VARIANT OF STATISTIC THEORY OF BRITTLE STRENGTH OF GLASS STRUCTURES AND THEIR OPERATE RELIABILITY

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ABSTRACT

The paper presents a variant of the statistic theory of brittle strength for determining the reliability of the built-up glass structures with foreign inclusions being under non-stationary long-term operating conditions. The theory allows to establish the relation between the short-term strength of glass specimens being in the stationary simple uniform stressed state and the long-term strength of glass structures under the non-stationary complicated stressed state. The theory in question is based on the thermofluctuation mechanism of material fracture being in conformity with the principles of the fracture linear mechanics on the experimental determination of the material strength parameters and also on the experimental and theoretical substantiation of the statistic function of specimen longevity distribution.

For determination of reliability, the built-up glass structure is separated schematically into the elements uniform by material, then differential division of the total stress tensor fields is carried out into the surface areas with quasi-uniform and simple stressed state which is equivalent to the specimen condition. After that the conception of a weak link is successively and repeatedly applied. In case of non-stationary stresses and strength parameters the time interval may be divided into sections having quasi-stationary stresses and parameters. After that the principle of equal reliabilities at the boundaries of adjacent time sections may be applied.

KEY WORDS

Glass structure, long-term strength parameters, the conception of a weak link, the principle of equal reliabilities.

INTRODUCTION

In order to ensure the reliability of the responsible built-up glass structures with foreign inclusions being manufactured and operated under non-stationary ambient conditions and changing external loading, a thorough analysis of their long-term strength should be carried out. It requires the expensive and long-term full-scale testing of a large statistic mass of glass structures. Besides, due to a large dispersion of the glass strength and also complexity and non-stability of the structures under stressed state, the results of testing are valid only for the type of manufactures in question of a definite period of production. Therefore the necessity arouse to work out some general methods for determining the long-term strength and reliability of glass structures based on evaluation of the strength of material and manufactures in the stressed state.

The problem of establishing the relationship of the glass specimen longevity upon the stress level and ambient conditions is elucidated in a number of papers. In particular, in the monographs and articles of Zhurkov (1953), Bartenev (1984), Pukh (1973), Razumovskaya (1983), Baker and Preston (1946), Charles (1958), Mould and Southwick (1959, 1960, 1961), Viderhorn (1969), Kerkhoff (1975). Nevertheless, the problem of interrelation between the material strength and the glass structure reliability remained unsolved.

This paper presents a variant of the statistic theory of brittle strength which establishes the above-said interrelation.

STATISTIC MODEL OF FRACTURE AND GLASS STRENGTH PARAMETERS

Suggested is the following statistic model of fracture of glass specimens made of the same kind of glass and by the same technology. A part of the surface area S_0 of these specimens is under the simple uniform stressed tensile state with maximum level of nominal stresses σ. Such stressed state is realized when the discs are loaded by the method of the axisymmetric bending (ASB).

Every specimen has in its zone S_0 a great number of surface microdefects (microcracks) with different orientation, size and degree of danger. Under the action of stress σ local stresses σ_l are formed at the base of the defects. At the temperature T° and humidity H the said local stresses activate the thermofluctuation process of link breakage in glass and propagation of microcracks into the glass depth. On the way of propagation of these defects into the disc bulk, the microstructure of the material of different degrees of danger is formed in the subsurface layer h in the form of internal microdefects and residual stresses of the second kind (in microzones). The growth of microcracks $l(t)$ for a period of time t is accompanied by the rise of local stresses $\sigma_l(t)$ which results in increasing the velocity of local stresses $\dot{\sigma}_l$. It may be assumed that the increment of value $\dot{\sigma}_l$ called forth by a small increment of local stress σ_l is in proportion to the value $\dot{\sigma}_l$ achieved by the given moment of time and to the increment σ_l. In differential form it can be written as $d\dot{\sigma}_l = g\dot{\sigma}_l d\sigma_l$, i.e.

$$\dot{\sigma}_l = D\exp(g\sigma_l). \tag{1}$$

When $\sigma_l(t)$ achieves the critical value σ_*, the microcrack begins to grow disastrously quickly during the time of Δt which may be neglected as compared to the time τ from the beginning of σ application to the point of the specimen fracture.

The fracture of the specimen (glass disc) occurs in the zone where the surface microdefects and microstructure of the layer h lead to the equality of $\sigma_l = \sigma_*$ for the minimum period of time at the given stationary σ. Therefore, the fracture of the specimen takes place due to the most dangerous combination of initial parameters of l_0 microcrack and the subsurface microstructure, and not because of the maximum initial value of one of these factors. It should be taken into consideration that for the majority of the glass manufacture production processes, the orientation of defects in various directions is equiprobable, so at the simple uniform omnidirectional tensile stressed state the geometrically equal microdefects are under the same initial conditions. Gradient $\pm\Delta\sigma(z)$ of stress σ (into the specimen bulk in the direction of z) is small within the limits of the layer h (h =0.25 - -0.5 mm). On the surface of glass discs, glass shells and other manufactures (excluding their ends) $l_0 = 1\text{-}10$ мсм; the critical parameter of microcracks l_* under stresses σ close to safe (σ_c) does not exceed 0.35 mm. Taking into account the fact that the characteristic thickness of glass specimens vary from several millimeters to several tens of millimeters and their curvature equals to several millimeters one can draw the conclusion that the evaluation of maximum local stresses σ_l both for glass specimens and glass manufactures can be carried out by using the model of a semi-infinite space with a surface crack. In this approximation, the self-similarity conditions (which are in fact the closure conditions for calculational models of linear fracture mechanics) are satisfied. Then, (Irvin,1948, Cherepanov, 1974)

$$\sigma_l = K_f \sigma l^{1/2} (2\pi r_0)^{-1/2}, \tag{2}$$

where K_f is the parameter determined by the body geometry and the crack shape, r_0 is the distance from the crack tip to the link-breakage zone ($r_0 \ll l$). Based on the formula $\tau = \int_{cK_0}^{cK_*} [\dot{K}(t)]^{-1} dK$ the longevity τ may be determined by the following expression (Margolin et al.,1988, 1991; Podstrigach et al., 1991)

$$\tau = (\wp q)^{-1} \exp(-qc\sigma l_0^{1/2}). \tag{3}$$

Here $\dot{K} = \wp\exp(qK)$, $c = K_f(2\pi r_0)^{-1/2}$, $K_0 = \sigma l_0^{1/2}$, $K_* = \sigma l_*^{1/2}$;

Q and q are the constants of the glass and the surface treatment process. The equation (3) is analogous to the well-known Zhurkov equation (1953), which at constant temperature T° has the form

$$\tau = A\exp(-\alpha\sigma), \tag{4}$$

where A and α are the material constants at the fixed temperature; $A=\tau_0\exp(U_0/kT^{\circ})$; $\alpha=\gamma/kT^{\circ}$; τ_0, U_0, γ, k – are the parameters of Zhurkov's equation. The comparison of (3) and (4) shows that these equations coincide for $A=1/Qq$ and $\alpha=qc\,l_0^{1/2}$. Formula (3) follows from the statistic model in consideration and expression (1), whereas the dependence of $\dot{K}$ upon K, Q and q has been proved experimentally. Hence, expression (4) has been substantiated by another method. Expression (3) may be written in the form

$$\lg\tau = \Pi - \Psi\sigma, \tag{5}$$

where $\Pi=-(\lg Q+\lg q)$ is the statistically averaged glass constant and $\Psi=q\lg e\,c\,l_0^{1/2}$ is the coefficient of glass static fatigue.

Based on the analysis of parameters q, K_f, τ_c and l_0 in the works of Margolin et al. (1988), Podstrigach et al. (1991) one can see that longevity τ is submitted to the logarithmically normal law. In these works were also determined experimentally the parameters of long-term strength with the help of the axisymmetric bending method for the electron-tube glasses of C52-1 and C93-2 brands: Π, Ψ_M is the median coefficient of static fatigue, ρ_W is the value for determining the specimen longevity logarithm dispersion, σ_c is the criterial safe stress.

For determining the long-term strength parameters, the glass discs fracture probability has been plotted against the longevity logarithm $W(\lg\tau)$ under constant stress σ. After that a family of relations have been plotted for various stresses. Then the data have been obtained for two levels of reliability ($R=R_M=0{,}5$ end $R=0{,}99$) required for plotting the two relations (5): $\lg\tau_M=\Pi-\Psi_M\sigma$ end $\lg\tau_{min}=\Pi-\Psi_{max}\sigma$. Parameters Π and Ψ_M were determined directly by them. Parameter $\rho_W=d^{-1}$, where d is the coefficient of proportionality for the expression $S_{\lg\tau}=d\sigma$, and $S_{\lg\tau}$ is a root-mean-square deviation of the longevity logarithm of specimens; ρ_W is also determined by relations (5). Determination of σ_c parameter is described in the next section.

RELIABILITY OF GLASS STRUCTURES UNDER STATIONARY AND CHANGING LOADS AND STRENGTH PARAMETERS

<u>Reliability of Specimens</u>. Based on the above-mentioned model of fracture and the strength parameters of glass discs in the simple homogeneous stressed state σ of the base area S_0, the reliability of specimens during the specified

operating time T_e may be determined by the following expression (Margolin et al.,1988) :

$$R_0=\begin{cases}1-\Phi(g_0), & \sigma>\sigma_c;\\ 1, & \sigma\le\sigma_c;\end{cases}\quad \Phi(g_0)=(2\pi)^{-1/2}\int_{-\infty}^{g_0}e^{-x^2/2}dx; \tag{6}$$
$$g_0=\rho_W\{\Psi_M+[(\lg T_e-\Pi)/\sigma]\}.$$

For the area S, taking into account the scale factor $M=S/S_0$ and the weak-link conception, the reliability should be $R_S=R_0^M$. The reliability of R_{n0} sampling from n specimens with different stresses σ_i in each of them ($i=\overline{1,n}$) and different area S_i having the simple uniform stressed state may be defined by the formula :

$$R_{n0}=(n)^{-1}\sum_{i=1}^{n}R_{i0},\ R_{i0}=[1-\Phi(g_{i0})]^{M_i}$$
$$M_i=S_i/S_0,\ g_{i0}=\rho_W\{\Psi_M+[(\lg T_e-\Pi)/\sigma_i]\}. \tag{7}$$

The safe criterial stress σ_c may be calculated by formula (6) where $S\gg S_0$ and $T_e>50$ years ($\lg T_{e,s}=9{,}0$).

<u>Reliability of Glass Structures under Stationary Conditions</u>. In order to determine the reliability of glass structures made of one kind of glass with complicated inhomogeneous tensor field with the total stressed state $\sigma^T(x,y)$, this field is divided into m zones with quasihomogeneous stressed state σ^T_ξ ($\xi=\overline{1,m}$). The division is carried out by spacings Δx and Δy after the coordinates of the surface of structures x and y. Taking into consideration the fact that in any system of coordinates the orientation of planes of microdefects propagation becomes equal through 180°, every ξ zone is divided into $K_\xi+1$, the angle interval $\varphi_r\pm\Delta\varphi_r/2$ is from 0 to 180° with quasisimple stressed state $\sigma^T_{\xi r}$.

$$\sigma^T_{\xi r}=[\sigma^T_{\xi 1}+\sigma^T_{\xi 2}+(\sigma^T_{\xi 1}-\sigma^T_{\xi 2})\cos 2\varphi_r]/2, \tag{8}$$

where $\sigma^T_{\xi 1}$ and $\sigma^T_{\xi 2}$ are the main stresses of ξ zone and φ_r is the angle between vectors $\sigma^T_{\xi r}$ and $\sigma^T_{\xi 1}$. Division spacings , and are selected allowing for the accuracy of obtaining the end result.

In compliance with the above-said, application for many times the weak-link concept and the formulae (6)-(8) results in obtaining the expression for determining the reliability of sampling from n structures when they are operated in the stationary conditions for a period of the arbitrary time t :

$$R_S(t)=\begin{cases}(n)^{-1}\sum_{i=1}^{n}\prod_{\xi=1}^{m_i}\prod_{r=0}^{K_\xi}[R_{0i\xi r}(t)]^{M_{i\xi r}}, & \sigma^T_{i\xi r}>\sigma_c;\\ 1, & \sigma^T_{i\xi r}\le\sigma_c.\end{cases} \tag{9}$$

Here $R_{0i\xi r}(t)$ is determined by formula (6) at $\sigma=\sigma^T_{i\xi r}$ and $T_e=t$; $M_{i\xi r}=S_{i\xi r}/S_0=\Delta x_{i\xi}\Delta y_{i\xi}\Delta\varphi_r^{\circ}/180^{\circ}S_0$.

Thus the interrelation has been established between the strength of glass specimens having simple uniform stressed state and the reliability of glass structures with complicated stressed state.

Reliability of glass Structures under Non-stationary Conditions. The dependence of reliability upon the time $R_v^v(T_e)$ under non-stationary conditions and strength parameters may be determined in the following way. The operating time range from 0 to T_e may be divided into small intervals $\Delta t_q (q=\overline{1,Q})$. Within these intervals the stresses and parameters are quasistationary. The spacing of division Δt_q should be chosen in view of the stress gradient and parameters, in order to ensure the accuracy of the end result.

It is also possible to divide the time range into small intervals $\Delta(\lg t_q)$. The changing stresses and parameters within the said intervals are substituted by their extreme values. For all that the amount of calculations is reduced.

The dependences $\Pi(t)$, $\Psi_M(t)$, $\rho_W(t)$, $\sigma_c(t)$ and $\sigma^T_{i\xi z}(t)$ are replaced by step functions having mean integral values of stationary parameters Π_q, Ψ_{Mq}, ρ_{Wq}, σ_{cq} and stresses $\sigma^T_{i\xi zq}$ in the intervals Δt_q. Time intervals in which $\sigma^T_{i\xi zq}(t) \leq \sigma_c(t)$ are neglected. Then the principle of equal reliability is applied for neighbouring intervals Δt_q and Δt_{q+1} beginning from $q=1$ in two stages (first, by the strength parameters, second, by stresses). In compliance with this principle, the equal reliability conditions should be observed at the end of the previous R^*_q and at the beginning of the next R^o_{q+1} time interval (Podstrigach et al., 1988, 1990, 1991). The said principle being applied, interval Δt_q may be substituted by interval $\Delta t^e_{q+1\,q+1}$ equivalent by reliability and having stress and parameters of Δt_{q+1} interval. As a result, the problem is r...uced to a stationary variant, first for two, then successively for all time intervals.

In compliance with the above-mentioned works the algorithm has been obtained for determining the structure reliability under non-stationary conditions of operation :

$$R_v^v(T_e)=(n)^{-1}\sum_{i=1}^{n}\prod_{\xi_i=1}^{m_i}\prod_{z=0}^{K_{i\xi}}(R_{oi\xi zq})^{M_{i\xi z}} \tag{10}$$

Here $R_{oi\xi zq}=R^*_{qqq}$, $R^*_{qqq}=1-\Phi\{\rho_{Wq}[\Psi_{Mq}+\frac{\lg(\Delta t^e_{qq}+\Delta t_q)-\Pi_q}{\sigma^T_{i\xi zq}}]\}$,

$$\Delta t^e_{qq}=\operatorname{ant}\lg[\frac{\sigma^T_{i\xi zq}}{\sigma^T_{i\xi zq-1}}(\lg\Delta t^e_{q-1q}-\Pi_q)+\Pi_q],\quad \Delta t^e_{q-1q}=\operatorname{ant}\lg\{\sigma^T_{i\xi zq-1}[\frac{\rho_{Wq-1}}{\rho_{Wq}}(\Psi_{Mq-1}+\frac{\lg(\Delta t^e_{q-1q-1}+\Delta t_{q-1})-\Pi_{q-1}}{\sigma^T_{i\xi zq-1}})-\Psi_{Mq}]+\Pi_q\}.$$

Reliability of Built-up Glass Structures. In order to determine the reliability of these structures they may be divided schematically into L nods consisting of the same kind of glass. Taking into account the independence of the nods reliability, the concept of a weak link is applied. The reliability of sampling from n built-up structures for a time period T_e is determined by the expression

$$R_{vB}^v(T_e)=(n)^{-1}\sum_{i=1}^{n}\prod_{\chi=1}^{L}\prod_{\xi_{i\chi}}^{m_{i\chi}}\prod_{z=0}^{K_{i\chi\xi}}[R^{(\chi)}_{oi\xi z}(T_e)]^{M_{i\chi\xi z}}, \tag{11}$$

where $\chi=\overline{1,L}$; $R^{(\chi)}_{oi\xi z}(T_e)$ is reliability of the glass not can be determined at $n=1$ by formula (9) under stationary and formula (10) under non-stationary conditions of operation. The strength parameters are established for every kind of glass.

Two criteria, σ_c and σ^d_R are employed in the suggested variant of the brittle strength theory. For stresses which maximum value does not exceed a criterial one ($\sigma_m \leq \sigma_c$) the reliability $R=1$ (defects do not propagate). When the stress exceeds the level σ_c, based on the above-said, the value of structural strength σ^d_R is determined for the specified value of reliability R^d. The value σ^d_R limits the level ($\sigma_m \leq \sigma^d_R$) of stresses which ensure the reliability R^d for a period of specified time of operation.

The theory was aprobated and introduced in the process of evaluation of reliability of glass shells of vacuum electronic devices. With that the problem was worked out pertaining to elaboration of operative methods and means (including forced testing) for evaluation of the structures reliability at the stage of elaboration and constructive-technological correction of vacuum electronic devices.

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