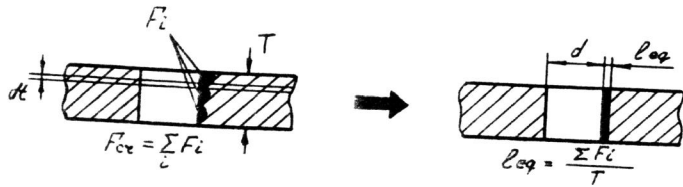


Fig. 1. Model presentation of small cracks.

To unify the description of small crack sizes at crack resistance analysis it is proposed to simulate the actual crack (or cracks) as a through-thickness one equal to the actual crack in respect of the area, as described by Zverev et al. (1984). Consider the adequacy and assumptions of the proposed approach by using the analysis of criterion relation for the body with a crack of arbitrary configuration. Write down an energy equation (Irwin, 1962):

$$d(U+H-W)/da = 0 \quad (1)$$

where  $U, H$  and  $W$  are the body strain energy, the external work and the energy of crack surface forming, respectively. Divide the body into layers of thickness  $da$  (Fig. 1) and use the following relations from Broek (1986)

$$d(U^*+H^*)/da = (\pi \cdot G_c^* \cdot d \cdot f_c)/E \quad (2)$$

$$dW^*/da = \gamma \quad (3)$$

where  $U^*, H^*$  and  $W^*$  are the above energies referred to the unit thickness;  $\gamma$  is specific density of fracture surface formation energy ( $\gamma = K_t^2/E$  in the case of the combined plane stress state);  $a$  is a crack front point coordinate;  $f_c$  is the function to account for layer interaction;  $G_c$  is the applied stress to failure. Then the criterion (1) can be written as

$$\frac{\pi \cdot G_c^*}{E} \cdot \int_0^l d \cdot f_c dt - \gamma \cdot \int_0^l dt = 0 \quad (4)$$

Assuming that layers affect each other insignificantly (due to an extensive plastic area near the crack front) the function  $f_c$  may be taken to be constant through the thickness and equal to 1. Relation (4) can now have a form of

$$\frac{\pi \cdot G_c^*}{E} \cdot \frac{F_{cx}}{T} - \gamma = 0 \quad (5)$$

where  $F_{cx}$  is the total crack area.

The relation (5) is similar to the criterion for the body with a through-thickness crack of length  $l$ , outlined from eq. (1), too (Broek, 1986)

$$\frac{\pi \cdot G_c^*}{E} \cdot l - \gamma = 0 \quad (6)$$

The multiplier  $F_{cx}/T$  in the left-hand side of relation (5) may be considered as the reduced through-crack length.

The test results confirming the opportunity to use the crack area as a damage measure are presented in fig. 2.

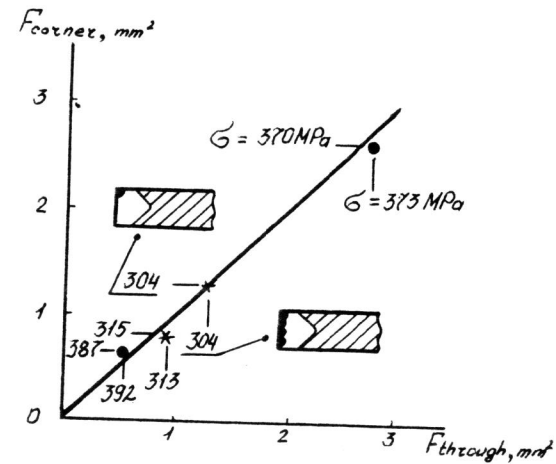


Fig. 2. Relation of areas of through- and corner cracks at similar values of  $G_c$ .

Each point here is to two specimens carrying the same failure load, but differing in shapes. The crack shape after the crack growth before the failure moment is also shown in this figure. It is seen that different initial crack shapes transform into similar shapes after the growth before at failure, this transformation covering the whole specimen thickness.

To describe the critical failure curve for small cracks the

function of crack resistance limit, introduced Morozov E.M. (Parton and Morozov, 1974), should be used:

$$I_c = K_c(\sigma_c) \quad (7)$$

that can due to Zverev(1984) be written as

$$I_c = K_c \left\{ 1 - \left[ \frac{\sigma_c}{(\varphi \cdot \sigma_s)} \right]^2 \right\}^{1/2} \quad (8)$$

where  $I$  is a function of crack-resistance limit (i.e. the limit values of  $K_c$ );

$$\varphi = \left[ 1 - (1 - \sigma_s/\sigma_c^0) / (1 + \rho^2/\rho^2) \right]^{-1}$$

is the factor to correct for the presence of a stress concentrator;  $\sigma_s$  is the ultimate strength;  $\sigma_c^0$  is the applied stress to failure of element without a crack;  $\rho$  is the radius at the stress concentrator tip;  $K_c$  is the critical stress intensity factor to be obtained by testing the usual specimens with large cracks at  $l/B = 1/3$ ;  $B$  is specimen width.

The predicted critical failure curves for various types of stress concentrators are shown in fig.3.

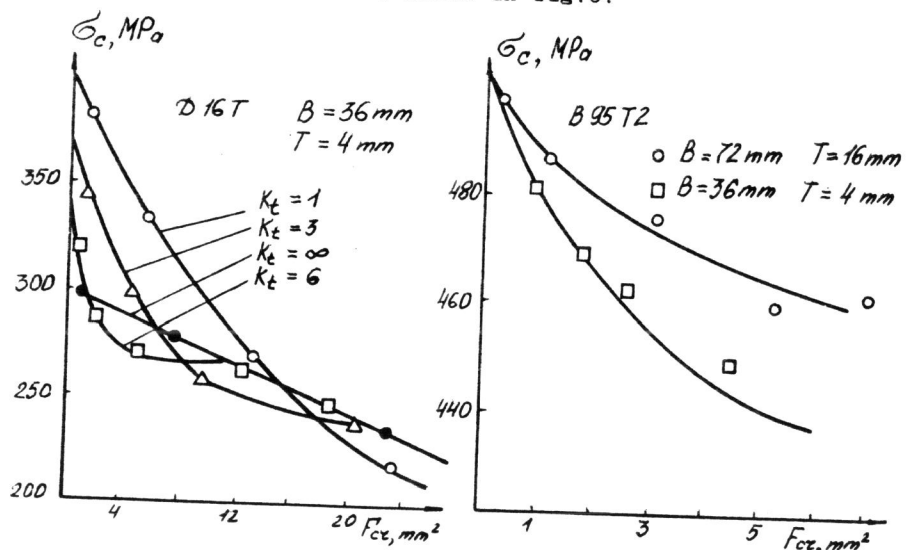


Fig.3. Specimen residual strength curve.

Here the following expression giving the maximum error of 5% in stress intensity factor of is used for the stress intensity factor of cracks near elliptical notches.

$$K = \sigma_c \cdot \sqrt{\pi \cdot \rho_c} \cdot \varphi \quad (9)$$

where  $\varphi = \varphi_1 \cdot \varphi_2 \cdot \varphi_3$ ;

$$\varphi_1 = [1 + (1.122 \cdot K_t - 1) / (1 + \rho/\rho)^{3.5}] \cdot [1 - (1 - \sqrt{1 + B/\rho}) / (1 + \rho^2/\rho^2)]$$

$$\varphi_2 = 1 / \{ 1 - (2 \cdot \rho/B) \cdot [1 - 1 / (1 + d^2 / [(B - \rho - B) \cdot (\rho + B) \cdot (K_t - 0.9)])] \}$$

$$\varphi_3 = \sqrt{\sec(\pi \cdot (\rho + B) / B)}$$

is the coefficient to

correct for the finite specimen size;

$\rho$  is the ellipse small semi-axis.

For obtaining the residual strength curve, all necessary parameters were taken from standard static crack resistance tests.

Such modeling of small crack shapes by the through thickness cracks was carried out to describe their growth kinetics. The kinetic failure curve for small cracks growing from the hole is shown in fig.4.

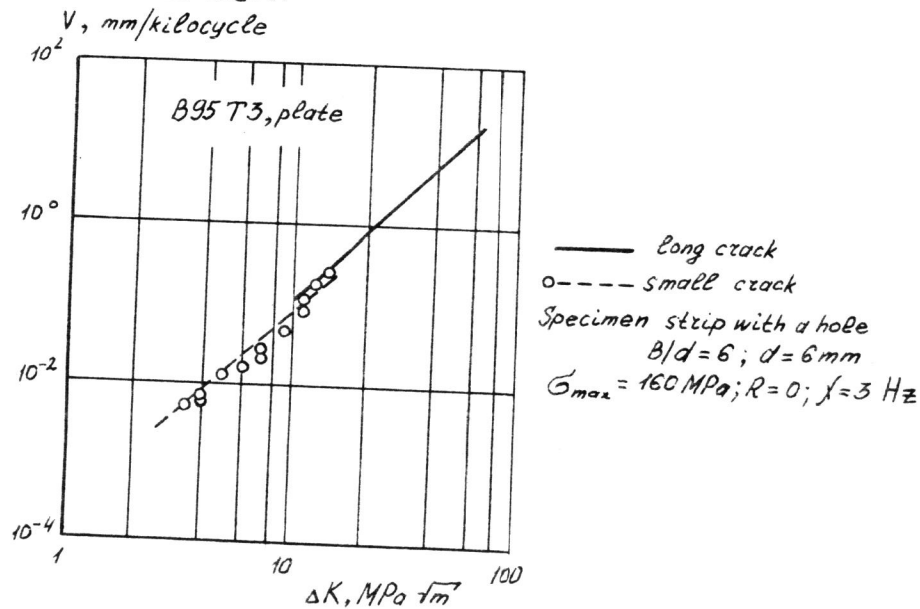


Fig.4. Long and small crack growth rates.

The method to measure the crack length during the cyclic loading (from  $F^2 = 0.05 \text{ mm}^2$ ) is described by Zverev et al.(1986). The comparison of kinetic failure curves for small and long cracks developed by Zverev et al.(1986) and in the present paper shows that the curves  $V = f(\Delta K)$  for small cracks may be considered as linear continuations of the middle part in the relationship  $V = f(\Delta K)$  for long cracks. This gives an opportunity to use the results of standard tests for the evaluation of small

crack growth rate.

To analyze the crack growth time in the structural elements from any initial flow to the failure it is necessary to draw the upper and the lower parts of kinetic failure curve for small cracks. To draw the upper part of this curve the modified Forman equation outlined by Makhutov et al. (1982) is suggested, considering the applied cyclic load level:

$$V = C \cdot \Delta K^m / [(1-R) \cdot I_c - \Delta K] \quad (10)$$

where  $I_c$  is the function of crack resistance limit from eq.(8). This curve is shown in fig.5

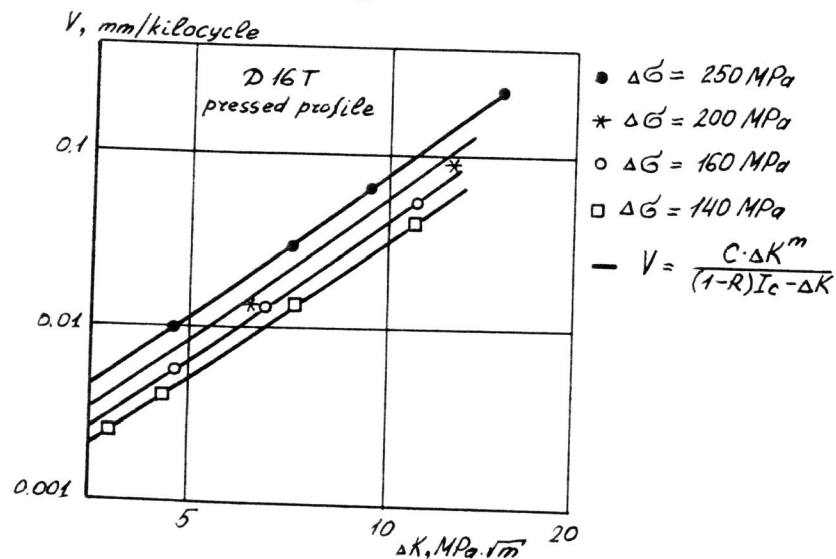


Fig.5 Approximation for  $V=f(\Delta K)$  using the modified Forman equation.

The lower part at  $F_{cr} < 0.05$  mm is proposed to be presented similar to Ivanov et al. (1987) as the line ( $v = \text{constant}$ ). Such presentation allows us to avoid the errors resulting from the indefinite form of the critical failure curve at the low values, and is conservative at crack growth time analysis. The final form of the critical failure curve is shown in fig.6.

So the above-mentioned method for crack-resistance evaluation in the presence of small crack including:

- crack shape model based on the equality of actual and modelled crack area;
- the usage of the function of crack-resistance limit;
- the usage of static and cyclic crack-resistance properties from the standard tests,

can be recommended for the evaluation of crack resistance, when the structure contains small cracks.

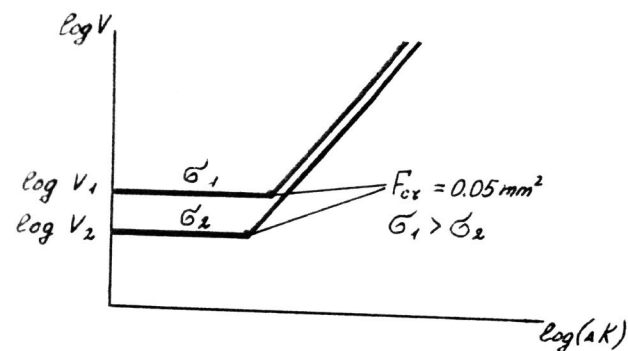


Fig.6. Crack growth rate simulation at crack area  $F_{cr} < 0.05$  mm

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