CORROSIVE MEDIUM ACCUMULATION NEAR CRACKS AND INCLUSIONS IN STRESSED-DEFORMED MATERIALS

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ABSTRACT

Within the elastically deformed model there has been given an estimate of corrosive medium degree concentration near fibres and cracks in the materials mechanically stressed. Represntation of active components' elements in the material has been performed according to the principles of thermodynamics unbalanced processes. Subordination of the consructed flows to continuity conditions made it possible to construct corresponding system of differential equations accounting simultaneous inter-relation of mechanical and physical fields. By applying the main statements of theory of elasticity and linear fracture mechanics there has been found analytical dependence on the concetration of corrosive medium near the surface of circular space, as well as near the fibres and cracks from the coefficients of stress intensity characteristics for local fields. As a result there has been estimated growth rate of the crack filled by the medium from stress coefficient, as well as there has been established the influence of interaction cracking defects on the corrosive medium accumulation near their tops.

KEYWORDS

Corrosive medium, fibre, inctusion, crack, flow, equation, destruction, concentration, coefficient of stress intensity.

URGENCY STATEMENT OF OVERALL PROBLEM AND INITIAL EQUTION SYSTEM

One of the most urgent and scantily researched problems in desructive mechanics is studing of corrosion cracking of materials in the field of mechanical stresses. Rather unfavourable in the process of this destruction occurs joint interaction of corrosive medium the deformed material near the defects like cracks, inclusions, spases, pits etc. The heterogeneities, mentioned above, are, as a rule, peculiar memory of corrosive medium in the vicinity of its peak-like or acute angle points. That is why, theoretic estimation of value accumulation of active components near the points of crack-like stress concentrates becomes of paramount importance nearly at all practical calculations on prevention of corrosive - mechanical dustruction. Let's examine the elastic body under the conditions of the plane problem theory of elastisity and reduced by curve-linear holes as well, bounded by the contours Γ_i, and interacting cracks and inclusions. It is considered that on the hole contours Γ_i or the cracks L_s there have been given zero stresses, on the inclusions, for instance, zero displacements. On the points, considerably remoted from those contours there interact perpendicular tension or compressive forces on the intensity p and $q=\eta p$ $(0\leq\eta<\infty)$, the forces p being quided by the angle α to the axis Ox of the main coordinate system xOy, connected with the elastic body in the point O; in the given points of the parameters of stressed-deformed state in the vicinities of the interacting cracks or inclusions or even pits; and also to estimate the concetration of the corrosive-active element, caused by interaction of crack-like deffects. Concentration of stressed field near the boundary of stress concentrator stimulates initiation of the flows and accumulation of physical substances, leading subsequently to corrosive cracking of carrying material. According to Onsager's principle (de Groot, 1964) the flux density of the n-th physical substance in medium is determined by means of the main and accompanying forces in the form of the linear combination

$$\vec{J}_n = \sum_{k=1}^{N} L_{nk}\vec{X}_k, \quad (n=1,2,\ldots,N), \tag{1}$$

where $\vec{J}_n$ is a finx of the n-th physical substance. L_{nk} are Onsager's Kinetic coefficients; $\vec{X}_k=-A_k\vec{\nabla}F_k$ are termodynamical forces (A_k are coefficients of proportionality depending on the other substances fluxes; $\vec{\nabla}$ is Gamilton's operator, F_i are the potentials corresponding to the thermodynamical forces). If we used the principle of continuity and considered the flows to be small- changeable, then, at the assignment of mechhanical stresses as a separate component from the relationship (1), we could receive the interrelated system of differential equations (Stashchuk, 1993). In case of the plane problem of theory of elasticity at the established regime of penetration of physical substances the corresponding system of differential equations assumes the form

$$\vec{\nabla}\sum_{k=1}^{N-1}\left[-L_{nk}A_k\vec{\nabla}F_k\right] + \vec{\nabla}A_n\left[F_{i,\,i=1,2,\ldots,n}\right]\vec{\nabla}\sigma = 0, \tag{2}$$

where σ is hydrostatical pressure near the space surface or in the vicinity of the crack or the inclusion that is determined by means of stress tensor on the relationship $\sigma=-\sigma_{ii}/3$. Solution of the received system of equations (2) can be made withinn the frames of the known boundary problems of mathematical physics. As an example we regard the flow of a physical substance, stimulated by the influence of mechanical forces

$$J_F = -A\left[\vec{\nabla}F + \frac{FB}{A}V_{eff}\vec{\nabla}\sigma\right], \tag{3}$$

where A is a transport substance coefficient B is its mobility (de Groot, 1964), and V_{eff} is the effective volume of interaction of active medium component and material. In the equation system (2) assumes the form

$$\vec{\nabla}^2F + k\vec{\nabla}\sigma\vec{\nabla}F = 0, \tag{4}$$

where $k=BV_{eff}/A$. Let's note that analogous equation for hydrogene diffusion into material in the field of mechanical stressed is represented in paper (Panasyuk, 1988). It's easy to show that whithin the polar coordinates (r,β) the general solution of the equation (4) is the following (Stashchuk, 1993)

$$F(r,\beta) = u(r,\beta)e^{f(r,\beta)}, \tag{5}$$

where the functions $f(r,\beta)$ and $u(r,\beta)$ satisfy the differential equations

$$\frac{\partial f}{\partial r} = -0.5k\frac{\partial\sigma}{\partial r}; \qquad \frac{\partial f}{\partial\beta} = -0.5k\frac{\partial\sigma}{\partial\beta}; \tag{6}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial u}{\partial r}\right] + \frac{1}{r^2}\frac{\partial^2u}{\partial\beta^2} - 0.25\left[\left(\frac{\partial\sigma}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial\sigma}{\partial\beta}\right)^2\right]u=0. \tag{7}$$

While concrete boundary conditions are given on the surfaces of the existing spaces or deffects of the crack-inclusion type and on the bounding body contour as well, the evident form of distribution function $f(r,\beta)$ can be determined.

THE CASE OF THE FILLED CIRCULAR HOLE

Let's examine the circular hole of the radius a in elastic isotropy plane, filled by corrosive-active medium with the concentration of the active element Co, the hole centre being aligned with the beginning of polar coordinate system (r,β). Then, hydrostatic stress at the given on plate infinity, stresses of the intensity p, coinciding in the direction of the effect with the polar straight line $\beta=0$, is determined (Muskhevishvili, 1966) by the relationship

$$\sigma = \frac{(1+\mu)p}{3}\left[\frac{2a^2}{r^2}\cos 2\beta - 1\right], \tag{8}$$

where μ is Pouson's coefficients. Having applied the method separation of variabilities to the equation (7) based on the relationships (6) and (8), the general solution (5) is represented by

$$F(r,\beta) = e^{t\cos 2\beta}\sum_{k=1}^{\infty}\lambda_n I_n(t)\cos 2\beta n, \tag{9}$$

where

$$t = -(1+\mu)pa^2k/3r^2, \tag{10}$$

λ_n are the coefficients subjected to determination, $I_n(t)$ are imaginary Bessel's functions. When it is required that boundary condition should be satisfied on the circular contour

$$\left.\frac{\partial F}{\partial r}\right|_{r=a} = m, \tag{11}$$

that corresponds to the penetrating of corrosive medium in materials and also assume that

$$F(r,\beta) \xrightarrow[z\to\infty]{} F_0, \tag{12}$$

then coefficients λ_n in the relationship (9) should be determined according to the recurrent system of algebraic equations

$$\begin{cases} \lambda_0 = F_0; \quad \lambda_1 = \dfrac{mI_0(t_a)}{I_1(t_a)} - 2\lambda_0; \quad t_a = -\dfrac{(1+\mu)pk}{3}; \\ \lambda_2 = \dfrac{-2mI_1(t_a)-(2\lambda_0+\lambda_1)I_0(t_a)}{I_2(t_a)} - \lambda_1; \\ \lambda_n = \dfrac{2(-1)^{n-1}mI_{n-1}(t_a)-(\lambda_{n-2}+\lambda_{n-1})I_{n-2}(t_a)}{I_n(t_a)} - \\ \quad - \lambda_{n-1}\ ; \quad n > 2. \end{cases} \tag{13}$$

However, when we assume a certain corrosive process to be performed discontinuosly, at accumullation in the material, to a certain level, the active component value, that corresponds, for instance, to flow J_F ($J_F\equiv 0$) absence, then from the relationship (3), we note immediately

$$F(r,\beta) = F_0 e^{-\frac{2k(1+\mu)p}{3}\frac{a^2}{r^2}\cos 2\beta} \tag{14}$$

Let's assume on the contour of circular hole, media interaction as chemical reaction

$$A_{CM} + B_c \longrightarrow Q_{\Pi}, \tag{15}$$

where A_{CM} and B_c are reagents, Q_{Π} is the product. Then according to the known equation (Uhlig, 1989) we receive change of Gibbs's energy

$$\Delta G = RT\ln\frac{\Pi}{c_M c}\ . \tag{16}$$

Here, and also from the above formula (14) we determine distribution of concentration Π product Q_{Π}, formed along the contour of the circular hole

$$\Pi = c_M c_0 \exp\left[\frac{\Delta G}{RT} - \frac{2k(1+\mu)p}{3}\cos 2\beta\right]. \tag{17}$$

The last relationship can be used for quantitative estimation of corrosive process on used for quantitative estimation of corrosive process on the circular boundary, stimulised by the field of external stresses.

CRACK-LIKE DEFFECTS

According to the papers (Berezhnitsky, 1983; Stashchuk, 1993) the components of stress tensors near deffect points like tough inclusions on cracks have the form

$$\sigma_{ij} = \frac{K_I}{\sqrt{r}}f_{ij}^{(I)}(\rho^*,\beta) + \frac{K_{II}}{\sqrt{r}}f_{ij}^{(II)}(\rho^*,\beta) + O(r^0), \tag{18}$$

where (r,β) are polar coordinates wich begin on top of the deffect, K_I and K_{II} are coefficients of stress intensity, parameter $\rho^*=-1$ in case of crack and $\rho^*=\varkappa$ for absolutely tough inclusion, where $\varkappa=3-4\mu$ in case of plane deformation and $\varkappa=(3-\mu)/(1+\mu)$ for plane stressed state. On this basis hydrostatic pressure σ in the vicinity of considered defect is determined by the formula

$$\sigma = \frac{2(1+\mu)}{3\sqrt{2\pi r}\rho^*}\left[K_I\cos\frac{\beta}{2} - K_{II}\sin\frac{\beta}{2}\right]. \tag{19}$$

Let's examine an infinite elastic body, reduced by linear crack or tough inclusion of the length $2l$, giving on the infinity intensity efforts p perpendicular to the deffect plane. Then coefficients of stress intensity will be

$$K_I = 0.25p\sqrt{\pi l}(3-\rho^*), \quad K_{II} = 0. \tag{20}$$

Substituting corresponding value of hydrostatic pressure σ into the equation (7) and solving it by the method of separation of variables, having satisfied at this boundary conditions

$$c = c_0 \text{ at } \beta=\pm\pi, \quad c = c_0 \text{ at } r \to \infty \tag{21}$$

we find, that

$$c = c_0 \exp\left[-\frac{2(1+\mu)BV_{eff}}{3\sqrt{2\pi}A\rho^*}K_I \cos\frac{\beta}{2}\right]. \tag{22}$$

At $\rho^*=-1$ from expression (22) follows distribution of chemical-active substance near cracks' points, and at $\rho^*=\infty$ near the top of absolute tough inclusion. Analogous result can be received for the interval between those boundary cases, namely, for the thin-walled elastic inclusion. Assuming the velosity of dissolutin reaction or chemical transformation in the peak point of crack is proportional to maximum concetration of the component that are present and also taking direct proportionality of dependence propagation crack velosity $dl/d\tau$ in time τ on the velosity of chemical reaction at its peak point, then based on the relationship (22) acounting interconnection (de Groot, 1964) between the parameters B, A, constant value R_b and absolute temperature T, to estimate velosity of a corrosive cracks growth we receive relationship

$$\ln\left(\frac{1}{c_0}\frac{dl}{d\tau}\right) = \frac{s_v}{R_b T f_T \sqrt{r_*}} K_I + b, \tag{23}$$

where $s_v=2(1+\mu)V_{eff}/3\sqrt{2\pi}$, f_T is correlative transport factor of active element, b is a certain constant value dependant on the process enthalpy ΔH, R_b, T etc., r is a certain stucture parameter of the material at the peak point of the crack, for instance, its circular radius. Comparison of the relationship (23) with experimental data has been in the monography (Stashchuk, 1993). To estimate the influence of interaction of deffects of crack type on the active medium concentration near their peak points as a test example was considered the case of two collinear cracks placed perpendicularly to tensions p. At this there were received numerical data of relative concentration of corrosive medium according to the following asymptotics

$$\frac{1}{p\sqrt{\pi l M}}\ln\frac{c}{c_0} = \frac{1}{\sqrt{r}}\left(1 + \frac{\lambda^2}{8} \mp \frac{\lambda^3}{16} + \frac{11}{128}\lambda^4\right)\cos\frac{\beta}{2}; \tag{24}$$

$$M = 2(1+\mu)BV_{eff}/3\sqrt{2\pi}A; \tag{25}$$

following from the formula (22) and supplements of the monography (Berezhnitsky, 1983). Sign "+" in the relationship (24) relates to the close placed peak points of collinear cracks, sign "-" relates to outer point. The calculation were made for the values $r=10^{-1}$, 10^{-2}, 10^{-3}, 10^{-4}, of different angles $0<\beta<\pi$, and also for different values of the parameter

$\lambda=l/d$, characterizing relative distance between the centres of cracks. As follows from numeric data, removing from the crack peak point to a distance equal one radius r order, the logarithm of relative concentration c/c_0 changes approximately 3 times. The zone on the cracks' extension along the connector between them is the most active at this. Convergence of the peak points filled in with the crack medium increases in quantity solution of active medium components in material. Analogous conclusions are received in the case of interacting dyads with each other of the type inclusion - crack and also in case of periodical and two-periodical systems of cracks and inclusios.

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