

# STOCHASTIC APPROACH OF THE RELIABILITY OF FIBREGLASS REINFORCED LAMINATED WOOD JOINTING PIECES FOR GLULAM CONSTRUCTION

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## ABSTRACT

This paper presents the study of the reliability of a joint specially designed for the construction with glued laminated timber. As the fire resistance is essential for this kind of construction, it is proposed to replace the metallic jointing plates by laminated wood jointing pieces glued to the beams. To improve the strength of such pieces, reinforcement by mean of fibreglass clothes has been developed. It has been shown that this technique improves the strength from 9 to 35 %, depending on the direction of loading. A stochastic approach of this design shows the allowable stresses in terms of probability of failure.

## KEYWORDS

Glulam - fracture - reliability - stochastic approach - fibreglass - composite

## INTRODUCTION

About Glulam Construction. Glulam construction is nowadays a very popular way to build large halls. This is particularly evident for sport halls, concert halls and generally culture oriented halls. It is also used for trade centers, or for industrial buildings. The advantages of this system of construction is that it allows very large spans in shapes according to the best strength and to new design ideas of architects. Wood is also very appreciated for its good acoustical properties.

But the main feature is that wood shows a very good strength to fire. It is well-known that wood has a bad reaction to fire, that is to say it is inflammable. But, as during a fire the temperature increases up to 600 or 800°C, the mechanical properties of the wood which is burning or heated are not affected, mainly because wood has a very high insulating power with respect to heat. Thus, in case of fire, it would take a long time before the frame of the structure collapses, which allows peoples to evacuate the building and firemen to fight safely against the fire inside of the building itself.

The same kind of deduction could be made for corrosion degradation of steel plates joints, which are very sensitive points in warehouses for chemicals or in swimming pools.

Consequently, we have tried to solve this problem in designing wood-based jointing pieces instead of the traditionnal steel jointing pieces. Those pieces are jointed to the beams by a multiple finger joint. In order to reinforce these pieces, we have proposed to make them from a wood-based composite reinforced with fibreglass clothes in the glue joints. This technique allows to increase significantly the stresses allowed in the finger joint.

Stochastic Approach of the Reliability of Wood-Based Jointing Pieces. Of course, the specific gravity of wood highly depends on the species, but also on the growing conditions of the trees, including climate. The consequence is that the specific gravity is very variable and, as the mechanical properties depends on it, they are also very variable. Cautious choice of the qualities of sawns for glulam construction can minimize this problem, but it remains very important and is a source of catastrophic failures in this kind of constructions.

To take into account this problem, designers use to employ rather large safety coefficient, the determination of which is mainly empirical. In this work, our purpose is to show how

they can be evaluated by a stochastic approach of the variability. It appears that the main difficulty of the approach is the size of the experimental sample, which is often small, due to the long duration and the cost of sophisticated tests. We have overpassed this difficulty by generating a large number of data using a Monte-Carlo method. But the results of this method also clearly depend on the significance of the original sample.

## EXPERIMENTS

**Materials properties.** All the experiments have been done on a material picked up in a glulam factory. This was a spruce (*Picea abies*). This species is very commonly used for glulam. It is chosen for its good homogeneity of specific gravity inside a tree, between 0.40 and 0.50 kg/dm<sup>3</sup> at 15% moisture content and a low volumic shrinkage (10%) for a variation in moisture content from 0 to 30%. The measured physical and mechanical properties are given in table 1.

Table 1. Physical and mechanical properties of the species used.

Property	H %	M <sub>v</sub> kg/dm <sup>3</sup>	E <sub>L</sub> MPa	σ <sub>b</sub> MPa	σ <sub>cp</sub> MPa	σ <sub>ct</sub> MPa	E <sub>t</sub> MPa	σ <sub>tp</sub> MPa	σ <sub>tt</sub> MPa	τ <sub>pf</sub> MPa
Mean value	12	0.44	13442	92	53	8.30	571	82	2.10	3.00
Standard deviation	0.46	0.04	2063	15.60	6.46	0.60	121.50	8.11	0.39	0.25
Coef. of variation (%)	3.96	9.26	15.35	16.90	12.28	7.26	21.27	9.93	18.56	8.34
Sample size	18	18	35	35	30	8	8	3	19	4

H : moisture content ; M<sub>v</sub> : specific gravity ; E<sub>L</sub> : bending Young's modulus in the grain direction ; σ<sub>b</sub> : bending fracture stress ; σ<sub>cp</sub> : compression fracture stress along the grain direction ; σ<sub>ct</sub> : compression fracture stress across the grain ; E<sub>t</sub> : compressive Young's modulus across the grain ; σ<sub>tp</sub> : tensile fracture stress along the grain ; σ<sub>tt</sub> : tensile fracture stress across the grain ; τ<sub>pf</sub> : shear fracture stress parallel to the grain.

**Experiments.** Our purpose was to estimate the behaviour of wood-based jointing pieces under different kind of loadings. Following this idea, two experiments have been designed :

- aligned jointing of two straight laminated beams submitted to pure bending,
- angle jointing of two straight laminated beams submitted to opening or closing moments.

In the first case, the behaviour of such beams was compared with that of similar single-piece beams. In the second case, as it appeared that the strength of the jointing piece was too low, similar experiments were conducted on the same kind of specimen realized with reinforced jointing pieces. The reinforced material was a multilayers composite made from wood plates and fibreglass clothes as described in Fig. 1.

Testing of this material has shown that the mechanical properties in the transverse direction with respect to the grain of the wood has been multiplied by 5.9. No value was obtained in the direction of the grain, but it was assumed that it was equivalent to that given in the case of solid wood. Also, no value was obtained for shear strength. The results are given in Table 2.

### Description of the experiments.

As all the components of the specimens are made from wood, a particular care was taken in the choice of the material to avoid major defects such as knots or cracks. Apart from the plates used to build the composite material, all other pieces were realized from lamellae of

10x55 mm<sup>2</sup> cross section. They have been cut in length of about 1170 mm. To build quite

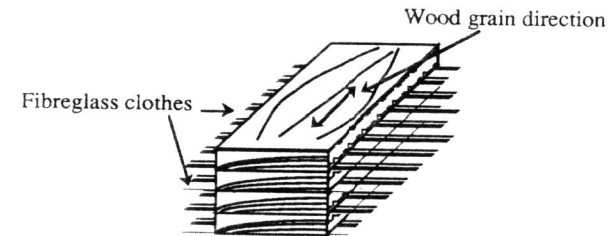


Figure 1. Realization of the composite material.

Table 2. Tensile strength across the grain of the composite material.

Mean value MPa	Standard deviation MPa	coef. of variation	sample size
12.39	1.85	14.92%	6

similar beams or jointing pieces, it is necessary to know the bending modulus of each lamellae. Every lamella has been submitted to a simple three-point bending test using a dead weight of 20 N and measuring the deflection of the beam after a constant delay. The results of this experiment are given in Table 3. Note that statistical analysis has been made using Gauss and Weibull's distributions.

Table 3. Statistical properties of the bending modulus of spruce.

Type of distribution	Mean value (MPa)	Standard deviation (MPa)	Coef. of variation	Sample size
Gauss	13702	2509	18.27%	261
Weibull	13724	2449	17.84%	261

As it can be seen, the differences between both distributions is not significant. For further analysis we will use the Weibull's distribution, as recommended by Bodig and Jayne (1982).

Six straight specimens, six angle specimens and six reinforced angle specimens have been realized. This corresponds to eighteen laminated beams of 70x40 mm<sup>2</sup> cross section, each made from seven 10 mm thickness lamellae. Three other beams of 140x40 mm<sup>2</sup> cross section have been made for the non-reinforced jointing pieces. To realize the most homogeneous sample, all the lamellae constituting these beams have been chosen with the aid of a computer programme so that the calculated bending modulus of all the beams are as near as possible. For all the beams, the difference between actual and calculated modulus was less than 9 %.

The three straight jointed specimens were realized in the following way (Fig. 2). The jointing piece has been directly cut in the middle of the beam. The multiple finger joints have been machined on each end of this piece and on one end of the two remaining parts of the original beam and these three parts have been bond together to constitute a beam.

The non-reinforced angle specimen have been made as previously described, except that the jointing piece has been cut in a 14 lamellae beam (Fig. 3). A particular attention was focused on the angle of the finger joint plane with respect to the orientation of the grain of the beam and of the jointing piece. Indeed, the goal was to minimize the stresses perpendicular to the

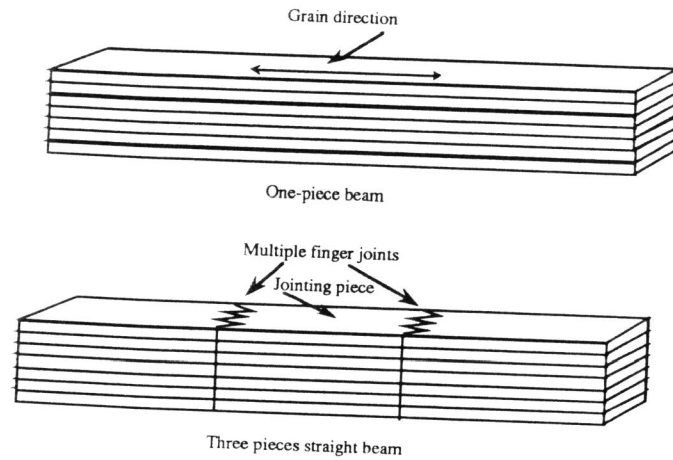


Fig. 2. Design of straight specimens

grain of the wood, which is the weakest direction of strength. It appears from strength of material calculations (Szűcs, 1991) that the angle which realize this condition is given by the bissectrice of the angle made by the grain direction in the beam and the grain in the jointing piece.

The reinforced angle specimens have been made in the same way as the previous one. In this case, as the jointing piece is made from a reinforced material instead of laminated wood, the angle of the finger joints plane which minimize the stresses in the laminated beam is 90° with respect to the grain direction in the beam.

In both cases, a special frame was designed to ensure a constant and convenient pressure during drying of the glue in the finger joint.

#### TESTING

Tests of straight beams (with or without jointing piece) has been proceeded through a 4 points bending device. The loading device was placed in an instrumented testing machine and the deflection of the pure bending part of the beam together with the applied load was recorded. For the beams with a jointing piece, the strain of each laminate in different places around the finger joint has been recorded to test the influence of this joint on the behaviour of the beam. The load was applied until total failure occurs.

The angle specimens (reinforced or non-reinforced) have been tested in a specially designed device. It was able to apply opening or closing loads. During the test, the relative displacement of the loading point with respect to the base line, the relative displacement of both ends of each branch of the V and the deflection of these branches has been recorded. These measurements were necessary to support the assumption which has been made during the analysis. Typical records are shown in Fig 4.

#### RESULTS

The results of the tests are analysed in terms of maximum stresses at defined places in the structure. In some cases, these places will correspond to the actual fracture location.

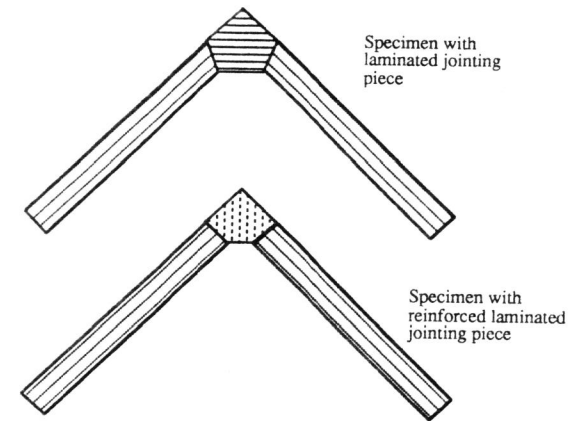


Fig. 3. Design of angle specimens, jointing piece reinforced or non-reinforced.

These stresses have been determined using simple strength of material analysis in all cases. Each experiment was repeated three times. All the tests were considered as valid. Results are given in table 4.

Table 4. Experimental results (fracture stress in MPa).

Experiment	Non jointed straight beams	jointed straight beams	Non reinforced V-beams (opening)	Non reinforced V-beams (closing)	Reinforced V-beams (opening)	Reinforced V-beams (closing)
N.1	82	61	31.78	36.03	41.09	42.41
N.2	84	78	33.85	38.44	45.39	39.14
N.3	82	58	31.71	35.27	44.52	38.23
Mean value	82.67	65.67	32.45	36.58	43.67	39.93
Standard deviation	1.15	10.79	1.21	1.66	2.27	2.20
Coef. of variation (%)	1.4	16.43	3.73	4.54	5.20	5.51

As a first analysis of these results, it can be seen that :

- the finger joint in straight beams induces a reduction in strength of about 20%. This result is consistent with tensile tests on such joints ;
- the reinforcement of the jointing piece in angle specimens leads to an increase of about 35% of strength in the opening case and 9% in the closing case. This shows the advantage in designing such pieces with a reinforcement.

#### STOCHASTIC ANALYSIS OF RESULTS

Monte-Carlo calculations. As it can be seen on the tables of results, the significance of mean and standard deviation is not very high for the number of results for each experiment is rather low, even if the quality of experiments and preparation of specimens are asserted and if the experimental device is very well prepared. Therefore, if there is a great confidence in our experiments and if we assume that statistics on this kind of material and structure follows a Weibull's law, it is possible to increase considerably the size of the sample for valid statistical calculations. This technique is known as the Monte-Carlo method.

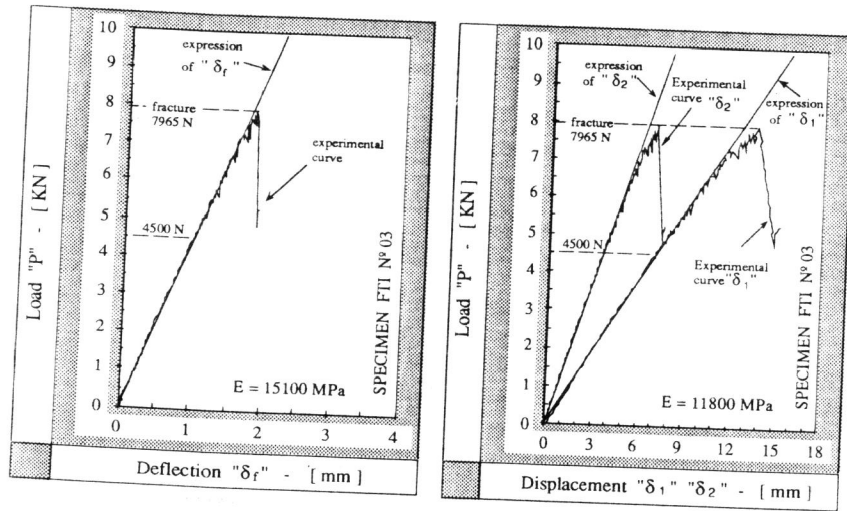


Fig. 4. Typical curves recorded for an angle specimen.

The relationship of the Weibull's distribution is recalled here :

$$F(x) = 1 - \exp\left[-\left(\frac{x - x_0}{x_a}\right)^m\right] \quad (1)$$

Where  $x_0$  is the lowest probable value of  $x$ ,  $x_a$  a standardization factor and  $m$  an exponent depending on the scattering of the data. This is the complete Weibull's law, but, usually a simplified one is used :

$$F(x) = 1 - \exp\left[-\left(\frac{x}{x_a}\right)^m\right] \quad (2)$$

Given the experimental value of  $m$  and  $x_a$  in the Weibull's law, a value of  $x$  is derived by taking a random value of  $F(x)$  ( $0 \leq F(x) \leq 1$ ) a great number of times, following the relationship :

$$x = \text{Exp}\left[\frac{\text{LnLn}\left(\frac{1}{1 - F(x)}\right) + m \cdot \text{Ln}(X_a)}{m}\right] \quad (3)$$

In our case, we have computed 999 values by this technique. The results of the calculation is given in table 5 in terms of standardisation factor  $\sigma_a$ , Weibull's exponent  $m$  and mean value. This mean is derived from the relationship :

$$\bar{\sigma} = \sigma_a \cdot \Gamma\left(1 + \frac{1}{m}\right) \quad (4)$$

where  $\Gamma$  is the "Gamma" function.

The safety factor. The concept of safety factor is derived from the stochastic analysis of the fracture of structures. Given a sample of tests and a distribution function of the results, the

safety factor  $f_s$  is defined as the ratio between the mean fracture stress over the stress  $\sigma$  corresponding to a given probability of failure :

$$f_s = \frac{\bar{\sigma}}{\sigma} \quad (5)$$

If  $P_f$  is the probability of failure at stress  $\sigma$ , the probability of survival  $P_s$  is :

$$P_s = 1 - P_f \quad (6)$$

Therefore, it can be easily shown that  $f_s$  is equal to :

$$f_s = \frac{1 \left(1 + \frac{1}{m}\right)}{\left[\text{Ln}\left(\frac{1}{P_s}\right)\right]^m} \quad (7)$$

Following this analysis, the results of the Monte-Carlo calculation has been analysed in terms of safety factor corresponding to two probabilities of failure, say  $10^{-4}$  and  $10^{-6}$ . These two probabilities correspond to the values allowed respectively for structures where there are few risks and where there are many risks for human life.

Table 5. Results and analysis from the Monte-Carlo calculation.

	Non jointed straight beams	jointed straight beams	Non reinforced V-beams (opening)	Non reinforced V-beams (closing)	Reinforced V-beams (opening)	Reinforced V-beams (closing)
$\sigma_a$ (MPa)	83.32	71.29	33.23	44.70	44.95	49.09
$m$	45.34	4.47	17.34	16.14	14.40	13.47
Mean (MPa)	82.30	65.03	32.23	43.26	43.35	47.24
Safety factor for $P_f = 10^{-4}$	1.21	7.16	1.65	1.71	1.83	1.91
Safety factor for $P_f = 10^{-6}$	1.34	20.06	2.15	2.28	2.52	2.68

From this table, it can be seen that there is a very good agreement between the experimental (see Tab. 4) and the calculated mean values. On the other hand, there is a large discrepancy between the  $m$  values (4.47 to 45.47). As  $m$  is a measure of the scattering of the data, this could indicate that some of the experiments are not very representative of the actual behaviour of the structures tested. From our experience, it seems that, for a carefully prepared experiment on a wood-based structure, a  $m$  value of 8 to 20 should be reasonable. This would signify that the experiments on non-jointed straight beams are "too" good and that those on the jointed straight beams are "too" scattered. The consequence of this is that the calculated safety factors in the first case are too low and in the second case are too high. In the four other cases, the safety factors are to be compared with those usually employed by the codes and standards.

For the non reinforced V-shaped joint (opening case), the DIN 1052 P1 standard uses the following relationship for the allowable stress :

$$\sigma_{\text{all}} = \sigma_{\text{all}}^{c//} - \left(\sigma_{\text{all}}^{c//} - \sigma_{\text{all}}^{c\perp}\right) \sin\alpha \quad (8)$$

where  $\sigma_{all}^{c//}$  and  $\sigma_{all}^{c\perp}$  are respectively, the allowable stresses in compression parallel and

perpendicular to the grain direction.  $\alpha$  is the angle between the direction of the wood fibers and the loads. As these stresses values, for the species here used, are respectively, 8.5 and 2 MPa, the angle being 22.5°, the allowable stress would be 6.01 MPa. The safety factor in this case is equal to 5.36. This corresponds to a probability of failure practically null, by comparison with 2.28 red in table 5. This shows that the standards, in this particular case, are very conservative.

Of course, this comparison is based on a stochastic analysis using very well selected experiments and having a rather low sample size. To be more realistic, it should be necessary to build a significant database directly from the fabrication of a manufacture. Only in this case, the safety factors would have an actual signification. But it remains true that, if the size of the sample is not large enough to obtain accurate Weibull's parameters, the Monte-Carlo method is always available.

#### CONCLUSION

This paper has described the realization and analysis of strength tests on joints which are or could be used in the glulam construction. It must be pointed out that the joints described here do not include any other material other than wood, excepted a thin layer of adhesive for jointing. This is very important in case of fire, as wood exhibits a good resistance to fire, with respect to steel, which is mainly used for bolts and plates in traditional joints.

The most important aspect of the design of the joint is the use of multiple finger joint which, if conveniently machined, provides a strength near to 80% of the solid wood one. For jointing of beams with a given angle, noticeable transverse and shear stresses appear which may result in catastrophic failure. To avoid this problem, it has been proposed to reinforce the material of the jointing piece with fibreglass clothes conveniently arranged.

But, as wood and wood-based composites exhibit a large scattering mainly due to variability in specific gravity, the results of the tests should be analysed from a stochastic point of view. Here, because of the needed time and the cost of the tests, each experiment has been repeated three times only. On this basis, it was not reasonable to build a realistic statistical analysis. To avoid this problem, it has been proposed to generate a great number of tests, from the existing ones and based on the Monte-Carlo method. The Weibull's distribution law was chosen as it is common now for wood properties. Parameters of this law has been derived and safety factors computed for each kind of experiment. It has been shown, that, apart of two experiments where there are non-realistic values, safety factors can be related to a given probability of failure. A comparison has been established in one case with the safety factor derived from standards. It has been shown that these standards led to conservative values of safety factors.

Finally, it can be said that a stochastic analysis of fracture of a structure, helped by a Monte-Carlo data generation method, if necessary, can be used to design a structure with respect to a given probability of failure.

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