A UNIFIED APPROACH TO THE CRACK PATH INSTABILITY PHENOMENA

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ABSTRACT

A unified study of the various dynamic instability problems associated with rapid crack propagation is attempted in the present work. The approach is based on the "twin-crack" model, according to which the geometry of the immediate vicinity of the tip of a rapidly propagating crack is better simulated by a pair of micro-cracks of random lengths and orientations than by a single point.

INTRODUCTION

It is known that the propagating crack tip is accompanied by a tuft of randomly oriented micro-cracks of various lengths [1,2], long before macroscopic crack path deviation is observed. Also it has been proved [3] that the classical static fracture criteria fail to predict the angle of the crack path deviation, when they are applied according to the mode introduced by Yoffe [4], i.e around a mathematical crack with a single point crack-tip.

Motivated by these observations Theocaris et al [5] introduced the *twin-crack* model, according to which the running tip is better described by a pair of two micro-cracks of equal lengths and inclinations relatively to the initial main crack direction. Thus, the prediction of the future crack direction necessitates the application of a suitable fracture criterion in the vicinity of a double crack tip. The *twin-crack* model was then further developed to study cases of partly asymmetrically pre-branched configurations [6,7], the study of which revealed an inherrent relation between the various types of directional instability, i.e. branching, kinking, curving, arrest and reinitiation.

In the present study the *twin-crack* model is generalized to encompass fully asymmetrically pre-branched geometries. For the application of the model a method introduced by Theocaris [8] for the calculation of the stress intensity factors (SIFs) is used. The SIFs obtained are corrected through the velocity factors given by Kostrov [9], and then the dynamic stress field of Freund and Clifton [10] is calculated. The stress field being known the T-criterion of fracture [11] in its dynamic version [12] is applied to predict the future paths of the two microbranches.

THEORETICAL CONSIDERATIONS

The "Twin-crack" Model

According to this model macroscopic crack path deviation is not governed by a single quantity (stress intensity factor, tangential stress, strain energy density etc) considered as critical. It is accepted that the phenomenon is characterized by a critical six-dimensional surface of the form:

$$f(\sigma_\infty,\ b/a,\ c/a,\ \varphi_B,\ \varphi_C,\ c)=\mathscr{C}$$

where σ_∞ is the externally applied stress, b/a, c/a the ratios of the lengths of the two micro-cracks (Fig.1), relatively to the one of the main crack, φ_B and φ_C the respective inclinations and c the velocity of the crack. For deviation to occur each of these factors must exceed a critical threshold. However, it is to be emphasized, that this condition is necessary but not sufficient for the phenomenon. It is observed, that a dual character is attributed to the phenomenon, consisted of *stochastic* and *deterministic* parts. The first is due to the presence of factors that cannot be prescribed *a-priori*, i.e. the microscopic level factors (lengths and orientations of the micro-branches) while the second one is represented by the externally induced (remote stresses) factors or externally exactly measured ones (crack velocity). Further, it is accepted that to predict the future crack path, a suitably selected fracture criterion should be applied around only two mutually influenced crack tips, since it is experimentally observed that from the whole tuft of microcracks, at the final steps before macroscopic deviations, only two dominate over the others.

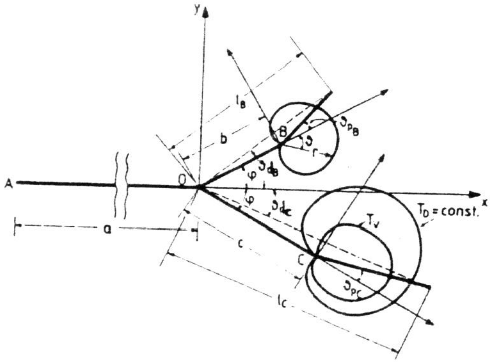

Fig.1: The geometry of the problem

In the present study the dynamic version of the T-criterion of fracture is selected, since its predictions approach better the experimental results [5]. This version is shortly described now.

The Dynamic version of the T-criterion of fracture

The T-criterion of fracture, initially proposed to study the static crack initiation problem, when applied in Dynamic Fracture Mechanics (accroding to the "*twin-crack*" model) is formulated as follows:

i) Each one of the two microbranches of inclination φ_i (i=B, C) relatively to the main crack direction, will propagate towards the direction θ_p^i, at which the dilatational strain energy density T_V, calculated along the Mises elastic-plastic boundary, reaches a maximum value T_V^{MAX}, provided that this value exceeds a critical threshold $T_{V,0}$, which is considered as dynamic material constant.

ii) This propagation will lead to succesful macroscopic deviation of the branch(es) for which the pair $(\varphi_i,\ \theta_p^i)$, i=B,C, does not lead the microbranch to be absorbed by the main crack.

For plane stress conditions the above assumptions are written as:

$$T_D(r,\theta)\Big|_{r=r(\theta)}=(1+v)(\sigma_{11}^2+\sigma_{22}^2-\sigma_{11}\sigma_{22})/3E=T_{D,0} \qquad (1)$$

$$\frac{\partial T_V(r(\theta),\theta)}{\partial\theta}\Big|_{\theta=\theta_o}=0 \quad \text{and} \quad \frac{\partial^2 T_V(r(\theta),\theta)}{\partial\theta^2}\Big|_{\theta=\theta_o}<0 \qquad (2)$$

$$T_V(r(\theta_o),\theta_o)\Big|_{\theta=\theta_o}=(1-2v)(\sigma_{11}+\sigma_{22})^2/6E=T_{V,0} \qquad (3)$$

$$\left.\begin{aligned}\theta_d^B&=\varphi+\sin^{-1}\Big\{\sin\theta_p^B\Big[(1-b^2/l_B^2\sin^2\theta_p^B)^{1/2}-\frac{b}{l_B}\cos\theta_p^B\Big]\Big\}\cong(\varphi_B+\theta_p^B)>0\\ \theta_d^C&=\varphi+\sin^{-1}\Big\{\sin\theta_p^C\Big[(1-c^2/l_C^2\sin^2\theta_p^C)^{1/2}-\frac{c}{l_C}\cos\theta_p^C\Big]\Big\}\cong(\varphi_C+\theta_p^C)<0\end{aligned}\right\} \qquad (4)$$

Calculation of the SIFs

The procedure adopted is the one developed by Theocaris [8]. The three cracks of Fig.1 are considered as independent and thus the Datshyshin-Savruk [13] method yields the following system:

$$\int_0^{a_n}\frac{g_n'(t)}{t-s}\,dt+\sum_{\substack{k=A,B,C\\ k\neq n}}\int_0^{a_k}\Big[M_{nk}(t,s)g'(t)+L_{nk}(t,s)\overline{g_k'(t)}\Big]dt=\pi(\sigma_{nk}-i\sigma_{tk}) \qquad (5)$$

with $0<s<a_n$ and n=A,B,C. Here a_k (k=A,B,C) are respectively the lengths a,b,c of the cracks OA, OB, OC. For the kernels it holds:

$$\left.\begin{aligned}M_{nk}(t,s)&=\overline{S_{nk}(t,s)}+S_{nk}(t,s)\exp[2i(\theta_k-\theta_n)]\\ L_{nk}(t,s)&=S_{nk}(t,s)\Big\{1-\frac{S_{nk}(t,s)}{\overline{S_{nk}(t,s)}}\exp[2i(\theta_k-\theta_n)]\Big\}\end{aligned}\right\} \qquad (6)$$

with $S_{nk}=\frac{1}{2}\{t-s\exp[i-(\theta_k-\theta_n)]\}^{-1}$, and $\sigma_{nk}-i\sigma_{tk}=-s[1+\exp(2i\theta_k)]/2$.

The single-valuedness condition for the displacement around the composite crack provides the additional equation:

$$\sum_{k=A,B,C}[\exp(i\theta_k)\int g_k'(t)dt]=0 \qquad (7)$$

The unknown functions g'(t) in Eqs(1,3) are proportjonal to the density of the dislocations along the three branches and they are related to the complex SIFs through the equations:

$$K_k=\sigma(2\pi a_k)^{1/2}\lim_{s\to a}[(a_k-s)^{1/2}\overline{g}_k'(s)] \qquad (8)$$

Since, however $\overline{g}_k'(s)$ can be written as :

$$\overline{g}_k'(s)=(a_k-s)^{1/2}[h_{k1}(s)+ih_{k2}(s)] \qquad (9)$$

where the real, $h_{k1}(s)$ and the imaginary, $h_{k2}(s)$, part of it have no singularities in $s=\alpha_k$, the complex SIFs are, finally, written:

$$K_k=\sigma(2\pi\alpha_k)^{1/2}[h_{k1}(\alpha_k)-ih_{k2}(\alpha_k)],\quad k=A,B,C \qquad (10)$$

The solution of the obtained complex singular integral system was achieved numerically by applying the Gauss-Legendre and Gauss-Lobatto integration rules. The Gauss-Lobatto one was applied to the integral equation for the branch OA with N=30 points of integration, and the Gauss-Legendre rule to the integral equations of branches OB, OC with the same number of integration points. Since the Gauss-Lobatto method has (N-1) collocation points arising from the linear system of 6N real unknowns, two real equations are missing, which are supplied by the condition of single-valuedness of the displacements in complex form. The reduced results of the method are plotted in Fig.2 for the case with b/c=1.00/0.50 and for three values of angle φ_B for both tips B and C.

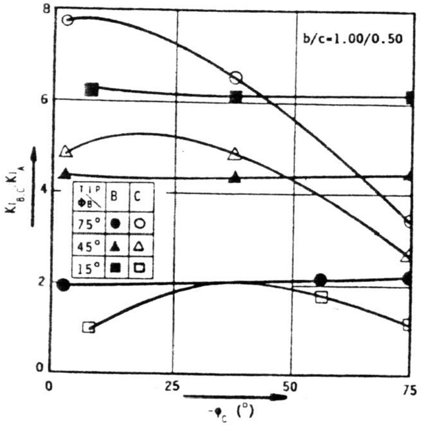

Fig.2: Reduced SIFs versus φ_C

The main advantage of the abovedescribed procedure is that it permits the direct calculation of SIFs at the tips of the branches without any extrapolation. Also, is proved to be accurate and stable for extreme geometries (small angles φ, and length ratios), which are of special importance for the present study. The results obtained are found to be in good agreement with experimental data (Theocaris and Blonzou [14]).

APPLICATION AND RESULTS

The above described procedure was applied for the case of 2124 AlLi Metal Matrix Composite (MMC). The angle φ_B of inclination of the microbranch OB was kept constant and equal to 15°. Two ratios for the lengths (OB) and (OC) were selected, i.e. b/c=1.00/0.50 and b/c=1.00/1.25. The ratio of the length of the main crack (OA) to the length (OB) was supposed constant and equal to 40, based both on experimental observations of fractured specimens with bifurcated cracks and on theoretical calculations indicating that the influence of this ratio is stabilised above this limit. Concerning crack velocity, c, three characteristic values were chosen, i.e. c/c_2=0.35, c/c_2=0.50 and c/c_2= 0.65 (c_2 is the velocity of distortional waves), since the value c/c_2=0.50 is considered as the critical limit above which cracks are bifurcated. The initial conditions are summarized as follows:

i) Crack OA propagates with constant velocity c in a thin sheet loaded at infinity normally to the crack axis.

ii) Tip O becomes suddenly stationary and two branches OB and OC of lengths b *and* c *and inclinations* φ_B *and* φ_C *appear* both *propagating with velocities equal to c.*

The advantage of the above formulation is that it permits direct comparison between the two possible states of crack tips, i.e. to continue running as a single tip (tip A) or to deviate from its straight path being bifurcated or kinked (tip O).

For any given geometry (b/c, φ_B, φ_C) and crack velocity c at each one of the three tips of Fig.1 there exists a direction towards which the local dilatational strain energy density, along the Mises elastic-plastic boundary, shows a maximum value $T_{V,max}^{i}$ (i=A,B,C). Thus the respective crack is expected, according to the T-criterion to propagate towards this direction. Which one of the two patterns (single A or double O) will be realized, depends on energy equilibrium considerations (as it will be discussed in the sequel). For the moment we have plotted in Figs 3(a,b) the maximum values of dilatational strain energy density of the tips

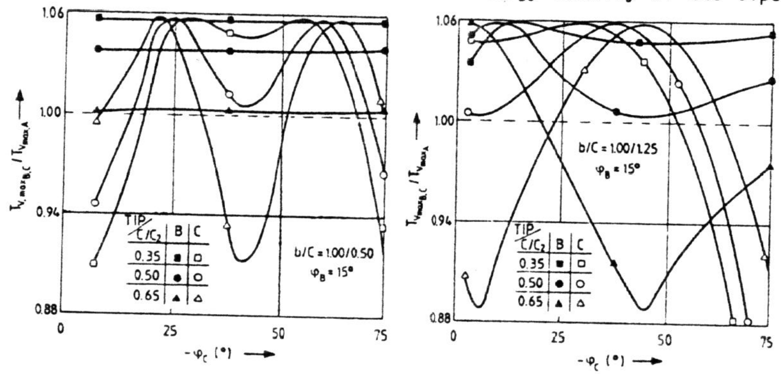

Fig.3: T_V^{MAX} versus angle φ_C for φ_B=15° and two length ratios

B and C, reduced to the respective maximum (and constant) value for tip A. In these figures we have drawn (with dashed line) the same quantity for the tip A. As it can be seen from these figures there exist certain regions of φ_C angles for which $T_{V,max}^{B}$ and $T_{V,max}^{C}$ are greater than $T_{V,max}^{A}$ implying, thus, that for such geometries and crack velocities the twin crack front O is more possible to appear and propagate than the single one, since the energy consumption is greater.

Up to this moment it seems that the instability phenomena present a completely stochastic behaviour, since the ratio b/c and the angles φ_i cannot be a priori known. However, the present model permits us to resolve this contradiction with common experience (that at least branching angle varies between narrow limits), by locating the optimum geometries through the additional reasonable assumption that only geometries where the sum of the dilatational strain

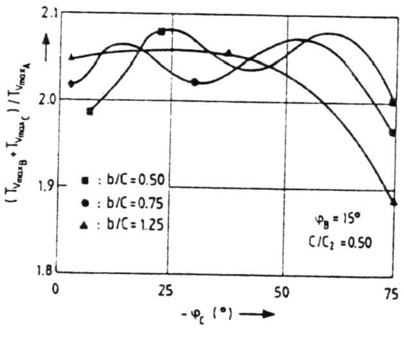

Fig.4: The sum $T_{V,max}^{B}+T_{V,max}^{C}$

energy consumption at the tips tips B and C becomes maximum. To locate these geometries we have plotted in Fig.4 the quantity $(Tv^{B}_{,max}+Tv^{C}_{,max})$, reduced to $Tv^{A}_{,max}$, versus angle φ_C, for three values of the ratio b/c, and for velocity equal to $0.5c_2$. It is clear that this quantity shows one or two discrete maxima (the number and location of which depends on the ratio b/c), corresponding, apparently, to the optimum combinations $(b/a, c/a, \varphi_B, \varphi_C, c)$ for macroscopic crack path deviation to occur.

The question, now, is to which direction each branch will propagate or, in other words, which kind of crack instability (branching, kinking, curving or arrest) will be realized. In Figs 5(a,b) the angular direction θ^B_p and θ^C_p, around each branch tip, where the maximum value of the dilatational strain energy density, $Tv^{B,C}_{,max}$, is attained, is plotted, versus inclination angle φ_C, and for the same constant quantities as in Fig.3. As it can be seen, both branches can propagate to either positive or negative

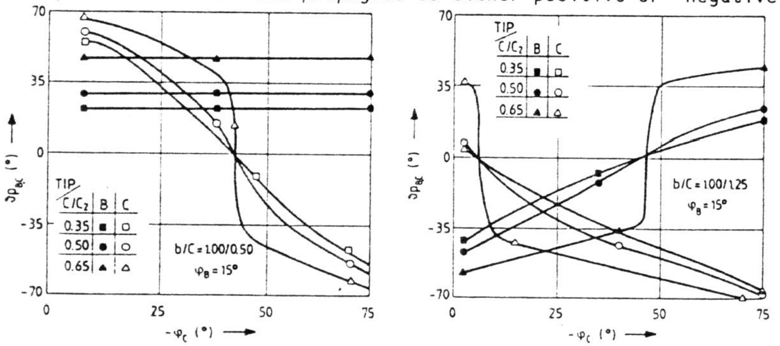

Fig.5: The propagation angle of the two microbranches

directions depending on their initial geometry and velocity. Whether the microbranch will survive or not depends on the specific relation between the angles φ and θ_p (Eq.4).

Summarizing, if we want to predict the future direction of a pair of microbranches of given geometry we proceed as follows:

i) In Fig.4 we locate the geometries of maximum dilatational strain energy density consumption.

ii) From Figs 3(a,b) we check whether both or one or none of them can propagate i.e. whether the condition

$$Tv^{B,C}_{,max} > Tv^{A}_{,max} \qquad (11)$$

holds or not.

iii) From Figs 4(a,b) we obtain the propagation angle $\theta^{B,C}_p$ of each microbranch and with the aid of Eq.(10) we calculate the final deviation angle $\theta^{B,C}_d$ of each microbranch. Depending on whether both or one or none of the microbranches survives and propagates independently and the final angle of deviation we can determine the kind of expected instability. (An analytical description of the possible combinations leading to all experimentally observed crack path deviation modes can be found in references [6] and [12]).

Applying the above procedure for the selected geometry, with $\varphi_B=15^\circ$ and for b/c=1.00/0.50 and velocity $c=0.5c_2$ we observe from Fig.4 that the optimum geometry corresponds to the combination

$(b/c, \varphi_B, \varphi_C, c)=(1.00/0.50,\ 15^\circ,\ -22^\circ, 0.50c_2)$. From Figs 3(a,b) we conclude that only microbranch 0B can survive and propagate independently, since the conditions of Eq.(4) are satisfied only for it, though both microbranches fulfil the equation $Tv^{i}_{,max} > Tv^{A}_{,max}$, (i=B,C). The angle at which the branch 0B will propagate is found from Fig.5 to be $\theta^B_p \cong 30^\circ$ resulting, thus, according to Eq.(4), to final deviation angle $\theta^B_d \cong 45^\circ$. Apparently the configuration under study resembles the phenomenon of macroscopic kinking, since only one branch survives, avoiding absorbtion by the main crack.

In a similar manner, the geometry with $\varphi_B=15^\circ$ and b/c=1.00/1.25 gives as optimum the combination$(b/c, \varphi_B, \varphi_C, c)=(1.00/1.25,\ 15^\circ,\ -30^\circ, 0.50c_2)$. In this case is microbranch 0C the one that survives and propagates independently fulfiling Eq.4 and the restriction that $Tv^{C}_{,max} > Tv^{A}_{,max}$. This configuration results to $\theta^C_p \cong 35^\circ$ resembling again the phenomenon of sudden crack kinking (with kinking angle $\theta_d \cong 65^\circ$).

Finally, studying the case with $(b/c, \varphi_B, \varphi_C, c)=(1.00/0.50,\ 15^\circ,\ -42^\circ, 0.50c_2)$ we conclude that both microbranches survive propagating independently, since both of them satisfy Eq.4 and the energy restrictions. The angles of propagation are found to be $\theta^B_p \cong 30^\circ$ and $\theta^C_p \cong 0^\circ$ and the respective deviation angles $\theta^B_d = 45^\circ$ and $\theta^C_d = -42^\circ$. This case corresponds to an almost symmetric branching with half angle of about 43°. However, we should emphasize that similar situations should not be expected to appear, since they do not correspond (from the energy point of view) to optimum geometric combinations. In fact, reported experimental cases with branching angles greater than 22° are ver seldom.

CONCLUSIONS

The expected behaviour of fast running cracks is studied by expanding the "twin-crack" model to cover fully asymmetrically prebranched crack-tip configurations. The a priori unknown lengths and orientations of the two microbranches act as a stochastic factor to the whole phenomenon introducing an uncertainty factor and arbitrary assumptions concerning their initial values results in macroscopic geometries resembling all macroscopically observed modes of dynamic crack path instability.

The main conclusion from the present study is that *there does not exist a well defined mechanical quantity controlling the instability phenomena.* Instead, a whole surface depending on both micro- and macroscopic factors, describes the running crack behaviour. The microscopic level factors, being a priori unknown, are "suppressed" into four macroscopically interpretable quantities, i.e.the length ratios, b/a and c/a, of the two microbranches and the initial directions of them, φ_i. Although, no predictions can be made on the exact values of these four quantities the phenomenon is proved to be not completely random (a fact experimentally verified) since the demand of the model for maximum consumption of dilatational strain energy density gives us the optimum geometries prefered by the nature of the phenomenon.

The final observation, extracted from the study of the problem of crack path instability through both symmetrical (refs [5,12]) and partly or fully asymmetrical (refs [6,7,12]) prebranched configurations, is that typical branching of the crack can be achieved under either symmetric or asymmetric initial geometries of the microbrances. On the contrary, the other kinds of crack path instability can be achieved only under asymmetric initial geometries and excess of available energy (second maximum of the dilatational strain energy density distribution). This phenomenon implies a kind of preference of the initial geometry of the microbranches (bifurcation is much more often observed experimentally compared to curving or kinking or arrest and reinitiation), indicating, thus, that a statistical approach of the phenomena is necessary in combination with the study of the divergence of materials from their assumed perfect structure. These divergences, either preexisting (voids, inclusions anisotropies) or externally induced due to the propagation of the crack, cannot be ignored.

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