COVARIANCE EFFECT OF BIAXIAL STRESS STATE ON FATIGUE LIFE CALCULATIONS

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ABSTRACT

In this paper influence of correlations between components of random stress tensor on the calculated fatigue life of the machine element has been analysed. The covariance matrix for components of biaxial stress state was considered. This matrix was determined on the basis of the measured strains in the element of the vibrating screen for aggregate. Computer simulation was realized with four criteria of multiaxial random fatigue. It has been shown that covariances between components of stress state tensor strongly influence the calculated fatigue life.

KEYWORDS

Biaxial random fatigue, fatigue criteria, service loadings, correlation analysis, covariance matrix.

INTRODUCTION

Many elements of machines and structures working under multiaxial service loadings undergo fatigue failures. For evaluation of life time of such elements suitable criteria of multiaxial random fatigue have been formulated (Macha,1984). They are used for determination of the equivalent stress $\sigma_{eq}(t)$, (t-time) or strain.

Next the well-known procedures for fatigue damage cumulation under uniaxial random loading can be applied. The algorithm for evaluation of fatigue life is based on many assumptions - at present they are verified by fatigue tests and simulation calculations. For instance, it is assumed that components of the random stress tensor are stationary and ergodic stochastic processes. From correlation theory of stochastic processes a problem of correlations between components of stress state tensor results. This paper shows how the calculated fatigue life of the material is influenced by correlations between stresses under plane stress state.

FATIGUE CRITERIA UNDER RANDOM BIAXIAL STRESS STATE

I. Criterion of the maximum normal stress in the fracture plane

$$\sigma_{eqI}(t)=\hat{l}_1^2\sigma_{xx}(t)+\hat{m}_1^2\sigma_{yy}(t)+2\hat{l}_1\hat{m}_1\sigma_{xy}(t) \qquad (1)$$

II. Criterion of the maximum normal strain in the fracture plane

$$\sigma_{eqII}(t)=[\hat{l}_1^2(1+\nu)-\nu]\sigma_{xx}(t)+[\hat{m}_1^2(1+\nu)-\nu]\sigma_{yy}(t)+2(1+\nu)\hat{l}_1\hat{m}_1\sigma_{xy}(t) \qquad (2)$$

III. Criterion of the maximum shear stress in the fracture plane

$$\sigma_{eqIII}(t)=(\hat{l}_1^2-\hat{l}_3^2)\sigma_{xx}(t)+(\hat{m}_1^2-\hat{m}_3^2)\sigma_{yy}(t)+2(\hat{l}_1\hat{m}_1-\hat{l}_3\hat{m}_3)\sigma_{xy}(t) \qquad (3)$$

IV. Criterion of the maximum shear and normal stresses in the fracture plane

$$\sigma_{eqIV}(t)=\{[\hat{l}_1^2-\hat{l}_3^2+K(\hat{l}_1+\hat{l}_3)^2]\sigma_{xx}(t)+[\hat{m}_1^2-\hat{m}_3^2+K(\hat{m}_1+\hat{m}_3)^2]\sigma_{yy}(t)$$
$$+[2(\hat{l}_1\hat{m}_1-\hat{l}_3\hat{m}_3)+2K(\hat{l}_1+\hat{l}_3)(\hat{m}_1+\hat{m}_3)]\sigma_{xy}(t)\}/(1+K) \qquad (4)$$

where: σ_{xx}, σ_{yy}, σ_{xy} - components of plane stress state, K - a material constant determined under cyclic loadings, ν - Poisson's

ratio, $\hat{l}_n$, $\hat{m}_n$, $\hat{n}_n$, (n=1,2,3) - mean direction cosines of principal stresses $\sigma_1(t) \geq \sigma_2(t) \geq \sigma_3(t)$. The expected fatigue fracture plane position is defined with these cosines.
From equations (1)-(4) it appears that the expected fatigue fracture plane position strongly influences the equivalent stress.
Under plane stress state a position of vector normal to the fatigue fracture plane can be described with one angle φ in relation to the Ox axis. Thus, direction cosines of stresses σ_1 and σ_3 are $l_1=\cos\varphi$, $m_1=\sin\varphi$, $l_3=-\sin\varphi$, $m_3=\cos\varphi$.
The fatigue fracture plane position is usually not known and for calculation of fatigue life its expected position is determined with one of these methods: the method of weight functions, the method of variance, the method of damage cumulation (Macha,1989). In this paper the method of variance was used (Będkowski,Macha, 1987). This method consists in searching a critical position in which variance of the equivalent stress reaches its maximum.

COVARIANCE MATRIX OF BIAXIAL STRESS STATE

For determination of the experimental covariance matrix of stress state components a vibrating screen for aggregate, working under service loadings, was chosen. Measurements of strains were made on a side wall of the screen where fatigue cracks had occured. For measurements strain gauges forming a rectangular rosette were applied. Signals from the bridges were registered and processed by a microcomputer measuring system. As a result of calculations the following covariance matrix of stresses $\sigma_{xx}(t)$, $\sigma_{yy}(t)$, $\sigma_{xy}(t)$ was obtained

$$\mu_x=\begin{bmatrix}\mu_{11} & \mu_{12} & \mu_{14}\\ \mu_{21} & \mu_{22} & \mu_{24}\\ \mu_{41} & \mu_{42} & \mu_{44}\end{bmatrix}=\begin{bmatrix}1.005*10^4 & 7.828*10^3 & 5.836*10^2\\ 7.828*10^3 & 8.664*10^3 & 3.790*10^2\\ 5.836*10^3 & 3.790*10^2 & 8.484*10^1\end{bmatrix} [MPa^2] \qquad (5)$$

From matrix (5) it results that stress state components are strongly correlated. Covariance between normal stresses, μ_{12}, plays the most important role. Also the values of covariance between normal stresses and shear stress, μ_{14} and μ_{24}, should be noticed; they are greater than variance of the shear stress μ_{44}.

CALCULATIONS AND RESULTS

For simplification eight types of covariance matrices were introduced. The covariance matrix including all the covariances was called Matrix 1. Next certain covariances of matrix (5) were neglected. It means that suitable elements of the matrix, $\mu_{ij}=0$, $(i\neq j)$, are assumed and we have the further types of matrices

$$\text{Type 1} \equiv \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad \text{Type 2} \equiv \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad \text{Type 3} \equiv \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix},$$

$$\text{Type 4} \equiv \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \quad \text{Type 5} \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}, \quad \text{Type 6} \equiv \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\text{Type 7} \equiv \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \quad \text{Type 8} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Fatigue life T_R was calculated in some stages. At the first stage variance of the equivalent stress $\mu_{\sigma eq}$ is calculated for each considered criterion and for eight types of covariance matrix. The expected direction of fatigue fracture plane was searched - values of the angle φ were discretely changed with a step $\Delta\varphi = \pi/180$ rad in the interval $0-\pi$ rad. Values of the angle φ for which variance of the equivalent stress $\mu_{\sigma eq}$ reaches its maximum determine the expected fatigue fracture plane position for a given type of covariance matrix.

At the next stage a time series of the equivalent stress, $\sigma_{eq}(t)$, with normal probability distribution and variance $\mu_{\sigma eq}$ calculated in the first stage is generated.

At the third stage cycles are counted with the rain flow method (Downing, Socie,1982). At the fourth stage fatigue life T_R is calculated according to Palmgren-Miner hypothesis.

The calculations were made for carbon steel for which fatigue limit σ_{af} = 230 MPa for $N_f = 10^6$ cycles, exponent of S-N curve m=8, ν=0.25, K=0.8979, Young modulus E=210 GPa. Observation time of stress history T_o=200 s.

The results of calculations of angle φ, determining the expected fatigue fracture plane position, and variance $\mu_{\sigma eq}$ for different types of covariance matrix of stress state components are shown in Table 1. Table 1 presents also the calculated fatigue life T_R for different types of covariance matrix (see Fig.1 as well).

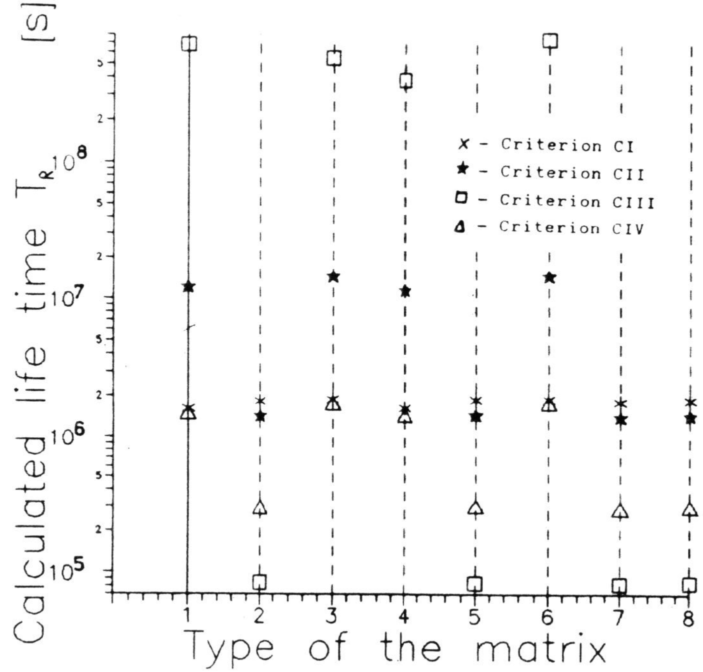

Fig.1 The calculated fatigue life T_R according to different types of covariance matrix.

ANALYSIS OF THE CALCULATION RESULTS

In the covariance matrix (5) stress variances are not equal and they can be arranged in the following way: $\mu_{11} > \mu_{22} > \mu_{44}$. However, $\mu_{12} \approx \mu_{22}$ and $\mu_{22} > \mu_{14} \approx \mu_{24} > \mu_{44}$.

Table 1 and Fig.1 show direct influence of correlations between stress components on the calculated fatigue life T_R. Values of T_R are arranged inversely in relation to values of variance $\mu_{\sigma eq}$ for each criterion.

Criterion (1) is the least sensitive to changes of a type of the covariance matrix. According to this criterion the least fatigue life is obtained for Matrix 1, i.e. min $T_{RI} = T_{RI}^{Typ1}$. The largest fatigue life is obtained for Matrix 8, i.e. max $T_{RI} = T_{RI}^{Typ8}$ - see Table 1. Also the ratio of the extreme values of fatigue life T_R is the smallest for this criterion - it is $\max T_{RI} / \min T_{RI} = 1.16$.

Criterion (3) is the most sensitive to changes of a covariance matrix type. According to this criterion a ratio of the extreme values of fatigue life T_R is very large and it is equal 9100. Acording to criteria (2) and (4) these ratios are 10.39 and 6.14 respectively.

According to criteria (2)-(4) the minimum values of fatigue life min T_R are obtained for Matrix 7 and $T_{RIII}^{Typ7} < T_{RIV}^{Typ7} < T_{RII}^{Typ7}$. The maximum values, max T_R, are obtained for Matrix 6 and here $T_{RIII}^{Typ6} > T_{RII}^{Typ6} > T_{RIV}^{Typ6}$.

Basing on the analysis of influence of a matrix type upon fatigue life T_R according to criteria (2)-(4) it is possible to divide the matrices into two groups. The first group includes Matrices 1,3,4 and 6 and here the fatigue life reaches large values. Matrices 2,5,7,8 belong to the second group and here values of T_R are smaller. Moreover, for the considered three criteria and the full Matrix 1 T_R is greater than for Matrix 8, i.e.

$$T_{RII}^{Typ1} > T_{RII}^{Typ8}; \quad T_{RIII}^{Typ1} > T_{RIII}^{Typ8}; \quad T_{RIV}^{Typ1} > T_{RIV}^{Typ8}.$$

Thus, a very important conclusion can be drawn here: the greater number of elements μ_{ij}, $(i \neq j)$ of the covariance matrix (5) is neglected, the less the fatigue life T_R is.

CONCLUSIONS

1. The considered algorithm for calculations of fatigue life of machine elements under multiaxial random loadings is strongly

Table 1. Values of the angle φ in [degrees] determining the expected fatigue fracture plane position, variance $\mu_{\sigma eq}$ in [MPa^2] and the calculated fatigue life T_R in [sec] for different types of covariance matrix of stress state components.

Criterion		Type of covariance matrix							
		Type 1	Type 2	Type 3	Type 4	Type 5	Type 6	Type 7	Type 8
CI	φ =	15	3	0	13	0	0	3	0
	$\mu_{\sigma eqI}$ =	10358	10118	10050	10335	10050	10050	10118	10050
	T_{RI} =	$1.623 \cdot 10^6$	$1.825 \cdot 10^6$	$1.887 \cdot 10^6$	$1.641 \cdot 10^6$	$1.887 \cdot 10^6$	$1.877 \cdot 10^6$	$1.825 \cdot 10^6$	$1.877 \cdot 10^6$
CII	φ =	12	2	177	12	0	0	3	0
	$\mu_{\sigma eqII}$ =	6932	10640	6687	7007	10591	6677	10662	10591
	T_{RII} =	$1.210 \cdot 10^7$	$1.419 \cdot 10^6$	$1.448 \cdot 10^7$	$1.146 \cdot 10^7$	$1.452 \cdot 10^6$	$1.459 \cdot 10^7$	$1.404 \cdot 10^6$	$1.452 \cdot 10^6$
CIII	φ =	4	1	83	10	89	0	2	0
	$\mu_{\sigma eqIII}$ =	3118	18720	3255	3491	18744	3058	18787	18714
	T_{RIII} =	$6.792 \cdot 10^8$	$8.416 \cdot 10^4$	$5.478 \cdot 10^8$	$3.813 \cdot 10^8$	$8.363 \cdot 10^4$	$7.485 \cdot 10^8$	$8.267 \cdot 10^4$	$8.430 \cdot 10^4$
CIV	φ =	171	162	157	172	159	159	162	152
	$\mu_{\sigma eqIV}$ =	10527	14574	10164	10604	14502	10155	14599	14501
	T_{RIV} =	$1.479 \cdot 10^6$	$2.943 \cdot 10^5$	$1.784 \cdot 10^6$	$1.443 \cdot 10^6$	$3.017 \cdot 10^5$	$1.792 \cdot 10^6$	$2.918 \cdot 10^5$	$3.018 \cdot 10^5$

influenced by changes of correlations between stress state components.

2. Criterion of the maximum shear stress in the fracture plane is the most sensitive to elimination of correlations between stresses. Criterion of the maximum normal stress in the fracture plane is the less sensitive to these changes.

3. For the full covariance matrix, taking into account all the correlations between stresses, the lowest calculated fatigue life was obtained according to criterion of the maximum shear and normal stresses in the fracture plane and the highest one - according to criterion of the maximum shear stress in the fracture plane.

4. From the obtained results it appears that some further experiments on correlations between components of stress state tensor while calculating fatigue life are necessary. The obtained results show directions of the future material tests.

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