PLASTIC STRIP MODEL SOLUTIONS FOR PLANE PROBLEMS OF FRACTURE MECHANICS

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ABSTRACT

In a loaded real plane with a crack, plastic deformations appear at the crack tips. If the plastic zone is comparable with the crack length, then for the studing of deformations of the plane the non-linear fracture mechanics should be used. The paper suggests general method of solving plane elasto-plastic crack problems by using plastic strip model. Plastic zone is modelled by inclined slip strips at the tip of a crack. It is assumed that the plane material is perfectly elasto-plastic and on the slip lines the yield criterion is satisfied. Thus, the problem of the development of plastic strips in a cracked plane is reduced to the boundary value problems of the elasticity theory for a region with kinked or branched cuts. The length and inclination angle of every lateral branch simulating of the plastic strip are determined in the process of solving the problem. The singular integral equation method is used to solve these problems. Numerical results are presented in the cases of an uniform tension at infinity or an edge crack in semi-infinite plane. Parameters of non-linear fracture mechanics, such as crack opening displacement (COD), length and inclination angle of plastic strip, are evaluated.

KEYWORDS

Non-linear fracture mechanics, plane problems, cracks, slip strips, infinite plate, semi-infinite plate, crack opening displacement, singular integral equation method.

The generalized plane stress state is realized when a thin plate is deformed by forces acting in its plane. Experimental investigations of plastic deformations of thin plates with opening mode cracks indicate that at first, when loading level is low, narrow plastic zones appear on the crack prolongation. When the loading is increased at the crack tip the secondary system of the very thin symmetric slip strips suddenly appears in the planes inclined at an angle of about 50^0 to the crack line. The primary and secondary plastic strips have different physical interpretation. The primary plastic strips appear on the plane of the maximal tensile stresses. In this case the slip occurs in the planes of maximal shear stresses, the planes are inclined at an angle of 45^0 to the plate surface. The secondary slip strips appear on the planes perpendicular to the plate surface with maximal shear stresses acting on the planes. Plastic strips will be modelled by means of the lines of displacements discontinuity; on the lines corresponding to the secondary slip strips the jumps of the shear displacements only will differ from zero. Then the problem on the plastic strips development in a thin cracked plate is redused to solving the elasticity theory problem for a branched cut (Panasyuk and Savruk, 1992; Panasyuk et. al., 1991).

For the first time the plastic strips model was used in papers for solving the problem of the elastic- plastic equilibrium of an infinite perfectly elastic-plastic plate with a straight crack under tensile stresses p directed perpendicularly to the crack line acting at infinity (Leonov and Panasyuk, 1959; Dugdale, 1960; Leonov et. al., 1962). In this case narrow plastic strips on the crack prolongation were substituted for fictitios cuts on whose faces tensile stresses equal to the yield stress σ_Y. Then the plastic zone length is given by formula

$$l_0/l=\cos(\pi p/(2\sigma_Y)),$$

where $2l_0$ is the crack length, $l-l_0$ is the plastic strip length.

It can be shown that this model satisfies the Treska yield criterion only when external forces are small enough. Let the crack faces be stress-free, and at infinity the plate be stretched by the forces p and q acting in the directions perpendicular and parallel to the crack line respectively. The maximal shear stress in the plane is determined by formula

$$\tau_{max}=|\sigma_Y\sqrt{l^2-l_0^2}\;z(z-\bar{z})/(\pi\sqrt{z^2-l^2}(z^2-l_0^2))+1/2(p-q)|.$$

The curve on Fig.1. shows quantities of the forces p and q when τ_{max} reaches the yield stress in shear $\tau_Y=\sigma_Y/2$ and plastic deformation should appear. It appears at a short distance from the crack tip. The increase of the external forces leads to the increase of the plastic zone. When this zone reaches the crack tip than the new slip strips can appear.

Let us assume that three plastic strips emanate from the crack tip. When the plate material is perfectly elastic-plastic and on the slip lines (contours L_k, $k=\overline{1,6}$) (Fig.2.) the Treska yield criterion is satisfied the

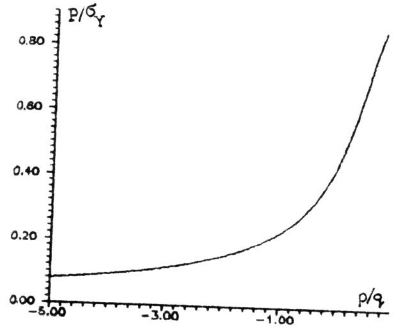

Fig.1.Dependence of the load p/σ_Y when τ_{max} reachs the yield stress in shear on p/q.

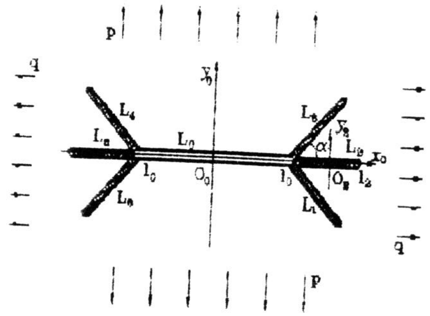

Fig.2.Slip strips at the tips of a crack in a plate, subjected to biaxial loading at infinity.

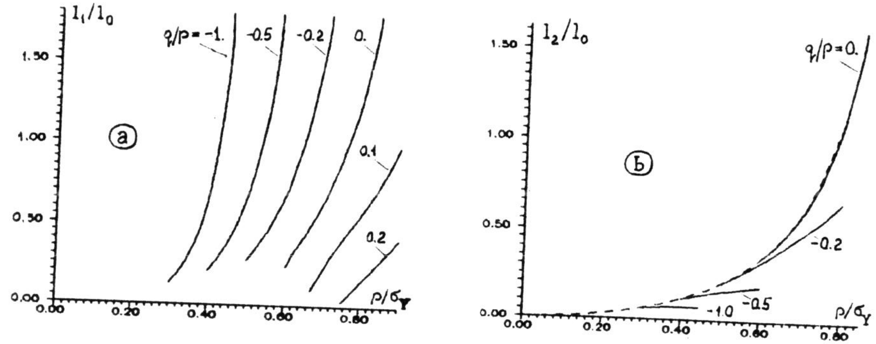

Fig.3.Variation of p/σ_Y with l_1/l_0 (a) and l_2/l_0 (b) (dotted line corresponds to the Dugdale solution).

following boundary conditions take place:

$$
\begin{aligned}
&N_0^{\pm}+iT_0^{\pm}=0, \quad t\in L_0;\\
&N_n^{\pm}+iT_n^{\pm}=\sigma_Y=2\tau_Y, \quad t\in L_n \quad (n=2,5);\\
&N_n^{+}=N_n^{-}, \quad v_n^{+}=v_n^{-}, \quad T_n^{\pm}=-\tau_Y, \quad t\in L_n \quad (n=1,4);\\
&N_n^{+}=N_n^{-}, \quad v_n^{+}=v_n^{-}, \quad T_n^{\pm}=\tau_Y, \quad t\in L_n \quad (n=3,6).
\end{aligned}
\tag{1}
$$

Here N_n and T_n are the normal and tangential components of stresses and v_n is the projection of the displacement vector onto the $0_n y_n$ axis, these values being specified on contours L_n $(n=\overline{0,6})$ related to the local coordinates system $x_n 0_n y_n$. The $0_n x_n$ axes $(n=\overline{1,6})$ pass along the contours L_n and form angles α_n with the $0_0 x_0$ axes, points 0_n $(n=\overline{1,6})$ are in the centres of contours L_n. The values with indices + or - refer to the left or right faces of the cut, $t=x_n+iy_n\in L_n$.

The boundary value problem (1) of the plane theory of elasticity for an infinite plate with a branched crack is reduced to solving the system of singular integral equations

$$
\begin{aligned}
\frac{1}{\pi}\begin{Bmatrix}\mathrm{Re}\\ \mathrm{Im}\end{Bmatrix}\Bigg\{ &\int_{-l_0}^{l_0}\Big[K_{n0}(t_0,x_n)+L_{n0}(t_0,x_n)\Big]g_0'(t_0)\,dt_0+\\
&+\int_{-l_1}^{l_1} i\Big[K_{n1}(t_1,x_n)-L_{n1}(t_1,x_n)-K_{n3}(t_3,x_n)+L_{n3}(t_3,x_n)+\\
&+K_{n4}(t_4,x_n)-L_{n4}(t_4,x_n)-K_{n6}(t_6,x_n)+L_{n6}(t_6,x_n)\Big]g_1'(t_1)\,dt_1+\\
&+\int_{-l_2}^{l_2}\Big[K_{n2}(t_2,x_n)+L_{n2}(t_2,x_n)+\\
&K_{n5}(t_5,x_n)+L_{n5}(t_5,x_n)\Big]g_2'(t_2)\,dt_2\Bigg\} =\\
&=\begin{bmatrix}-p, \ n=0; \ 2\tau_Y-p, \ n=2\\ -\tau_Y+(p-q)\sin\alpha\cos\alpha, \ n=1\end{bmatrix}, \quad x_n\in L_n
\end{aligned}
\tag{2}
$$

to determine the unknown functions $g_k'(t_k)$ $(k=\overline{0,2})$. Here

$$
K_{nk}(t_k,x_n) = \frac{e^{i\alpha_k}}{2}\left[\frac{1}{T_k-X_n}+\frac{e^{-2i\alpha_n}}{\overline{T}_k-\overline{X}_n}\right],
$$

$$
L_{nk}(t_k,x_n) = \frac{e^{-i\alpha_k}}{2}\left(\frac{1}{\overline{T}_k-\overline{X}_n}-\frac{T_k-X_n}{(\overline{T}_k-\overline{X}_n)^2}e^{-2i\alpha_n}\right),
$$

$$
T_k=t_k e^{i\alpha_k}+z_k^0, \quad X_n=x_n e^{i\alpha_n}+z_n^0,
$$

$$
z_k^0=l_0+l_k\exp(i\alpha_k) \ (k=\overline{1,3}), \quad z_k^0=-l_0+l_k\exp(i\alpha_k) \ (k=\overline{4,6}),
$$

$$
\alpha_1=\alpha, \ \alpha_2=0, \ \alpha_3=-\alpha, \ \alpha_4=\alpha-\pi, \ \alpha_5=-\pi, \ \alpha_6=\pi-\alpha.
$$

The unknown functions $g_k(t_k)$ are expressed in terms of the jumps of the displasement vectors on the contours L_k

$$
g_k(t_k)=(E/4)(u_k^{+}-u_k^{-}), \ k=0,2, \quad g_1(t_1)=(E/4)(v_1^{+}-v_1^{-}),
$$

where E is the Young's modulus, u_n is the projection of the displacement vector onto the $0_n x_n$ axis.

For the determination of the plastic strips lengthes $2l_1$, $2l_2$ we have two conditions

$$
g_1'(l_1)=0, \quad g_2'(l_2)=0, \tag{3}
$$

orientation angle α will be obtained considering that slip strips L_1, L_3, L_4, L_6 are oriented so that their length l_1 takes the maximal value: $l_1(\alpha)\to\max$.

The numerical solution of the system of integral equations (2) with the specified parameters l_1, l_2 and α was found by the mechanical quadratures method (Savruk, 1981). We couldn't obtain good results at the moment of the new slip strips appearance, that is why we suggest this results only for the case $l_1/l_0>0.1$.

The numerical results show, that when new slip strips are small they are inclined at an angle of about 60^0 to the crack line. The increase of the tensile stresses leads to the decrease of this angle, and its magnitude is about 45^0 when $(p-q)/\sigma_Y\to 1$. The influence of the lateral slip strips L_1, L_3, L_4, L_6 development and the force acting in parallel to the crack direction on the plastic strips lengthes $2l_1$, $2l_2$ and crack opening displacement $\Delta=\pi E(v_0^{+}-v_0^{-})/(8l_0\sigma_T)$ is shown on Fig.3 and Fig.4 (solid lines). Dotted lines in this figures correspond to the Dugdale solution.

Now let the stresses p be directed at angle γ to the axis x (Fig.5), $q=0$. The solution of such problem is obtained for the case when only one plastic strip emanates from crack tip (Panasyuk et. al., 1991). The numerical results for this

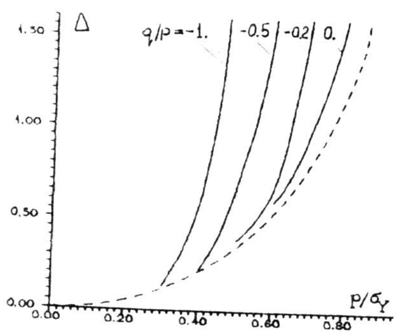

Fig.4. Variation of p/σ_Y with crack opening displacement $\Delta = \pi E(v_0^+ - v_0^-)/(8 l_0 \sigma_Y)$ (dotted line corresponds to the Dugdale solution).

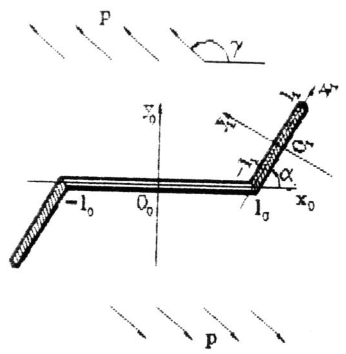

Fig.5. Slip strips at the tips of a crack in a plate, subjected to uniaxial loading at infinity.

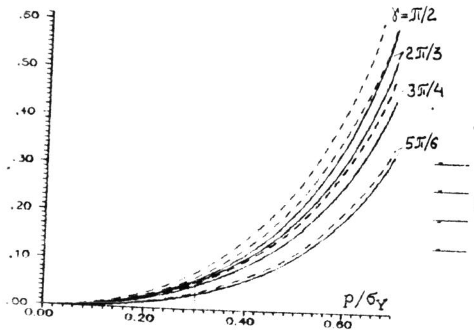

Fig.6. Variation of p/σ_Y with l_1/l_0 for the cases of an infinite plane (solid lines) and a semi-infinite plane (dotted lines).

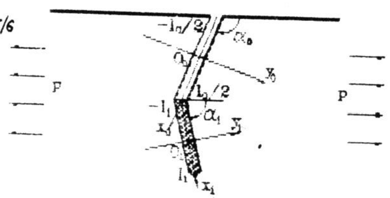

Fig.7. Edge crack with plastic zone.

strip length $2l_1$ (Fig.6) show that it is maximal in the case of simmetric loading ($\gamma=\pi/2$) at any level of loading ($p/\sigma_Y<1$). When the angle between the tension direction and the crack line is decreased the length $2l_1$ is also decreased. The orientation angle α_1 is chosen in the way providing limitation of the shear stresses at the tip of plastic strip.

The similar numerical results (Fig.6, dotted lines) are obtained for the slip strip at the tip of an arbitrary oriented edge crack in a semi-infinite plane (Fig.7).

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