APPLICATION OF T*-INTEGRAL TO THE CRACK PROPAGATION ANALYSIS UNDER THE QUASI-STATIC AND DYNAMIC LOADING

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ABSTRACT

A computation analysis of a stable crack growth by ductile separation in the conditions of quasi-static and highspeed monotonic loading, based on the **T***-integral application, is made. An experimental computational method of $\mathbf{J_R}$ determination requiring no accurate fixing of incipient growth of cracks is offered. On the basis of the proposed approaches the verification of applying $\mathbf{J_R}$-resistance curve criterion to the analysis of a stable crack growth in the specimens loaded according to different schemes is carried out.

KEY WORDS

T*-integral, crack, crack growth rate, stable crack growth, unstable crack growth, J_R-approach.

At present, to analyse the stability of quasi-static crack growth the criterion of $\mathbf{J_R}$-curves and tearning modulus is applied. The essence of J_R-approach lies in the assumption that the fracture process which occurs at the stably propagating crack tip controlled by two parameters, viz., the crack length increment **ΔL** and the Cherepanov and Rice **J**-integral introduced for the non-linear elastic body. In other words, it is supposed that **J(ΔL)** uniquely defines the resistance to the stable crack growth irrespective of the applied load type on condition of monotonic loading character and the specimen geometry. At the same time, many studies have pointed out the vulnerability of the above approach and, in particular, the non-invariance of $\mathbf{J_R}$-curves to the type of loading and the specimens geometry. Therefore, many papers have been published recently, which are devoted to the $\mathbf{J_R}$-approach modification by means of introducing energy integrals of different types (Atluri, 1982; Brust et al., 1986). The most significant results have been obtained by using the so-called T*-integral. At the same time methodology of its computation, validity and the field of application for the problems pertaining to the ductile crack growth are not practically examined.

STABLE CRACK CROWTH

To describe the stable crack growth Atluri (1982), Brust et al. (1986) used the parameter **T***:

$$T^* = \lim_{\Delta \to 0} \int_{\Gamma_\Delta} [(U+K)n_1 - t_i \frac{\partial u_i}{\partial x_1}] d\Gamma \qquad (1)$$

where $\mathbf{U} = \int \sigma_{ij} d\varepsilon_{ij}$ is the strain energy density; **K** – kinetic energy density; $\mathbf{t}_i$ – projection of the force vector on the contour Γ_Δ to the $\mathbf{X}_i$-axis; $\mathbf{u}_i$ – displacement vector components; $\mathbf{n}_1$–the projection to the $\mathbf{X}_1$-axis of the positive unit normal to the Γ_Δ (Fig.1*a*); the $\mathbf{X}_1$-axis along the crack axis; Γ_Δ- integration path enveloping the crack tip. The computational and experimental work performed by the Atluri, Brust on the compact tension specimens and the edge crack specimens resulted in the following. Observations for the stationary crack under monotonic loading in the conditions of the elasto-plastic strain the parameters **T*** and **J**-integral (computed by the external path) coincide. Along with the crack propagation the **J**-integral increases continuously, whereas **T*** grows up to a certain constant $\mathbf{T^*}_{const}$ level and does not change with the further increase of **ΔL**. Note that for different materials and specimens the value of $\mathbf{T^*}_{const}$ varies in the range of (2,5–10)$\mathbf{J}_{IC}$.

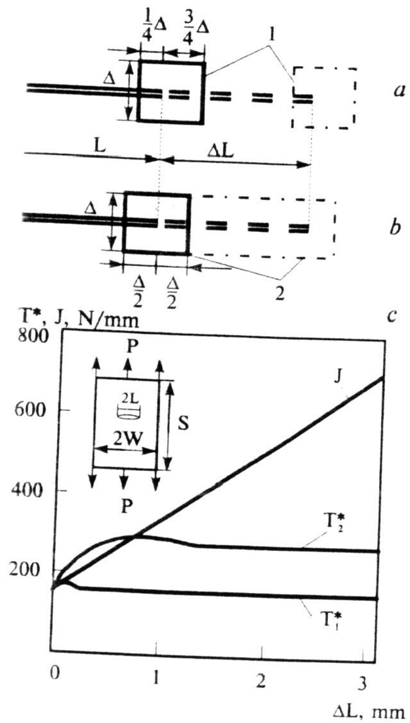

Fig.1. Other paths integration stationary (——) and moving (- - • - -) crack (*a, b*) **T***-, J- resistance curves (*c*)

Another important problem considered by Brust (1986) is the analysis of the incipient growth of a stationary crack after unloading and reloading. The author states that with the **T*** corresponding to the moment of unloading as the critical value under the incipient regrowth of a crack the corresponding computed loading $\mathbf{P}_{IC}$ is roughly equal to the loading obtained during the experiment ($\mathbf{P}_{IC} \cong 0{,}5\mathbf{P}$, where **P** is the loading at the moment of the beginning of the specimen unloading). If **J**-integral or the crack opening angle criterion is incipient regrowth of the crack, then $\mathbf{P}_{IC} \cong \mathbf{P}$, which does not agree with the experimental data. Note that in the above works **T*** is computed by rather a specific integration path Γ_Δ, which stretches with the crack propagation (Fig.1*b*), whereas from the definition of the integral **T*** by the eqn (1) it follows that Γ_Δ (Fig.1*a*) must be used. The analysis of the above results gives rise to the following questions. What is physical meaning of the parameter **T***; what is the reason Atluri, Brust and other choose the integration path given in Fig.1*b*; how to make use of **T*** integral for the analysis of the fracture process. The absence of invariance dependence **T* (ΔL)** on the specimen type forms the last question.

To answer these questions as well as to analyse the applicability of **T*** for the description of the stable crack propagation under monotonic loading we have make some computations. By means of FEM (Kostylev and Margolin, 1990) the elastic-plastic problem on the crack propagation under the plane strain conditions is solved. In all the present analyses, we use specimens with the dimensions **S**=400 mm, 2**W**=200 mm, 2**L**=100 mm and properties of the material corresponding to the steel 15CrMoVa at **T**=20°C: **E**=2·10^5 MPa, ν=0.3, $\mathbf{J}_{IC}$=162 N/mm. The nonlinear stress vs strain curve is described by the dependence σ_i=(520–596)·$(\varepsilon_i^P)^{0{,}494}$ MPa. It is supposed that the elementary act of the crack growth occurs when the local fracture criterion is satisfied at the crack tip. Ductile fracture of the material at the crack tip is defined by the dependence $\sigma_m(\varepsilon_i^P)$ (Karzov et al., 1989) (σ_m–the hydrostatic component of the stress tensor). In case if the loading of the material at each point of the future crack occurs by one and the same dependence $\sigma_m(\varepsilon_i^P)$, then the crack propagation criterion is providing of self-similarity of the local stress-strain state (SSS) at the tip of the moving crack. Thus, the numerical simulation of the ductile crack growth is conducted by observing the self-similarity of the local SSS at its tip which is provided by means of the appropriate external loading selection. The computation of **T*** is performed by two types of paths (Fig.1*a, b*) with $\Delta = \Delta_o$ providing the convergence of integrals: $\mathbf{T^*}_{\Delta=\Delta o-\delta} = \mathbf{T^*}_{\Delta=\Delta o}$, where $\delta<\Delta_o$.

The results obtained (Fig.1*c*) allow to conclude that the parameter $\mathbf{T^*}_1$ uniquely controls SSS at the tip of the moving crack; to describe the SSS by means of $\mathbf{T^*}_2$, the dependence $\mathbf{T^*}_2(\Delta\mathbf{L})$ should be used. Fig.1*c* gives the dependence of **J**-integral computed by the external path. It is evident that the rise of $\mathbf{T^*}_2$ with the increase of **ΔL** is related to the material unloading processes occuring at the surface of the moving crack. The unique controlling of the local SSS by the parameter $\mathbf{T^*}_1$ in the stable crack propagation process is evidently caused by the fact that in the small path Γ_Δ (Fig.1*a*) enveloping only the moving crack tip, mainly the monotonic loading process occurs whereas the unloading is practically absent. Under these conditions, as it is well known, the SSS is uniquely related to the strain energy of the material **U** which in turn leads to the singlevalued correspondence of $\mathbf{T^*}_1$ to the local SSS at the moving crack tip (analogous to the SSS connection at the crack tip with J-integral for a non-linear elastic body).

The next question we examine pertains to the validity of **T*** as the criterion under the arbitrary loading. In spite of the satisfactory agreement of computation and experimental data obtained by Brust (1986), note that in the general case the application of **T*** as the criterion under the arbitrary loading is problematic. Let us prove this assertion using the nonstationary loading as an example. Assume that under the nonstationary (in particular, cyclic) loading the fracture criterion of the material at the crack tip is the

$$T^{*}=\lim_{\Delta\to 0}\int_{\Gamma_\Delta}(Un_1-t_i\frac{\partial u_i}{\partial x_1})d\Gamma=J_{1c} \qquad (2)$$

For the cyclic steady material by the end of each cycle SSS on the path Γ_Δ is one and the same in particular, $t_i\frac{\partial u_i}{\partial x_1}$), therefore, (2) is expressed in the form

$$\lim_{\Delta\to 0}\int_{\Gamma_\Delta} Un_1 d\Gamma=\lim_{\Delta\to 0}\int_{\Gamma_\Delta} t_i\frac{\partial u_i}{\partial x_1}d\Gamma+J_{1c}=const \qquad (3)$$

N_f being the notation of the number cycles to fracture, eqn. (3) is rewritten in the form

$$\lim_{\Delta\to 0}\int_{\Gamma_\Delta} U_{cycle}N_f n_1 d\Gamma=const \qquad \text{or} \qquad N_f\lim_{\Delta\to 0}\int_{\Gamma_\Delta} U_{cycle}n_1 d\Gamma=const \qquad (4)$$

If small **Δ** is fixed and taken as equal to Δ_c- parameter of the material or as it is often called «zone process» then (4) is similar to Sih's criterion (Sih, 1974) – the critical strain energy density at some distance from the crack tip. Taking into consideration the fact that under the cycling loading the strain energy density U_{cycle} is equal to the irreversibly dissipated energy per cycle, (4) comes to the condition of the elementary volume damage at the crack tip which is presented in the following form:

$$N_f U_{cycle}=const=U_c \qquad (5)$$

It is assumed from equation (5), that all the dissipated energy goes for the damage. It is seen from work (Troshenko, 1981) that some part of the dissipated energy goes for the deformation and only part of it for the damage of the material, the portion of the energy going for the damage depending on the level of the total dissipated energy and the loading character (quasi-static, cyclic, etc). Thus, the results qiven in Troshenko's work do not allow to consider (5) and, therefore, (2) to be valid for the use under the nonstationary loading. Evidently, the application of Brust. **T*** as the local criterion for the analysis of the crack initiation under reloading after unloading leads necessarily to the use of specific path Γ_Δ (Fig.1*b*) to obtain the agreement between experimental and computation data.

It has been already pointed out that the application of J_R-approach is based on J_R-curves invariance to the type of the loading. All the evidence of the invariance or its absence is based on different experimental studies. At the same time, the variability of experimental data leaves the question of J_R-curves invariance open. Using **T***-integral and its property: **T*(ΔL)=const**, consider the behaviour of J_R-curves under the stable crack growth in the different specimens (center-cracked in tension, single edge notch in tension, three-point bend). The computation technique of the stress-strain state and **J, T*** parameters, specimens dimensions **(S, W, L)** and the stress-strain properties of the material are taken identical to

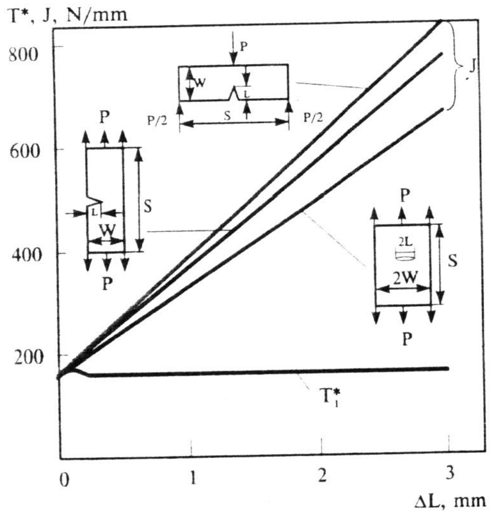

Fig.2. T^*_i- and **J**-resistance curves for other specimens

the first problem. In the crack growth process, the unloading along its surface occurs resulting in **J**-integral becoming noninvariant with respect to the integration path. Therefore, to compute **J**-integral the integration path passing along the external specimen boundary which corresponds to the value of **J**-integral is computed on the basis of the diagram **P-u** which is used to obtain J_R-curves. Fig.2 shows that the dependences **J(ΔL)** obtained for different loading schemes differ essentially, the maximum difference reaching 30 % with **ΔL**=3 mm. Thus, the application of J_R-curve obtained for any loading scheme leads to considerable errors in the estimate of the stable crack growth even under the monotonic type of loading in case of an arbitrary construction geometry.

THE TECHNIQUE OF DETERMINATION OF J_{IC}

This paper presents experimental and computation technique to determine J_{IC} using the diagram **P–ΔL** obtained for one specimen. The method is based on the constancy of the parameter T^*_i after the crack-growth initiation, i.e., on the single valued correspondence of the diagram **P–ΔL** to the condition $T^*_i(\Delta L)=const=J_{IC}$. To demonstrate this method consider the following example. Let the dependence **P–ΔL** be obtained as the result of the experimental work on the stable crack growth, a reliable definition of the dependence **P–ΔL** being made with $\Delta L>\Delta L_B$ (Fig.3*a*, curves **BC** or **BC'**) due to the technical complexity of fixing small increments of the crack length. The problem is to define the true dependence **P–ΔL** in the section $\Delta L<\Delta L_B$ (along the known curves **BC** or **BC'**) from the condition $T^*_i(\Delta L)=const$ which allows to define the crack initiation loading P_{IC} and the corresponding values of J_{IC}. Suppose that with $\Delta L<\Delta L_B$, the determing dependence **P–ΔL** corresponds to the curve **AB** (the curve **ABCC'** is computed beforehand by means of FEM from the condition $T^*_i(\Delta L)=const$ for the center-cracked specimen). It is clear that the definition process is the iterative process. In the first approximation, the dependence **P–ΔL** (curve **BC**) is extrapolated arbitrarily in section $\Delta L<\Delta L_B$: e.g., **FB** or **DB** (Fig.3*a*) ($P_{IC}^{(1)}$ – the loading corresponding to the crack initiation in the first approximation). Further, by means of FEM the elastic-plastic problem is solved, the dependence **P(ΔL)** along **FBC** or **DBC** being

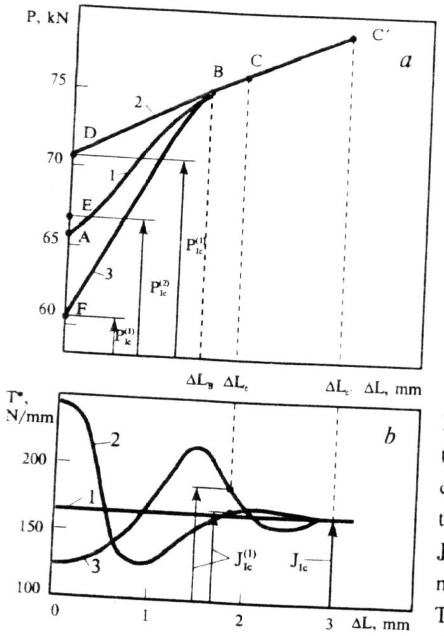

Fig.3. Schematic illustration method of definition J_{IC}: load vs crack growth (*a*), corresponding T^*_I -resistance curves (*b*)

observed. In the process of the stable crack growth the value of T^*_I-integral is calculated. The results of the computation are given in Fig.3. It is see that with the crack propagation and approximation of **FBC** and **DBC** to **ABC** the dependences $T^*_I(\Delta L)$ obtained under the loading along **FBC** and **DBC** converge to the values of $J_{IC}^{(1)}$ close to the defining J_{IC}. In the second approximation, the loading $P_{IC}^{(2)}$ is defined by means of FEM, which corresponds to the crack initiation by the condition $T^*_I(P=P_{IC}^{(2)})=J_{IC}^{(1)}$. Then interpolate **P(ΔL)** between the points **E** and **B** and repeat the procedure of defining $T^*_I(\Delta L)$ using the curve **EBC** similarly to the first approximation. It is clear that the value $J_{IC}^{(2)}$ is close to J_{IC} than $J_{IC}^{(1)}$. In other words, J_{IC} is where **n**-number of approximation converges to J_{IC}. Thus, by means of the above method parameter J_{IC} is determined more accurately even if the data about the primary region of dependence **P–ΔL** is unreliable. It will be noted that this method is iterative only with a small extension **BC** when $\Delta L_C<2\Delta L_B$. When $\Delta L_C>\Delta L_B$, the first approximation $J_{IC}^{(1)}$ with $\Delta L=\Delta L_C$, practically defines the J_{IC} (Fig.3*b*).

METHOD OF DEFINING CRITICAL LOAD P_c

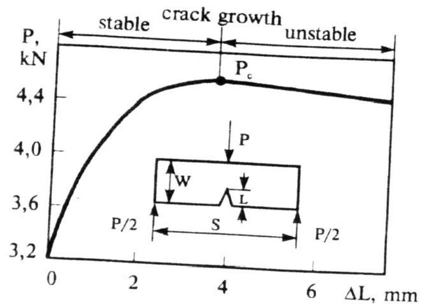

Fig.4. Load vs crack growth of bending specimen

The computation of the critical load P_c as well as the definition of J_{IC} is performed on the basis of a single valued connection conception of the diagram **P–ΔL** with **T*(ΔL)=const=**J_{IC}. Fig.4 gives the example of computing **P(ΔL)** both on the ascending and descending branches, the properties of the material and specimen dimensions are identical to the previously obtained ones, except J_{IC}=49 N/mm. The maximum value P_c of **P(ΔL)** corresponds to the unstable crack propagation initiation. It is evident that here the quasistatic analysis cannot be applied and it is necessary to use an approach allowing for the dynamic processes occuring in the structural element caused by the crack propagation with the final rate. The existance of the unstable crack growth is related to the probable absence of the catastrophic fracture of the structure due to the unsteadily growing crack stop caused by the following factors: pressure reduction in the pressure vessel resulting from the gas or liquid etchihg through an open crack; the presence of favourable non-homogeneous form fields of residual stress and the changed properties of material on the crack propagation path.

DYNAMIC CRACK PROPAGATION

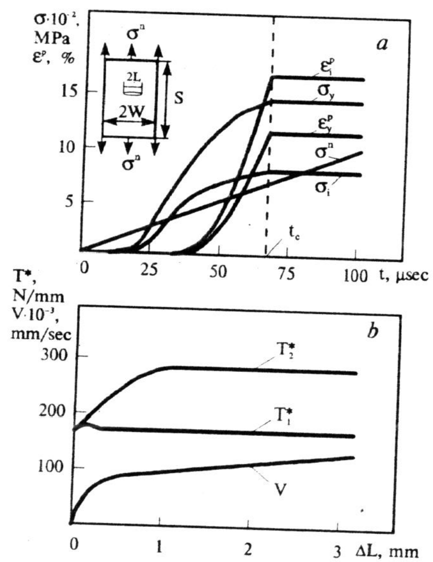

Fig.5. Applied load σ(t) and SSS at the tip crack (*a*), **T***, **v** - resistance curves (*b*)

The dynamic crack growth may be caused both by the unstable crack growth and its development under the highspeed loading of the structure. It is evident that in one and the same case the algorithm of the crack kinetics computation is one and the same. To study the behaviour of the parameter **T*** and its application to the analyses of the dynamic crack growth the following experiments are made: a center-cracked panel undergoes a dynamic loading by the rule $\sigma^n = \sigma(t)$. As soon as the critical SSS is reached in the crack tip, which corresponds to the condition **T*=**J_{IC} (**T*** is computed with regard for the kinetic energy by the formula (1)), the crack initiation and its propagation in the conditions of the increasing external loading occur (Fig.5*a*). The crack propagation criterion is the self-simulation of the SSS in the crack tip, which is performed by means of the crack growth rate selection **V=dL/dt≅ΔL/Δt**. The SSS computation is performed by FEM in the dynamic elastoplastic conditions, the crack growth similation is carried out according to the technique (Kostylev, Margolin, 1990). The SSS kinetics, **V** and **T***, computed for different types of integration paths is given in Fig.5. It is clear that to provide conditions for self-simulation of the SSS in the moving crack tip its growth rate **V** should increase continuously with the given loading character. Dependences **T*(ΔL)** possess the same peculiarities that occur in the case of quasi-static loading. T^*_1 behaves most steadily which makes it useful for the numerical simulation of the dynamically growing crack, the rate of the crack growth being determined from $T^*_1(\Delta L)=const=J_{IC}$. Since T^* is the function of V, this non-linear equation is solved only by the iteration method. The recurrence formula (Kostylev and Margolin, 1990) to define V in the elastic body is

$$V_{n+1}=C_R-(C_R-V_n)/(G_n/2\gamma)^m \quad (6)$$

where C_R is the Rayleigh wave rate, G_n is the elastic energy release rate on n iteration, 2γ is the effective surface energy. (6) with the rates $V<<C_R$ which corresponds to the case of the ductile fracture requires the exponent m to be chosen correctly to provide $V>0$, which results in a complex definition process. In the case of the ductile fracture the following recurrence formula of the rate definition is propouse:

$$V_{n+1}=V_n(T^*_n/J_{IC})^m \quad (7)$$

CONCLUSION

1. The computation method of stable and unstable crack growth under the quasi-static and hiqhspeed monotonic loading based on the application of **T*-integral** is developed.

2. The configuration of integration path of **T*-integral** with the parameter **T*** uniquely controlling the stress-strain state at the moving crack tip is substantiated.

3. The new method for defining J_{IC} by the experimental data obtained on one specimen is offered, which does not require the exact incipient growth of the crack fixing.

4. J_R-curves are proved to depend on the structural element loading scheme.

5. The application of **T*-integral** to define the critical load P_c is demonstrated.

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