STRESSED STATE OF ANISOTROPIC CYLINDRICAL SHELLS WITH CUTS

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ABSTRACT

In the present analysis singular integral equations of the problem on the elastic equilibrium of anisotropic cylindrical shell with the cut oriented along a screw curve segment are obtained.For this purpose the relations of both Love theory and Timoshenko theory are used.The cut is considered as allocation locus of initial stress sources and their densities are bound up with displacement discontinuities and the shell normal element rotations.On the basis of numerical solution of the obtained equations.the shell anisotropy influence on the stress intensity factors is examined for single and interacting cracks oriented along the curvature line.

KEYWORDS

Anisotropic nonshallow cylindrical shell, arbitrilâry oriented cut, Love and Timoshenko theories, system of singular integral equations.

GOVERNING RELATIONS

Introduction.The examination of the shell anisotropy influence on the stressed state in the vicinity of a crack is carried out,in the main, for orthotropic shells (see i.e. review in the work by Osadchuk et al.(1986).In the work by Lubchak et al. (1985) the Kierchhoff-type shallow cylindrical shells with circumferential crack were considered for the case of general anisotropy (the elastic symmetry plane of the shell material perpendicular to the median surface is absent or its track is inconsistent with the principal curvature lines).Below the stressed state problem for anisotropic nonshallow cylindrical shell with cracks is investigated.

Anisotropic Cylindrical Shell with Initial Stresses. Refer a median surface of the shell to the coordinate system $\alpha_1 O \alpha_2$, where α_1, α_2 are the distances assigned to the shell radius R oriented along ruling straight line and guiding circular, respectively. The formal vectors of the stressed-strained state parameters are introduced:

$$\bar{n}_1 = (N_1, S, N_2, \underline{Q_1, Q_2}); \quad \bar{n}_2 = b_1(M_1, H, M_2, \underline{0,0});$$

$$\bar{\varepsilon}_1 = (\varepsilon_{11}, \varepsilon_{12}, \varepsilon_{22}, \underline{-\varepsilon_{13}, -\varepsilon_{23}}); \quad \bar{\varepsilon}_2 = b_1^{-1}(\varkappa_{11}, 2\varkappa_{12}, \varkappa_{22}, \underline{0,0});$$

$$\bar{u} = (u_1, u_2, w, \underline{\gamma_1, \gamma_2}) \; ; \; b_1 = \sqrt{3}/h \qquad (1)$$

Here N_i, Q_i, M_i are normal force, transverce shear force and bending moment in the cross-section α_i=const, respectively; S, H are symmetrisized shearing force and twisting moment, respectively (in particular, in the case of cylindrical shell S is the shear force in the axial cross-section); ε_{ij} are shell membrane strains, ε_{i3} are transverce shear strains; $\varkappa_{ij}$ are bending strains (i,j=1,2); u_i are displacements along $O\alpha_i$ line assigned to R; w is the deflection assigned to R, γ_i are rotations of the shell normal element around the line $O\alpha_{3-i}$, h is the half-thickness of the shell. Here and after the underlined quantities must be omitted for the case of Love theory. Formulas, written for this case, exlusively, are denoted by the word " Love " after the corresponding numerical designation. The equilibrium and compatibility equations are:

$$\sum_i \hat{N}_i \bar{n}_i = 0 \; ; \; \bar{\varepsilon}_i = \bar{\varepsilon}_i^{e} + \bar{\varepsilon}_i^{0} = \hat{N}_i^{T} \bar{u} \quad (i=1,2) \qquad (2)$$

Here $\bar{\varepsilon}_i^{e}$ are elastic strains, $\bar{\varepsilon}_i^{0}$ are free from stresses strains, uncompatibility of which affects the stresses in the shell (Osadchuk, 1985), $\hat{N}_i$ are the operator matrix of the dimensions 5×5 in the case of Timoshenko theory and 3×3 in the case of Love theory (such objects are denoted with the sign "∧" under corresponding letter), superscript "T" denotes the transposition. Non-zero matrix elements are the following:

$$\{\hat{N}_1\}_{i,\,j+i-1} = -\{\hat{N}_1\}_{3,\,j+3} = \{\hat{N}_2\}_{i+3,\,j+i-1} = \partial_j \; ;$$

$$\underline{\{\hat{N}_1\}_{33} = \{\hat{N}_1\}_{25} = -\{\hat{N}_1\}_{i+3,\,i+3} = 1;} \quad \underline{\{\hat{N}_2\}_{22} = \partial_1} \qquad (3)$$

$$\{\hat{N}_2\}_{2,\,j+1} = (1+\delta_{1j})\partial_j ; \{\hat{N}_2\}_{3,\,j} = -(1+\delta_{2j})\partial_1^{3-j}\partial_2^{j-1} \qquad (3\text{-Love})$$

where δ_{ij} is Kronecker symbol, $\partial_i = \partial/\partial\alpha_i$. Here and after the letter subscripts become 1,2 if their range is not indicated. The elements of operator matrix are interpretted as polynomial forms with constant coefficients above variables ∂_1 and ∂_2, if some operations above this matrix are executed:

$$\partial_1^m \partial_2^n = \partial^{m+n}/\partial\alpha_1^m \, \partial\alpha_2^n \; ; \; (m,n=0,1,2,\ldots<\infty, \partial_i^0=1) \qquad (4)$$

The relations between forces-moments and elastic strains are received from the analysis of the shell elastic energy.

$$\bar{n}_i = \hat{C}\,\bar{\varepsilon}_i^{e} \quad , \quad \hat{C} = \begin{bmatrix} \hat{A} & \hat{0} \\ \hat{0} & \hat{B} \end{bmatrix}$$

$$\hat{A} = \begin{bmatrix} C_{11} & C_{16} & C_{12} \\ C_{16} & C_{66} & C_{26} \\ C_{12} & C_{26} & C_{66} \end{bmatrix} \qquad \hat{B} = - \begin{bmatrix} C_{55} & C_{45} \\ C_{45} & C_{44} \end{bmatrix} \qquad (5)$$

Here $\hat{0}$ are the zero matrix, C_{ij} are the shell elastic constants (Ambartsumian, 1974). The forces and moments are expressed by displacements and free from stresses strains:

$$\bar{n}_i = \hat{M}_i \bar{u} - \hat{C}\bar{\varepsilon}_i^{0} \; ; \qquad \hat{M}_j = \hat{C}\,\hat{N}_j^{T} \qquad (6)$$

The system of equations in displacements is obtained by substitution of (6) in the equilibrium equations:

$$\hat{L}\,\bar{u} = \sum_i \hat{M}_i^{T} \bar{\varepsilon}_i^{0} \; ; \quad \hat{L} = \sum_i \hat{N}_i \hat{M}_i . \qquad (7)$$

Partial solution of this system is introduced in the form:

$$\bar{u} = \sum_i \hat{L}^{-1} \hat{M}_i^{T} \bar{\varphi}_i \; , \qquad (8)$$

where $\hat{L}^{-1}$ is matrix of cofactors to matrix $\hat{L}$ such that $\hat{L}^{-1}\hat{L} = \hat{E}\,\det\hat{L}$ ($\hat{E}^{-1} = \hat{E}$), $\bar{\varphi}_i$ are vectors of resolving functions, which satisfy the equations :

$$D^N \bar{\varphi}_i = \bar{\varepsilon}_i^{0} \; ; \; D^N = \det \hat{L} \, , \qquad (9)$$

their order N is equal to 10 in Timoshenko theory case and in Love case it is equal to 8. The integral representations for the resolving functions are built as convolutions of free strains with 2π-periodic above variable α_2 fundamental solution $G(\alpha_1, \alpha_2)$ equation (9) (Osadchuk et al., 1991). The forces and moments expressions by resolving functions are obtained by substitution of (9) and (8) in (6)

$$\bar{n}_i = \sum_j \hat{K}_{ij} \bar{\varphi}_j \; ; \quad \hat{K}_{ij} = \hat{M}_i \hat{L}^{-1} \hat{M}_j^{T} - \delta_{ij}\hat{C} D^N \; . \qquad (10)$$

The Shell with Arbitrarily Oriented Crack. Let the shell contain a cut, the track Γ of which on the median surface is the segment of a screw line, creating the angle φ with the coordinate line α_1=const. The parametric equation of the cut line Γ is given in the form $\alpha_i=\xi_i(\lambda)=(-1)^i\nu_{3-i}\lambda+\alpha_i^0$; where $\nu_1=\cos\varphi$, $\nu_2=\sin\varphi$, λ is assigned to R arc Γ length (its sign is the same as for segment), which is measured from its center (α_1^0,α_2^0); $|\lambda|\le\lambda_0$, where λ_0 is a halflength of Γ, assigned to R. We shall assume the shell is affected by the external load and equal by the value and contrarilly directed forces and moments are applied to the crack edges so that the edges don't contact between themselves. The stressed state of the shell is presented as the sum of principal stressed state, caused by the external load in the shell without a cut and exited state, which is caused by the crack presence (Osadchuk, 1985). The principal stressed state is assumed to be known. According to this, the boundary conditions on the crack edges for the excited state are written in the form:

$$\sum_i \hat{V}_i \bar{n}_i = \bar{t}|_\Gamma = \bar{t}^1-\bar{t}^0 = \bar{f} \; ; \; \bar{f} = (f_1,f_2,f_3,f_4,f_S) \; ; \qquad (11)$$

where t, t^0, t^1, respectively, are the vectors of generalized forces and moments for excited and principal stressed states and total one, which are applied to the crack edges. The prhycical nature of these vectors is such: f_1, f_2 are generalized normal and shearing forces, respectively, f_3 is generalized transverce shear force; f_4, f_S are bending and twisting moments, respectively. Here $\hat{V}_i$ are matrices with the dimentions 5×5 in Timoshenko theory case and 4×3in Love case. The non-zero components of the matrices are of the form:

$$\{\hat{V}_1\}_{mn} = C_1^{-1}\{\hat{V}_2\}_{m+3,n} = p_{mn}, \quad \{\hat{V}_1\}_{3,m+3} = \nu_m \qquad (12)$$

$$\{\hat{V}_2\}_{mn} = \overline{C_1\nu_1\nu_{3-m}(p_{2n}+\delta_{2n})} \; ; \{\hat{V}_2\}_{m+2,n} = C_1(p_{1n}\partial_\nu^{2-m}+\delta_{2n}p_{2n}\partial_\tau)$$

where

$$(12\text{-Love})$$

$$p_{11}=\nu_1^2 \; ; \; p_{13}=\nu_2^2 \; ; \; p_{22}=\nu_1^2-\nu_2^2 \; ; \; \tfrac{1}{2}p_{12}=-p_{21}=p_{23}=\nu_1\nu_2$$

$$\partial_\nu=\nu_1\partial_1+\nu_2\partial_2 \; ; \; \partial_\tau=\nu_1\partial_2-\nu_2\partial_1 \; ; \; m=1,2; \; n=1,2,3; \; C_1=b_1R.$$

Taking into account that forces and moments remaine continuous when they cross the crack line, and displacements and rotations undergo the discontinuities of the first kind. They differentiate as generalized functions on the basis of (2) so that the following expressions for free strains caused by the cut are obtained:

$$\bar{\varepsilon}_j^{\,o}(\alpha_1,\alpha_2)=\int_{-\lambda_0}^{\lambda_0}[\bar{\upsilon}(\lambda)]\,\hat{V}_j^T\,\delta(\alpha_1-\xi_1(\lambda),\,\alpha_2-\xi_2(\lambda))\,d\lambda\,, \qquad (13)$$

where the discontinuities of the vector components are denoted by the square brackets and their geometric matter is following:

$$\upsilon_1=\nu_1u_1+\nu_2u_2 \; ; \; \upsilon_2=\nu_1u_2-\nu_2u_1 \; ; \; \upsilon_3=-w$$
$$-\upsilon_4=\nu_1\gamma_1+\nu_2\gamma_2 \; ; \; -\upsilon_S=\nu_1\gamma_2-\nu_2\gamma_1 \qquad (14)$$

$$\upsilon_4=\partial_\nu w+\nu_1^2\upsilon_2-\nu_2^2\upsilon_1 \qquad (14\text{-Love})$$

Integral representations for generilized forces and moments are built by the substitution of (10) in (11):

$$\bar{t}(\alpha_1,\alpha_2)=\int_{-\lambda_0}^{\lambda_0}[\bar{\upsilon}(\lambda)]\,\hat{T}\,G(\alpha_1-\xi_1(\lambda),\,\alpha_2-\xi_2(\lambda))\,d\lambda \qquad (15)$$

where

$$\hat{T}=\hat{W}\hat{L}^{-1}\hat{W}^T-(\sum_i\hat{V}_i\hat{C}\hat{V}_i^T)D^N, \quad \hat{W}=\sum_i\hat{V}_i\hat{M}_i \qquad (16)$$

The system of integral equations for determining the unknown discontinuities is obtained by equating these representation values to the given ones on the crack edges according to the conditions (11). The method of reducing the obtained system to the system of integral equations with singular Caushy kernels is the same as one for isotropical shells (Osadchuk, 1985). According to the expression (16) the components of matrix $\hat{T}$ are follows :

$$\{\hat{T}\}_{pq}=\sum_{s=1}^{N/2-1}X_{pq}(2s+1+a_s-b_s)\,\partial_\tau^{b_s} \quad ; \quad p,q=\overline{1,N/2} \qquad (17)$$

where $X_{pq}(g)$ are homogeneous operators of the order g, and they have a form of linear combination of monomials such as in (4) under $m+n=g$

$$a_s=1-d_1+2d_2 \; ; \; b_s=\delta_{s4} \; ; \qquad (18)$$
$$a_s=b_1=d_1 \; ; \; b_2=d_2 \; ; \; b_3=0 \qquad (18\text{-Love})$$

Here $d_1=1+\delta_{p3}+\delta_{q3}$; $d_2=\delta_{p3}\delta_{q3}$. Then on the basis of singlevaluedness conditions for the displacements and of rotations in the cut tips we decrease the order of differential operators (17) by transferring the action of derivatives along the crack line ∂_τ from the integral representation kernels (15) on their densities with the help of integration by parts. The singular parts of the kernels are extracted in the evident form with the help of the fundamental solution of equation (9), where only the higher derivatives (of the order N) (Osadchuk, 1985) remaine. As a result of some manipulations we get the system whose form is analogical to the systems of integral equations, discribing the elastic equilibrium of isotropic and transversally isotropic shells.

$$F_p(x) = \int_{-\lambda_o}^{\lambda_o} g_q(\lambda) \left[\frac{A_{pq}}{\lambda - x} + K^o_{pq}(\lambda - x) \right] d\lambda , \quad (|x| \le \lambda_o) \qquad (19)$$

In the case of Timoshenko theory:

$$F_p(x) = f_p(x) , \; g_q(\lambda) = d[v_q(\lambda)]/dx , \; p,q = \overline{1,5} \qquad (20)$$

These relations are true in the case of Love theory, except

$$F_3(x) = \int f_3(x)dx , \; g_3(\lambda) = d^2[v_3(\lambda)] / d\lambda^2 \qquad (20\text{-Love})$$

where f_3 is a shearing force, $g_3=w$ is the deflection. In the case of general anisotropy for nonshallow shells all the constants $A_{pq}=A_{qp}\neq 0$. Only in the case of Timoshenko theory $A_{p3}=A_{3p}=0,(p\neq 3)$, and the corresponding kernels have a logarithmic singularity. The unknown solutions must satisfy the known conditions obtained from the singlevaluedness for displacements and rotations in the crack tips.

RESULTS AND CONCLUSION

The system of integral equations (19) was solved numerically for anisotropic cylindrical shells of the Love type with the system of cracks oriented along curvature lines. The solution was introduced in the form:

$$g_p(\lambda) = g^0_p(\lambda)\left[1 - (\lambda/\lambda)^2_0 \right]^{-1/2} \quad (\overline{p = 1,4}), \qquad (21)$$

where $g^0_p(\lambda)$ is continuous function. In this connection the conditions of absence of the crack edges contact (Osadchuk, 1985) were controlled. On the basis of this solution the axially symmetric forces and moments distribution in the vicinity of crack tips was built and the values of dimensionless stress intensity factors for the quantities t_p from Eqv.(11) were found by the formulas:

$$K_p^{*\pm} \overset{def}{===} \sqrt{2} \lim_{\rho\to+0} \rho^{1/2+\delta_{3p}} t_p(\xi_1(\lambda),\xi_2(\lambda)) = -\sum_{q=1}^{4} A_{pq} g^0_q(\pm\lambda_0) \qquad (22)$$

where $\lambda=\pm(\lambda_0+\rho)$. The shells are considered in the case when the axes of material orthotropy are inconsistent with the shell curvature lines. The matrix of the shell elastic constants (5) is determined by the set of parameters (Ambartsumian, 1974):

$$E_1, E_2, \nu_1, \nu_2 \quad (E_2\nu_1 = E_1\nu_2), \; G_{12}, \; \varphi_0 , \qquad (23)$$

Here $E_i, \nu_i (i=1,2), G_{12}$ are elasticity moduluses, Poisson ratios and the shear modulus, respectively; φ_0 is an angle between the axis of the shell and the orthotropic axis with a larger value of the elasticity moduluses $(E_1>E_2)$. The Figures show the cha-

racteristic relations for $K^*_p(\varphi_0)$ $(p = 1,2)$ under h/R=0.01. The solid curves correspond to the material with the parameters E_1=14.9; E_2=0.6; G_{12}=0.4; ν_1=0.31, shading ones correspond to the material with the parameters E_1=6.25; E_2=2.12; G_{12}=0.9; ν_1=0.25, where the values E_1, E_2, G_{12} are assigned to 10^4MPa. In Fig.1 the graphes for the shell with the periodic system of seven collinear circumferential cuts are shown. The curves 1,2,3 correspond to the extension (with the boundary conditions as $f_p = -\delta_{2p}$; $K^*_1>1$; $K^*_p \le 0$); the curves 4,5,6 correspond to the torsion ($f_p=-\delta_{1p}$; $K^*_2>1$; $K^*_1 \le 0$). The parameter of the crack length λ_0 becomes 0.2 (curves 1,4); 0.4 (curves 2,5) and 0.44 (curves 3,6). In Fig.2 the graphes for $K^*_p(\varphi_0)$ $(p=1,2; K^*_1>1; K^*_2\le 0)$ for the shell with two collinear longitudinal cuts in the case of

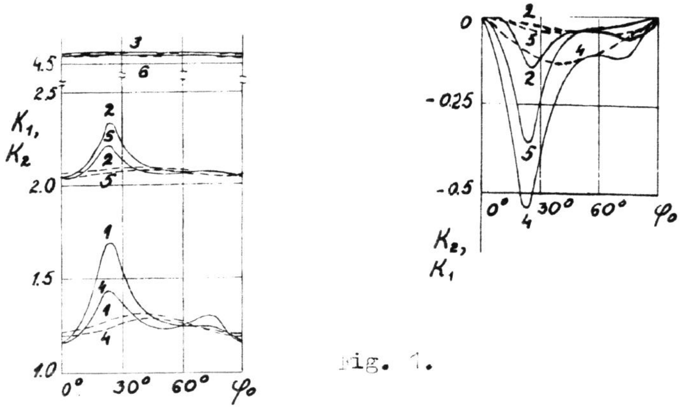

Fig. 1.

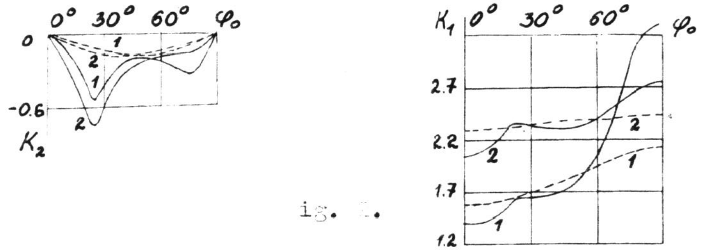

Fig. 2.

inner pressure ($f_p=-\delta_{1p}$) are shown.The halflength of cuts is $\lambda_0=0.2$;the distances between the cut centers is equal to $d_0=2.2\lambda_0$.The curves 1 correspond to the external cut tips,the curves 2 correspond to the internal ones.On the whole one can make the following conclusions about the influence of the anisotropy on the stress intensity factors:

1. For the shells in contrast to the planes under the boundary conditions on the cut edges $t_p=-\delta_{rp}$ ($r=1,2$; $p=1,2,3,4$) all stress intensity factors are non-zero and they depend on anisotropy of the shell.Besides the values K_1^* and K_2^* may be comparative,the values K_3^* and K_4^* have the order of values hK_r^*/R under $\lambda_0\leq 1$.The dependence of the stress intensity factors on the cut length and shell curvature has approximately the same nature as in the case of isotropic shells.

2.For the collinear cuts case the dependence of the stress intensity factors in the neighboring cut tips on the distance between them has the same nature,on the whole,as for isotropic shells (the stress intensity factors increase sharply when the distance d_0 decreases).The case of circumferential orientation of the cuts under the considered boundary conditions is the exception:K_{3-r}^* is approaching to zero.At the extrance of shell anisotropy in the case of the longitudional collinear cuts the dependence $K_r^*(d_0)$ may have substantially non-monotone nature. The smaller is the value d_0,the smaller is the dependence of the stress intensity factors on the shell anisotropy in the internal cut tips.

3.The value of the error caused by application of the shallow shells theory may depend substantially from the shell anisotropy and, in particular, from the value on angle φ_0 .

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