A NUMERICAL MODEL FOR PITTING CORROSION KINETICS IN PRESSURIZED PIPELINES

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ABSTRACT

Corrosion failures of pipelines carrying sour gas are often attributed to stress corrosion cracking. Nevertheless, general corrosion, leading to gradual level wall thinning, and pitting corrosion have also to be taken into account when making lifetime predictions for such pipelines. Pitting corrosion progresses around various part-through surface flaws such as installation mechanical damage, external corrosion stains or grooves under disbonded coatings on line pipes, the internal corrosion stretching along the pipe bottom (so-called 'stream' corrosion), etc. One of the principal factors affecting on the corrosion expansion rate is the level of the local stress-strain state along the corrosion front. Strain and stress values around the surface local damage can considerably exceed the appropriate values averaged across the unsound wall. Consequently, it is necessary to be able to trace the corrosion front evolution to evaluate the remaining lifetime and the current strength of the pipeline segment subjected to the local damage. In this article , a numerical model for kinetics of the localized corrosion expansion is developed. The model involves a new effective recurrent technique for determining an elastic/plastic solution to the problem of stress and strain fields disturbed by the damage.

KEYWORDS

Pipeline reliability, remaining strength, remaining lifetime, pest corrosion, corrosion kinetics, part-through surface flaw.

RECURRENT TECHNIQUE FOR CORROSION FRONT EVOLUTION

Stress-strain state of strained metal affects on corrosion metal loss rate in accordance with the well known equation:

$$V = V_0 (K\varepsilon_p + 1)\exp(\sigma V/RT) \qquad (1)$$

where σ is the hydrostatic constituent of the stress tensor, V is the molecular volume, R and T are the universal gas constant and the metal's temperature, K is a constant, ε_p is an average value of the plastic constituent of the deformation, v_0 is the general corrosion rate for the stress free metal. The actual values of K and v_0 depend on both the pair 'metal - sour environment' and the temperature value.

It is clear that the corrosion metal loss rate will not be constant along the corrosion front because the stress-strain state around the corrosion damage locality is essentially heterogeneous. Let us consider that we are able to calculate effectively the stress-strain picture around the surface damage with an arbitrary shape. Then, a time-step recurrent technique, making it possible to trace the corrosion front evolution, can be presented as follows. Time is divided into rather small equal intervals. In the first step of the recurrent procedure, the stress-strain state around the damage with some initial shape is determined. After that, using the equation (1), the corrosion front spreading rate is calculated at each point of the damage surface. Before the second recurrent step, the new corrosion front position is calculated after that all the procedure is restarted and so on.

The necessity to describe in detail the stress-strain state around the part-through surface damage having an arbitrary shape is the most difficult part in the proposed model. An effective numerical approach to this problem is developed below. The essence of the approach to the problem of elastic/plastic analysis for the part-through flaw locality is, first, to find an approximate solution within the elastic theory of shells with varying wall thickness and, second, to perform a local elastic/plastic correction to the rough shell solution. It is important to note here that plastically deformed fields around the damage are supposed nowhere to embrace the wall thick layer entirely. For solving the appropriate boundary shell problem an effective recurrent technique is developed in the next subsection. This approach involves splitting the differential operators for equilibrium equations into two constituents one of which is just appropriate operators in the constant wall thickness shell theory. The result is so called 'external' shell solution. The elastic solution constructed within the shell theory is obviously invalid in the damage locality. The 'external' elastic shell solution needs some 'internal' elastic/plastic correction over the damage region. To overcome this difficulty some 'internal' boundary problem is formulated for an infinite strip containing an infinite groove with some fixed profile which copies the local profile of the actual flaw. Boundary conditions for the 'internal' problem are taken from the associated 'external' shell solution.

<u>'External' Shell Problem</u>. The stress field in the internally pressurized infinite cylindrical shell with no defects is homogeneous and given by:

$$\sigma_x^0 = \nu PR/h_0 \qquad \sigma_y^0 = PR/h_0 \qquad \sigma_{xy}^0 = 0 \tag{2}$$

where P is operating pressure, R and h_0 are radius and nominal wall thickness of the cylindrical shell, ν is Poisson's ratio, x and y are axial and hoop cylindrical coordinates.

Now, let us consider a pipeline containing some part-through surface flaw. The equilibrium equations for the shallow cylindrical shells can be written as follows:

$$\begin{aligned} &\frac{\partial N_1}{\partial x} + \frac{\partial S}{\partial y} + X = 0 \\ &\frac{\partial S}{\partial x} + \frac{\partial N_2}{\partial y} + Y = 0 \\ &\frac{N_2}{R} + \frac{\partial^2 M_1}{\partial x^2} - 2\frac{\partial^2 T}{\partial x \partial y} + \frac{\partial^2 M_2}{\partial y^2} - Z = 0 \end{aligned} \tag{3}$$

where generalized membrane stresses (N_1, N_2, S), bending moments (M_1, M_2) and torsional moment (T) are connected with displacement components (u, v, w) by the well known formulae:

$$\begin{aligned} N_1 &= A(1-f)\left(\frac{\partial u}{\partial x} + \nu\frac{\partial v}{\partial y} + \nu\frac{w}{R}\right) & M_1 &= B(1-g)\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) \\ N_2 &= A(1-f)\left(\nu\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{w}{R}\right) & M_2 &= B(1-g)\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right) \\ S &= A(1-f)\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\frac{1-\nu}{2} & T &= -B(1-g)(1-\nu)\frac{\partial^2 w}{\partial x \partial y} \end{aligned} \tag{4}$$

Here A and B denote some constants whereas f and g are functions of both x and y:

$$A = Eh_0/(1-\nu^2) \qquad B = Eh_0^3/[12(1-\nu^2)]$$
$$f = 1-h/h_0 \qquad g = 1-(h/h_0)^3$$

By using the above relationships (4) between stress and displacement components, the equilibrium equations (3) can be transformed to some compact form in terms of the displacement field (note: the differential operators are disintegrated here into two constituents one of which is just correspondent operators of the elastic theory of constant wall thickness shells):

$$L_{ik}u_k - D_{ik}u_k + X_i = 0 \qquad (i,k=1,2,3) \tag{5}$$

where the following dimensionless new values are introduced:

$$X_1 = R^2X/(Ah_0) \qquad X_2 = R^2Y/(Ah_0) \qquad X_3 = -R^2Z/(Ah_0)$$

$$u_1 = u/h_0 \qquad u_2 = v/h_0 \qquad u_3 = w/h_0$$

And the matrices of the differential operators L_{ik} and D_{ik} are:

$$L_{ik} = \begin{pmatrix} \dfrac{\partial^2}{\partial x^2} + \dfrac{1-\nu}{2}\dfrac{\partial^2}{\partial y^2} & \dfrac{1+\nu}{2}\dfrac{\partial^2}{\partial x\partial y} & \nu\dfrac{\partial}{\partial x} \\ \dfrac{1+\nu}{2}\dfrac{\partial^2}{\partial x\partial y} & \dfrac{\partial^2}{\partial y^2} + \dfrac{1-\nu}{2}\dfrac{\partial^2}{\partial x^2} & \dfrac{\partial}{\partial y} \\ \nu\dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & 1 + C\nabla^2\nabla^2 \end{pmatrix} \tag{6}$$

$$D_{ik} = \begin{pmatrix} \dfrac{\partial}{\partial x}f\dfrac{\partial}{\partial x}+\dfrac{1-\nu}{2}\dfrac{\partial}{\partial y}f\dfrac{\partial}{\partial y} & \nu\dfrac{\partial}{\partial x}f\dfrac{\partial}{\partial y}+\dfrac{1-\nu}{2}\dfrac{\partial}{\partial y}f\dfrac{\partial}{\partial x} & \nu\dfrac{\partial}{\partial x}f \\ \nu\dfrac{\partial}{\partial y}f\dfrac{\partial}{\partial x}+\dfrac{1-\nu}{2}\dfrac{\partial}{\partial x}f\dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial y}f\dfrac{\partial}{\partial y}+\dfrac{1-\nu}{2}\dfrac{\partial}{\partial x}f\dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y}f \\ \nu f\dfrac{\partial}{\partial x} & f\dfrac{\partial}{\partial y} & f+C\left[\dfrac{\partial^2}{\partial x^2}g\left(\dfrac{\partial^2}{\partial x^2}+\nu\dfrac{\partial^2}{\partial y^2}\right)+\dfrac{\partial^2}{\partial y^2}g\left(\dfrac{\partial^2}{\partial y^2}+\nu\dfrac{\partial^2}{\partial x^2}\right)+2(1-\nu)\dfrac{\partial^2}{\partial x\partial y}g\dfrac{\partial^2}{\partial x\partial y}\right] \end{pmatrix} \tag{7}$$

Here x and y denote dimensionless coordinates: $x=x/R$, $y=y/R$; and C is a constant: $C=h_0^2/(12R^2)$.

The problem written in the form of the equations (5) is ready for solving by a recurrent technique. The main thought of the recurrent procedure given below is to present the solution as an expansion, each term of which represents a solution to a certain problem:

$$u_k = u_k^{(0)} + u_k^{(1)} + \ldots + u_k^{(j)} + \ldots \qquad (k=1,2,3) \tag{8}$$

Each term of the above series has to meet the correspondent equation of the following set:

$$\begin{aligned} L_{ik}u_k^{(0)} &= -X_i && \equiv X_i^{(0)} \\ L_{ik}u_k^{(1)} &= D_{ik}u_k^{(0)} && \equiv X_i^{(1)} \\ &\ldots\ldots\ldots\ldots \\ L_{ik}u_k^{(j)} &= D_{ik}u_k^{(j-1)} && \equiv X_i^{(j)} \\ &\ldots\ldots\ldots\ldots \end{aligned} \tag{9}$$

To avoid too many problems with the boundary conditions, let us consider an infinite set of regularly spaced identical flaws. If the distances between nearby flaws ($2x_\infty$ in x-direction and $2y_\infty$ in y-direction) are much bigger than the flaw dimensions, it is clear that there is no interaction between them. Now, each term of the series (8) as well as the right side parts of the equations (9) can be presented as double Fourier series:

$$u_k^{(j)} = \sum_{n,m=-\infty}^{\infty} \gamma_k^{(j)}(n,m)\exp[i\pi(nx/x_\infty + my/y_\infty)] \tag{10}$$

$$X_i^{(j)} = \sum_{n,m=-\infty}^{\infty} \chi_i^{(j)}(n,m)\exp[i\pi(nx/x_\infty + my/y_\infty)] \tag{11}$$

The initial approximation $u_k^{(0)}$ corresponds to the solution for the unsound pipeline. In order to determine Fourier coefficients $\gamma_k^{(j)}(n,m)$ in the 'j' step of the recurrent procedure, we substitute the Fourier series (10) and (11) in the proper equation of the set (9), after that each couple of integer numbers (n,m) receives its own autonomous system of three linear algebraic equations

$$l_{ik}(n,m)\gamma_k^{(j)}(n,m) = \chi_i^{(j)}(n,m) \qquad (i,k=1,2,3) \tag{12}$$

where $l_{ik}(n,m)$ is a matrix of complex values given by

$$l_{ik}(n,m) = \begin{pmatrix} -\left(\dfrac{n^2}{x_\infty^2}+\dfrac{1-\nu}{2}\dfrac{m^2}{y_\infty^2}\right)\pi^2 & -\dfrac{1+\nu}{2}\dfrac{nm\pi^2}{x_\infty y_\infty} & \dfrac{in\pi\nu}{x_\infty} \\ -\dfrac{1+\nu}{2}\dfrac{nm\pi^2}{x_\infty y_\infty} & -\left(\dfrac{m^2}{y_\infty^2}+\dfrac{1-\nu}{2}\dfrac{n^2}{x_\infty^2}\right)\pi^2 & \dfrac{im\pi}{y_\infty} \\ \dfrac{in\pi\nu}{x_\infty} & \dfrac{im\pi}{y_\infty} & 1+C\pi^4\left(\dfrac{n^2}{x_\infty^2}+\dfrac{m^2}{y_\infty^2}\right)^2 \end{pmatrix} \tag{13}$$

The values $\chi_i^{(j)}(n,m)$ in the right hand sides of the equations (12) are to be calculated in the preceding 'j-1' step of the recurrent procedure in accordance with the definition $X_i^{(j)} \equiv D_{ik}u_k^{(j-1)}$ as follows:

$$\chi_i^{(j)}(n,m) = \sum_{k=1}^{3} \sum_{\lambda,\mu=-\infty}^{\infty} d_{ik}(n,m,\lambda,\mu)\gamma_k^{(j-1)}(\lambda,\mu) \tag{14}$$

The complex matrix d_{ik} in the above equation depending on the four integer numbers is given by:

$$d_{ik} = \varphi(n-\lambda,m-\mu)\left(\begin{array}{ccc} -\left(\frac{n\lambda}{x_\infty^2}+\frac{1-\nu}{2}\frac{m\mu}{y_\infty^2}\right)\pi^2 & -\left(\frac{n\mu\nu}{x_\infty y_\infty}+\frac{1-\nu}{2}\frac{m\lambda}{x_\infty y_\infty}\right)\pi^2 & \frac{in\pi\nu}{x_\infty} \\ -\left(\frac{m\lambda\nu}{x_\infty y_\infty}+\frac{1-\nu}{2}\frac{n\mu}{x_\infty y_\infty}\right)\pi^2 & -\left(\frac{m\mu}{y_\infty^2}+\frac{1-\nu}{2}\frac{n\lambda}{x_\infty^2}\right)\pi^2 & \frac{im\pi}{y_\infty} \\ \frac{i\lambda\pi\nu}{x_\infty} & \frac{i\mu\pi}{y_\infty} & 1+\frac{\psi(n-\lambda,m-\mu)}{\varphi(n-\lambda,m-\mu)} \\ \multicolumn{3}{c}{C\pi^4\left[\frac{n^2}{x_\infty^2}\left(\frac{\lambda^2}{x_\infty^2}+\nu\frac{\mu^2}{y_\infty^2}\right)+\frac{m^2}{y_\infty^2}\left(\frac{\mu^2}{y_\infty^2}+\nu\frac{\lambda^2}{x_\infty^2}\right)+2(1-\nu)\frac{nm\lambda\mu}{x_\infty^2 y_\infty^2}\right]} \end{array}\right) \tag{15}$$

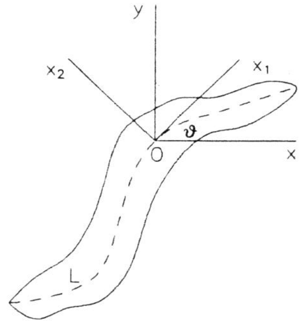
where $\varphi(n,m)$ and $\psi(n,m)$ are the Fourier coefficients for the functions $f(x,y)$ and $g(x,y)$:

$$f(x,y) = \sum_{n,m=-\infty}^{\infty} \varphi(n,m)\exp[i\pi(nx/x_\infty + my/y_\infty)] \tag{16}$$

$$g(x,y) = \sum_{n,m=-\infty}^{\infty} \psi(n,m)\exp[i\pi(nx/x_\infty + my/y_\infty)] \tag{17}$$

Determining Fourier coefficients from the sequence of the autonomous systems (12) completes the solution of the 'external' shell problem in terms of the displacement field. After that all membrane stresses and moments are easily determined by using the equations (4).

'Internal' Boundary Problem. Let us consider the second case of flaws having narrow oblong shapes and stretching along some line L (Fig.1). The line L is the projection of the 'river bottom'

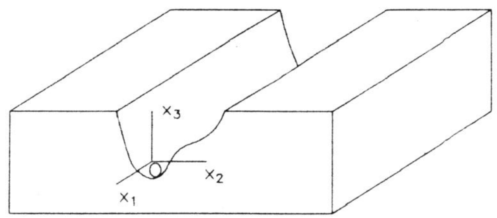


path to the mid-surface of the pipe. The 'river bottom' path is unequivocally determined by the damage relief. The line L is assumed to be smooth, and the variance of the flaw profile along the line L to be rather gradually.

Fig.1. Boundary contour of the surface flaw stretching along the line L (plan view).

The 'external' solution for this type of flaws is valid only out of the flaw, and it needs a serious correction when speaking about the damage locality. Let us assume that we found the stress field everywhere, including the line L, in terms of the shell theory as it was described in the previous section. That would mean that the solution to the appropriate 'external' problem is found. In order to find the 'internal' correction to the 'external' solution, which would be valid over the plane crossing perpendicularly the line L, it is necessary to specify some 'internal' problem related to this cross-section. For this purpose the local Cartesian system of coordinates is introduced as follows (Fig.1):

$$x_1 = x\cos\vartheta + y\sin\vartheta, \quad x_2 = -x\sin\vartheta + y\cos\vartheta$$



The 'internal' problem is formulated for an infinite strip with the thickness equaled to the wall thickness. The strip width have to be no less than double wall thickness plus the flaw width. The strip contains an infinite groove with a permanent profile which is identified with the actual flaw profile in the considered section (Fig.2).

Fig.2. The fragment of the infinite strip for which the 'internal' problem is specified.

The lower and the upper surfaces of the strip as well as the groove surface are stress free. Normal and tangent stresses along the side surfaces of the strip are to be taken from the 'external' shell solution so that their generalized membrane stresses and bending and torsional moments would be equal to the correspondent quantities over the pipe cross-section, oriented along the tangent to the line L at the point O (Fig.1). So boundary conditions over the side surfaces of the strip have to be taken as follows:

$$\sigma_{22}(x_3) = N_n/h_0 + 12M_b x_3/h_0^3, \qquad \sigma_{23}(x_3) = 0,$$

$$\sigma_{21}(x_3) = N_t/h_0 + 12M_t x_3/h_0^3 \qquad (18)$$

where

$$N_n = N_1 \sin^2\vartheta + N_2 \cos^2\vartheta + S\sin 2\vartheta, \qquad N_t = -\frac{1}{2}(N_1 - N_2)\sin 2\vartheta - S\cos 2\vartheta$$

$$M_b = M_1 \sin^2\vartheta + M_2 \cos^2\vartheta + T\sin 2\vartheta, \qquad M_t = \frac{1}{2}(M_1 - M_2)\sin 2\vartheta + T\cos 2\vartheta$$

The above-cited 'internal' problem with the boundary conditions (18) can be expanded into a plane-strain and an anti- plane constituents:

(a) boundary conditions for the plane-strain state are:

$$\sigma_{22}(x_3) = N_n/h_0 + 12M_b x_3/h_0^3, \qquad \sigma_{21} = \sigma_{23} = 0$$

(b) boundary conditions for the anti-plane state are:

$$\sigma_{21}(x_3) = N_t/h_0 + 12M_t x_3/h_0^3, \qquad \sigma_{22} = \sigma_{23} = 0$$

There are many effective computer programs, based on the different boundary element methods, for the solving of plane problems in solid mechanics (Banerjee and Butterfield, 1981; Crouch and Starfield, 1983).

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