REINFORCEMENT OF A CRACKED PLATE BY A SYSTEM OF PARALLEL STRINGERS

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ABSTRACT

Stress and strain state and limiting equilibrium of a plate reinforced by a system of parallel stringers and weakened by an arbitrarily located curvilinear crack are investigated. The plate is subjected to biaxial tensile stresses at infinity, and self-equilibrating forces are assumed to act at the faces of a crack. The stringers are joined with the plate by rivets, which are simulated by round rigid inclusions of small radius. Interaction between the stringers and the plate at attachment points is substituted by action of unknown point forces applied at the rivet centres. The boundary value problem is reduced to a singular integral equation (SIE) in a displacements jump derivative. Unknown point forces involved in this equation are found from compatibility conditions for displacements of corresponding points of the stringers and the plate. Stress intensity factors (SIF) and critical load in the plate are determined on the basis of numerical solution of the SIE.

KEYWORDS

Fracture mechanics, stress intensity factors, cracked plate, stringer, rivet, singular integral equations method.

REDUCTION OF THE PROBLEM TO INTEGRAL EQUATIONS

Consider an infinite plate of thickness h weakened by a curvilinear crack L and strengthened by a system N_2 of parallel stringers (Fig.1). All the stringers are supposed to be of constant cross-sections, to work only under tension-compression conditions and to be joined with the plate by means of N_3 rivets the radii values of which are small as compared with those of other plate linear dimensions such as the crack length, the stringer interspacings and the rivet interspacings. The rivets are simulated by rigid inclusions which fill up the corresponding holes in the plate and stringers. The cracked plate with stringers is under generalized plane stress conditions.

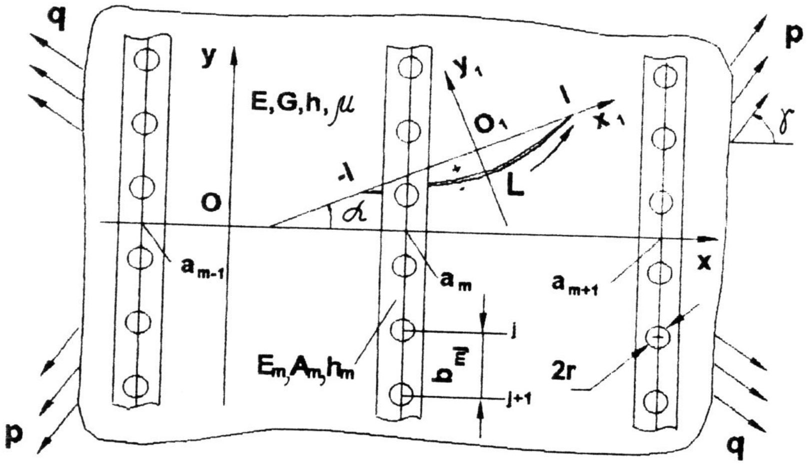

Fig.1. Scheme of reinforcement of a plate containing a curvilinear crack by a set of parallel stringers.

We use the basic Cartesian coordinate system, which is introduced so that the Oy axis is parallel to the stringers, and the local $x_1O_1y_1$ one to which the crack contour L refers. The relationship between the point coordinates in the basic coordinate system and those in the local coordinate system is determined by

$$z = z_1 \exp(i\alpha) + z_0, \quad z = x + iy, \quad z_1 = x_1 + iy_1, \tag{1}$$

where z_0 is the origin coordinate of the local coordinate system. Mutually perpendicular stresses p and q act at infinity in the plate plane, the stresses p being directed at an angle of γ with the Ox axis. Self-equilibrating stresses

$$N^{\pm}(t) + i\, T^{\pm}(t) = p_1(t), \qquad t \in L, \tag{2}$$

where N and T are the normal and tangential stresses respectively, are specified at the faces of the crack. The index + (-) denotes the boundary value of the function at L when approaching the latter from the left (right).

The effect of the stringers on the plate is substituted by

the effect of point forces $Y_{m\nu}$ ($m = \overline{1,N_2}$, $\nu = \overline{1,N_3}$), applied at the rivet centers ($z^0_{m\nu}$ points) in direction parallel to the Oy axis (Cherepanov and Mirsalimov, 1969). To deduce the SIE, these forces are supposed to be known. Their calculation will be considered later. The boundary condition (2) for the crack faces can be expressed as

$$\Phi(T_1') - \overline{\Phi(T_1')} + \exp(-2i\alpha)(d\bar{t}_1'/dt_1')\left[T_1'\overline{\Phi'(T_1')} + \overline{\Psi(T_1')}\right] =$$
$$= p_1(t_1'), \qquad T_1' = t_1'\exp(i\alpha) + z_0, \quad t_1' \in L. \tag{3}$$

The complex potentials $\Phi(z)$ and $\Psi(z)$ are (Savruk, 1981)

$$\Phi(z) = \frac{1}{2\pi}\int_L \frac{e^{i\alpha} g_1'(t_1)}{T_1 - z}dt_1 - \frac{i}{2\pi h(1+\varkappa)}\sum_{m=1}^{N_2}\sum_{\nu=1}^{N_3}\frac{Y_{m\nu}}{z - z^0_{m\nu}} + (p+q)/4;$$

$$\Psi(z) = \frac{1}{2\pi}\int_L\left[\frac{e^{-i\alpha}\overline{g_1'(t_1)}}{T_1 - z}d\bar{t}_1 - \frac{\bar{T}_1 e^{i\alpha} g_1'(t_1)}{(T_1 - z)^2}dt_1\right] - \tag{4}$$

$$- \frac{i}{2\pi h(1+\varkappa)}\sum_{m=1}^{N_2}\sum_{\nu=1}^{N_3}\left[\frac{\varkappa}{z - z^0_{m\nu}} - \frac{z^0_{m\nu}}{(z - z^0_{m\nu})^2}\right]Y_{m\nu} - \frac{1}{2}(p-q)e^{-2i\gamma};$$

where

$$g_1'(t_1) = \frac{2G}{i(1+\varkappa)}\frac{d}{dt_1}\left[\left[u_1(t_1) + iv_1(t_1)\right]^+ - \left[u_1(t_1) + iv_1(t_1)\right]^-\right] \tag{5}$$

is the derivative of the displacement jump of u + iv on the crack contours ($t_1 \in L$) in the local coordinate system; $\varkappa = (3-\mu)/(1+\mu)$; μ, G are Poisson's ratio and the plate shear modulus respectively; $T_1 = t_1\exp(i\alpha) + z_0$.

Satisfying the boundary condition (3) with the aid of the complex potentials (4) gives the SIE as

$$\frac{1}{\pi}\int_L\left\{K_1(t_1,t_1')\, g_1'(t_1)dt_1 + L_1(t_1,t_1')\,\overline{g_1'(t_1)}d\bar{t}_1\right\} +$$
$$+ \frac{i}{2\pi h(1+\varkappa)}\sum_{m=1}^{N_2}\sum_{\nu=1}^{N_3}M_1(z^0_{m\nu},T_1')Y_{m\nu} = p_1(t_1') + F_1(t_1'), \quad t_1' \in L, \tag{6}$$

where $\quad K_1(t_1,t_1') = \dfrac{e^{i\alpha}}{2}\left[\dfrac{1}{T_1 - T_1'} + \dfrac{e^{-2i\alpha}}{\bar{T}_1 - \bar{T}_1'}\dfrac{d\bar{t}_1'}{dt_1'}\right];$

$$L_1(t_1,t_1') = \frac{e^{-i\alpha}}{2}\left[\frac{1}{\overline{T_1}-\overline{T_1'}} - \frac{(T_1-T_1')e^{-2i\alpha}}{(\overline{T_1}-\overline{T_1'})^2}\frac{d\overline{t_1'}}{dt_1'}\right];$$

$$M_1(z,t_1') = \frac{1}{z-T_1'} - \frac{1}{\overline{z}-\overline{T_1'}} - e^{-2i\alpha}\left[\frac{\ae}{\overline{z}-\overline{T_1'}} - \frac{z-T_1'}{(\overline{z}-\overline{T_1'})^2}\right]\frac{d\overline{t_1'}}{dt_1'};$$

$$F_1(t_1') = -\frac{1}{2}\left[p+q-\frac{d\overline{t_1'}}{dt_1'}(p-q)\,e^{2i(\gamma-\alpha)}\right].$$

A solution of the SIE (6) must satisfy an additional condition, namely

$$\int_L g_1(t_1)dt_1 = 0, \qquad (7)$$

which provides uniqueness in the displacements on traversing the contour L.

Unknown point forces $Y_{m\nu}$ ($m=\overline{1,N_2}, \nu=\overline{1,N_3}$) may be defined on the basis of conditions for compatibility of displacements of stringer elements and those of corresponding plate points. To do this, each stringer is arbitrarily devided into N_3-1 of elements. For each stringer element Hooke's law can be written down as (Cherepanov and Mirsalimov, 1969; Poe, 1971)

$$P_{mj} = \frac{E_m A_m}{b_{mj}}\left[v_m(z^1_{mj}) - v_m(z^1_{mj+1})\right], \quad m=\overline{1,N_2}, \ j=\overline{1,N_3-1}. \qquad (8)$$

Here, $P_{mj} = \sum_{\nu=1}^{j} \overset{s}{Y}_{m\nu}$ are the forces, which stretch the jth element of the mth stringer; b_{mj} is the length of the corresponding element; $\overset{s}{Y}_{m\nu} = -Y_{m\nu}$ ($\nu=\overline{1,N_3}$) are the corresponding point forces in stringers; A_m, E_m are the cross-section area and Young's modulus of the mth stringer respectively; $z^1_{mj} = z^0_{mj}+ir$ are the coordinates of points at the edges of the rivets; r is the rivet radius; $v_m(z^1_{mj})$ is the vertical displacement of the corresponding stringer point.

Since each of the stringers is in equilibrium, some additional conditions have to be fulfilled:

$$\sum_{\nu=1}^{N_3} \overset{s}{Y}_{m\nu} = 0, \qquad m=\overline{1,N_2}. \qquad (9)$$

A complex-valued displacement vector for the plate is found from (Muskhelishvili, 1966)

$$2G\left[u(z)+iv(z)\right] = \ae\,\varphi(z) - z\,\overline{\Phi(z)} - \psi(z), \qquad (10)$$

where $\varphi(z)=\int\Phi(z)dz$, $\psi(z)=\int\Psi(z)dz$, and the complex potentials $\Phi(z)$ and $\Psi(z)$ can be obtained from the equation (4). As a result, we obtain

$$2Gv(z) = \frac{1}{2\pi}\,Im\int_L\left\{f_1(t_1,z)\,g_1'(t_1)dt_1 + f_2(t_1,z)\,\overline{g_1'(t_1)dt_1}\right\} + $$

$$+\frac{1}{2\pi h(1+\ae)}\sum_{m=1}^{N_2}\sum_{\nu=1}^{N_3} f_3(z,z^0_{m\nu})\,Y_{m\nu} + F_4(z), \qquad (11)$$

where $f_1(t_1,z) = e^{i\alpha}\left[\overline{\ln(T_1-z) - \ae\ln(T_1-z)}\right]$;

$$f_2(t_1,z) = e^{-i\alpha}(T_1-z)/(\overline{T_1}-\overline{z});$$

$$f_3(z,z^0_{m\nu}) = -2\ae\ln|z-z^0_{m\nu}| - Re\left[(z-z^0_{m\nu})/(\overline{z}-\overline{z^0_{m\nu}})\right];$$

$$F_4(z) = Im\left[\frac{\ae-1}{4}(p+q)\,z + \frac{p-q}{2}e^{2i\gamma}\,\overline{z}\right];$$

$Im(\ )$, $Re(\ )$ are the imaginary and real parts of the complex function respectively.

Taking into consideration the equations (8),(9) and (11), the displacement compatibility conditions can be expressed as

$$\begin{cases} Im\int_L\left\{\Delta f_1(t_1,z^1_{mj})\,g_1(t_1)dt_1 + \Delta f_2(t_1,z^1_{mj})\,\overline{g_1(t_1)dt_1}\right\} + \\ +\sum_{\nu=1}^{N_3}\left[\frac{1}{1+\ae}\Delta f_3(z^1_{m\nu},z^0_{m\nu}) + \frac{2\pi b_{mj}\delta_{\nu j}}{(1+\mu)b\lambda_m}\right]\frac{Y_{m\nu}}{h} = -2\pi\,\Delta F_4(z^1_{mj}); \qquad (12) \\ \sum_{\nu=1}^{N_3} Y_{m\nu} = 0, \qquad m=\overline{1,N_2}, \quad j=\overline{1,N_3-1}. \end{cases}$$

Here, $\Delta f(z^1_{mj}) = f(z^1_{mj}) - f(z^1_{mj+1})$; $\delta_{\nu j} = \begin{cases}1 & \text{for } \nu\leqslant j,\\ 0 & \text{for } \nu > j;\end{cases}$

$\lambda_m = A_m E_m/(bhE)$ is the relative stiffness of the mth stringer; $b=\min\{b_{mj}\}$; E is Young's modulus of the plate; the functions of $f_i(t,z)$ $(i = 1,2,3)$, $F_4(z)$ are obtained from the equation (11).

Thus, the set of equations (12) together with the SIE (6) and additional conditions (7) provide a complete system of equations to determine the function $g_1'(t_1)$ $(t_1 \in L)$ and the $N_z = N_2 \times N_3$ of point forces $Y_{m\nu}$ $(m = \overline{1,N_2};\ \nu = \overline{1,N_3})$.

ANALYSIS OF NUMERICAL RESULTS

A numerical solution of the integral equations (6), (7) and (12) can be obtained by a method of mechanical quadratures (Savruk, 1981). A parametric equation for the L contour in the local coordinate systems may be expressed as

$$t_1 = \omega(\xi), \quad \xi \in [-1, 1] \quad (t_1 \in L). \tag{13}$$

The SIF are determined by the relation

$$K_I^{\pm} - i K_{II}^{\pm} = \mp \sqrt{\pi|\omega'(\pm 1)|}\, u(\pm 1)/\omega'(\pm 1), \tag{14}$$

$$u(\xi) = g_1'(t_1)\omega'(\xi)\sqrt{1-\xi^2}, \quad \omega'(\xi) = d(\omega(\xi))/d\xi;$$

where symbols − and + indicate the crack beginning $(z=\omega(-1))$ and the crack end $(z=\omega(+1))$ respectively.

To determine the critical loads p_c, we use the σ_θ-criterion (Panasyuk, 1968). As a result,

$$p_c/p = K_{Ic}/\left[K_I^{\pm}\cos\frac{\theta_c}{2} - 3K_{II}^{\pm}\sin\frac{\theta_c}{2}\right]/\cos^2\frac{\theta_c}{2}, \tag{15}$$

where K_{Ic} is the critical value of SIF K_I, θ_c is the angle which characterizes an initial direction of the crack growth.

The effects of stringer stiffness, crack geometry, crack orientation and loading type on variation of the SIF and the limiting equilibrium of the plate were investigated. An assumption was made that all the stringers had the same stiffness $(\lambda_m = \lambda)$ and the same quantity of rivets (N_3), the distance between the rivets being of constant value $(b_{mj} = b)$. Since disturbance of the stress and strain state of the plate containing a crack is local, only a finite quantity of stringers and rivets was used in calculations. Calculations were performed for straightline $(\omega(\xi) = l\xi)$ and parabola-like $(\omega(\xi) = l\xi - id(\xi^2-1))$ cracks for $\mu = 0.3$.

The results obtained show as follows.

1. The more the rivets in stringers, the larger the critical loads should be applied to the crack faces. When the quantity of rivets attains 10 further increase beyond this number does not influence the situation at the crack tips.

2. Stiffness of stringers has an essential effect on variations of the stress and strain state and the limiting equilibrium of a plate containing a crack. With increasing quantity of stringers, the influence of this parameter becomes more appreciable.
3. As to the crack geometry, weakly curved cracks are the most dangerous rather than straightline or strongly curved ones.
4. Until the crack tip is at a definite distance (of about 0.2l) from the stringer, values of the SIF and critical load remain almost constant. The SIFs begin to decrease abruptly (the critical load begins to increase), when the crack grows out under the stringer and advances the same distance away from the stringer. Then the normalized SIFs $K_{I,II}^{\pm}/(p\sqrt{\pi l})$ begin to increase (the normalized critical load p_c/p begins to decrease) being lower (the critical load being higher) than those for the initial crack (Fig.2). Similar conclusions were made earlier for a plate with a straightline crack perpendicular to a stringers (Poe, 1971).

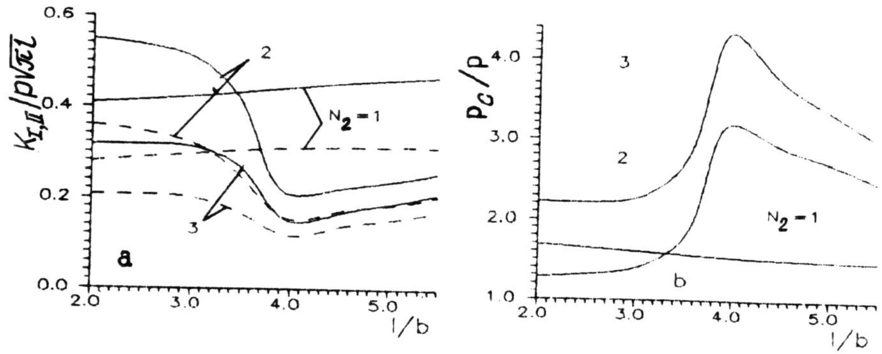

Fig.2. Dependence of the normalized SIF $K_i/(p\sqrt{\pi l})$ (a) (solid and dashed lines refer to i=I and i=II respectively) and the critical load p_c/p (b) on length l/b of an inclined (α=0.59) straightline crack for a plate subjected to uniaxial stretching at infinity and reinforced by one $(N_2=1,\ a_1=0)$, two $(N_2=2,\ a_1=-3b,\ a_2=3b)$ and three $(N_2=3,\ a_1=-3b,\ a_2=0,\ a_3=3b)$ stringers at $\lambda=2, r=0.1b, N_3=12$.

5. A general picture of variation of the SIF and the critical load as a function of the crack orientation (the angle α) has been found to be similar to that for a plate with no stringers. In particular, for uniaxial stretching at infinity critical forces are minimum not for a crack which is perpendicular to direction of the force action

but for a crack which is inclined at a definite angle. This angle varies with varying stiffness of stringers. For biaxial stretching, the largest critical load is seen to be at $\alpha = 0$ (Fig.3a). With increasing the angle α, the load level decreases and, at $\alpha \to \pi/2$, it approaches the corresponding value of p_c for a plate with no stringers.

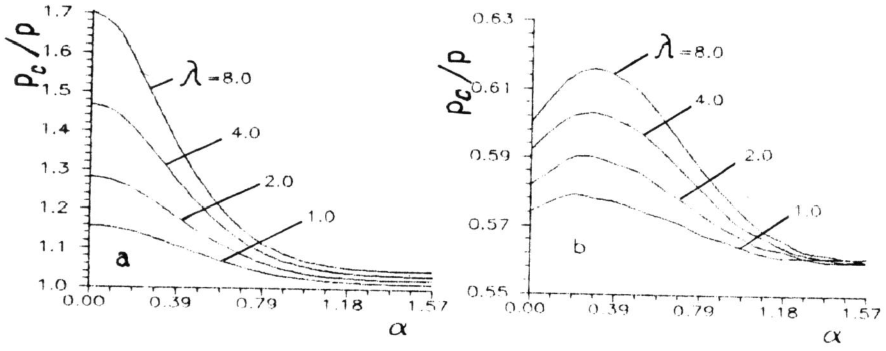

Fig.3. Dependence of the normalized critical load p_c/p on an orientation (an angle α) of a straightline crack for a plate subjected to biaxial (q=p) stretching at infinity ($\gamma = \pi/2$) (a) and when normal and tangential load is applied to crack faces (b) at $l = 2.4b$, $r = 0.05b$, $N_3=10$, $N_2=2$, $a_1=-3b$, $a_2=3b$.

If, in addition to normal stresses, there are also tangential ones at the faces of a crack (Fig.3b), the critical load value will be maximum at $\alpha \approx 0.3$. Also, these tangential stresses cause a large decrease in the critical load value.

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