NUMERICAL METHODS OF FRACTURE MECHANICS PARAMETERS CALCULATION*

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ABSTRACT

This paper deals with the determination such the linear and nonlinear fracture mechanics parameters as the stress intensity factor K, the energy J-integral and others by numerical methods. The various fracture criteria are discussed.

KEYWORDS

Crack, fracture, finite element, fracture criteria, elasticity, plasticity, numerical methods.

It is well-known that the cracks bodies calculations presuppose the knowledge of the stress-strain state near the crack tip. The fracture criteria deal not only with the components of the stress-strain state but the fracture mechanics parameters as well, which evaluate this state in a certain way. In the first place these parameters include the stress intensity factor (SIF) K, the energy integral J which seems to be most perspective in the nonlinear fracture mechanics, then the crack opening displacement and others (Parton and Morozov, 1985). All of them are widely used.

It is quite enough to know SIF to formulate the fracture criterion in the field of the linear fracture mechanics. For it determination the analitical and numerical methods of the theory of elasticity are used. As a matter of fact the analitical methods (for isotropic and homogeneous media) have already been exhausted; the possible variety of rather simple bodies forms and load schemes has been investigated and now voluminous reference literature about SIF exists which has been created on the base of these solutions. Still there is not enough reference data to meet the need in factors K. In this connection the effective

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numerical methods have been worked out not limited by the body's geometry and the type of the load.

Among the most popular numerical methods the finite element method (FEM) (Morozov and Nikishkov, 1980) and the boundary element method rank no 1 and no 2; in certain cases their combination is also effective.

The detailed technical description of the algoritmic modes and programming of the fracture mechanics parameters calculation methods which are mentioned below is not considered our priority here. Our purpose is to point out the possible continuity and interdependence of the calculated criterial characteristics what in its turn may enable the perfection and universalization of the numerical procedures themselves. Besides one shouldn't lose the sight of the final purpose of the calculation which is making up the criterial equation:

$$K(P,\ell) \le K_{\ell im} = \{K_{Ic}, K_c, \sqrt{EJ_c}, I_c\}, \qquad (1)$$

its right, experimental part is defined with relatively large and as a rule an indetermined error. As a result the labour-consuming pursuit for numerical calculations precision of the left parts of the criterial correlations is becoming groundless and, precisely speaking, the calculations error of fracture mechanics parameters in the limit of 10 percents may be considered as quite satisfactory.

The finite element method for the stress intensity factors determination is realized by various calculated ways - there are direct K determination methods on stresses and displacements asymptotic near the crack tip with extrapolation on the crack tip; more accurate energy methods, such as the method of the virtual crack extension, the method of compliance, the method of the immediate integration on the small contour near the crack tip, the method of the equivalent domain integration by Nikishkov and Atluri (1987) and others. Weight functions method by Bueckner and Rice has been widely used of late. This method is good enough for technical applications so far as it allows to define factor K for the given body's geometry on condition that only the stresses distribution in the crackless body and the weight functions are known (Nikishkov and Chernysh, 1989).

It seems appropriate to note here, that the weight functions method is derived from the variation principle of the cracks theory offered as egrly as middle 60, by one of authors, which may be presented in our purposes for the plane problem in the form of:

$$\frac{\partial}{\partial \ell}\int_0^{\ell}(2\gamma - p_i u_i)\,dx = 0, \qquad (2)$$

where l - the crack length (or semilength in simmetrical task); 2γ - the specific fracture work; $p_i = -\sigma_{ij} n_j$;

σ_{ij} - the stress on the crack line with the normal n_j in the entire body due to the definite load; u_i - displacement of the crack sides due to load p_i.

It follows from condition (2) that

$$2\gamma - \int_0^{\ell} p_i \frac{\partial u_i}{\partial \ell}\,dx = 0 \quad or \quad 2\gamma - G = 0.$$

In this equation G is the energy flow in the crack tip; by Irvin $G = K^2/E$. Therefore,

$$\frac{K^2}{E} = \int_0^{\ell} p_i \frac{\partial u_i}{\partial \ell}\,dx. \qquad (3)$$

If we now introduce two systems of the loads one of them is given (and for which it is needed to define K), and the second is a standard one("unit", $K^{(1)}$ is known for that), so using superposition principle and Betty theorem we may obtain from equation (3)

$$\frac{2}{E}\left(K_I K_I^{(1)} + K_{II} K_{II}^{(1)}\right) = \int_0^{\ell^+} p_i \frac{\partial u_i^{(1)}}{\partial \ell}\,dx + \int_0^{\ell^-} p_i \frac{\partial u_i^{(1)}}{\partial \ell}\,dx \qquad (4)$$

Here the lower index Roman numerals stand for factors K for I and II crack deformation type; the upper index (1) notes values of the standard load; l^+ and l^- are the upper and the lower sides of the crack.

The obtained expression (4) is in accordance with the weight functions method (which is the complex of the considered known functions $E/K^{(1)} \cdot \partial u_i^{(1)}/\partial \ell$) and allows to determine K from the arbitrary load p by simple integration of the known functions on the crack surface.

The influence functions method supposes the similar operation of integration on the crack surface for factor K determination in the arbitrary point of the volume crack front. The essence of the method includes the preliminary determination of the stress intensity factor K from the unit loads distributed on all the element nodes adjoining the crack surface.

The weight functions method and the influence functions method require the spade work therefore, their application is justified in only case of relatively typical constructional elements with various calculating regimes (for example, pipings, construction buildings, pressure vessels).

Jaking into account the above-mentioned unexactness of the calculated equation to the input value pricision the simple line spring model is worth considering (Akimkin and Nikishkov, 1991). The method is based on the substitution of a three-dimensional problem with an unthrough crack for the problem of the shell theory - a plate or a shell with a through crack and the additional pivotal elements which compensate the compliance change caused by the crack surface increase.

the edge length near the crack front. While carrying out the equivalent domain integration method the linear S-function on the front for the corner nodes and the quadratic one for the middle nodes is applied. The calculated weight functions for the point of the crack front $\theta = \pi/2$ (the deepest point) are given on fig.1 in the normalized way

$$\bar{h}_2 = h_2 \Phi c \sqrt{\pi a}$$

where Φ is a full elliptical integral of the second type. These weight functions are singular on the crack front along the length of one element, that is in accordance with the middlement part of K calculating by the equivalent domain integration method. The comparison with the Benchmark Editorial Committee results (1980) of the normalized stress intensity factors for tension and bending cases calculated by the weight functions integration on the surface crack showed that the difference is not over 3 %.

The problem on the three-dimensional elastic-plastic energy integral definition for the semielliptical crack in a plate under tension has been solved. The calculations have been made for the semielliptical crack with the semiaxes relation a/c = 2/3, 1/2 and 1/3. The discrete model comprises 128 finite elements and 743 nodes. The nonlinear material behaviour was modelled on the base of the plastic flow theory with isotropic hardening. The iterating scheme of the initial stresses has been used for solving of the nonlinear equations. The energy integral values along the crack front for various load levels have been defined by the equivalent domain integration method. The calculations have been done for the ideally elastic-plastic material with E = 0 and for the line-hardening material with E = 0.1*E. The comparison of the J-integral value change along the crack front a/c = 2/3 for elastic and elastic-plastic behavior of material under the load $\sigma = \sigma_T$ is shown on fig.2.

The stress intensity factor in the joint zone of the branch pipe with the energy reactor body under the break-down situation threat has been calculated. The general problem scheme is shown on fig.3. The reactor body and the considered branch pipe have been heated to temperature of 300 C in the stationar regime. There assumed a rupture of the first contour pipe, as a result the system of the zone break-down cooling begins functioning and the cold water starts flowing through the branch pipe. The water fills the part of the branch pipe and gets in the reactor body, where the barrier blocks its way, and at a consequence the layer of the cold water is generated below the joint of the branch pipe with the reactor body. It is believed that the surface semielliptical crack with a/c = 2/3 and the depth of 0.25th body reactor's wall thickness is located in the joint zone.

The nonisothermal elastic environment model with the dependence of the phisical-mechanical properties on temperature has been taken for calculation. The load scheme represents the nodal forces from the internal pressure, the

increase of the plastic zone size in front of the crack tip leads to need of calculation of such fracture mechanics parameters as the strain intensity factor M_ε in elastic-plastic field, the crack opening displacement δ, the energy integral J, the crack resistance limit I_c.

All those parameters proceed from the presence of the plastic zones near the crack tip and in any case reflect the material ability to irreversible energy spending on the plastic strain work, or even the plastic strain phenomenon itself. Thus the interdependence between the nonlinear fracture mechanics parameters looks as followes:

$$M_\varepsilon = \left(\frac{2\pi J}{\sigma_* F_m}\right)^{1/(1+m)} \left(\frac{\sigma_*}{\sigma_T}\right)^{1/m} \tilde{\varepsilon}, \qquad \sigma_i = \sigma_T$$

$$I_c = \begin{cases} \sigma_T M_{\varepsilon c}^{1/p}, & \sigma_i < \sigma_T \\ \sigma_T M_{\varepsilon c}^{1/p} (\sigma_i/\sigma_T)^{\alpha}, & \sigma_i \geq \sigma_T \end{cases}$$

$$E J_c = \Phi I_c^{2},$$

$$J = m_1 \sigma_T \delta,$$

Here $\tilde{\varepsilon} \sim 1$, $\alpha = (m-1)/[m p(1+m)]$, σ_i is the nominal stress intensity, - the yield limit, E - elastic modulus, $\varepsilon_T = \sigma_T/E$, $p = [2 - 0.5(1-m)(1-\sigma_i/\sigma_T)](1+m)^{-1}$, $m_1 = (2^{m/2} \alpha F_m)/[(\sigma_T/E)^m (2\tilde{u})^{1+m}]$.

Usually for the numerical calculations the power dependence for schematization of the material deformation diagram is taken. It is accepted here

$$\sigma = \sigma_T [\varepsilon/(\alpha \varepsilon_T)]^m = \sigma_* \varepsilon^m, \quad 0 \leq m \leq 1.$$

Thus, the calculated criterial characteristics turn out to be interdependent and therefore there is no principal difference what criterium may be used for the description of the fracture beginning and for obtaining the unknown critical parameters of the task from the fracture criteria. The choice of the criterious is dictated by the defining value calculation comfort and to a certain extent by the subjective views of a calculator.

There obtained the solutions of some problems with the help of finite element method and various methods for fracture parameters determination. In particular, there has been solved the problem of the weight functions definition for a plate with a semielliptical surface crack based on the equivalent domain integration method. This approach differs from the virtual crack extension method so as it does not require the finite-differencial approximations application for the determination of the displacement derivations on the crack length.

The crack sizes are a/c = 0.5, a/t = 0.25. The discrete model of the task consisted of 106 quadratic isoparametrical elements with 626 nodes. There disposed the singular degenerative elements with the middle nodes on the quarter of tension from the pressure on the cover of the reactor body and the ficticious nodal forces caused by temperature. The discrete model covers 443 quadratic elements and 2233 nodes (fig. 3). The stress intensity factor calculation has been done by the direct asimptotic method on displacements, by the virtual crack extension method and also by using the influence functions for a semielliptical crack in a plate. The calculation results are shown on fig. 4.

There has been done the three-dimensional elastic-plastic calculation of the stress-strain state and the fracture mechanics parameters for the steam-generator collector perforate zone under the break-down conditions. In the stationar regime the collector is under internal and external pressure, the collector's walls are heated with a minor temperature gradient through their thickness. Under the threat of the germetization violation of the second contour the external pressure falls to zero and the outer surface itself is essentially cooled.

The fracture mechanics parameters have been calculated for the following axial cracks: through-wall, semielliptical through-wall and semielliptical with a quarter wall thickness depth cracks situated in the middle of the perforate zone. The elastic-plastic material model with the isotropic hardening is taken for calculation; the perforate field is considered as the homogeneous orthotropic medium with the prominent radial direction, therefore the problem has been solved in cylindrical coordinates. The effective elastic orthotropic parameters are found from the condition of equality of the average displacement values in entire and perforate bodies. The energy J-integral is determined by the virtual crack extension method which proves to be effective dealing with the elastic-plastic problem.

The discrete model of the body with a semielliptical through-wall crack consists of 322 three-dimensional isoparametric quadratic elements with 1737 nodes (fig.5). Fig. 6 shows the J-integral change on upper through-wall crack front under pressure and temperature for 4 steps. Comparing the obtained J-integral value with the corresponding material characteristic J_c, we may draw the conclusion about the crack starting on the upper front outer surface.

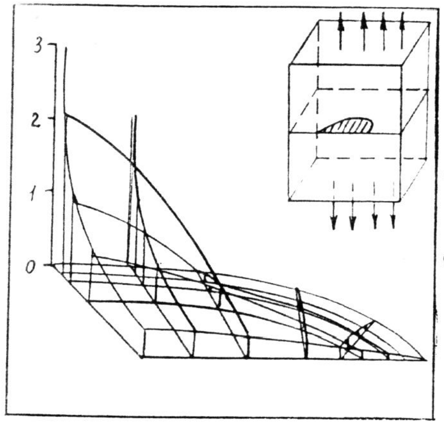


Fig.1. Weight fuctions for the deepest point of semi-elliptical crack

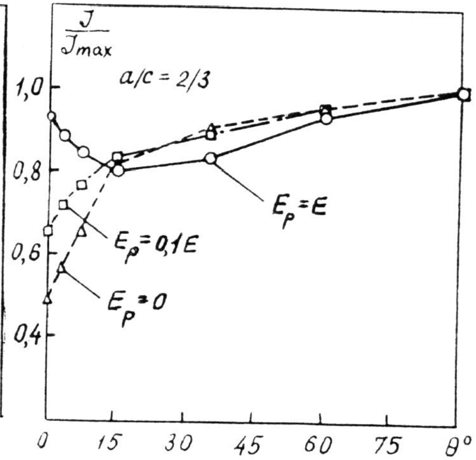


Fig.2. J-integral for semi-elliptical crack in elastic and elastic-plastic bodies

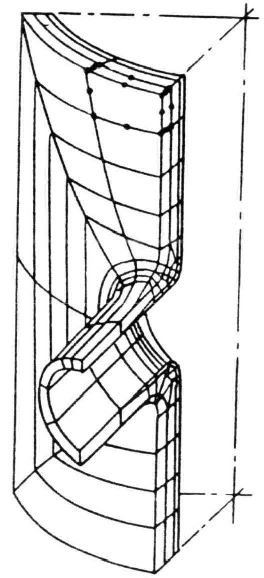


Fig.3. Discrete model for joint zone of branch pipe with energy reactor body

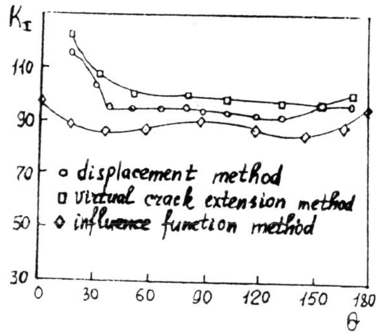
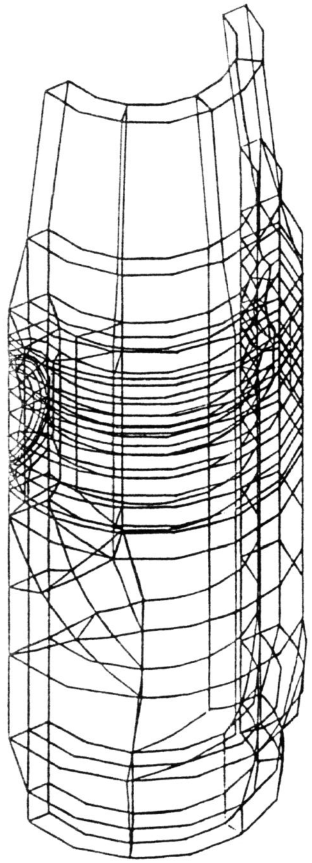


Fig.4. The crack intensity factor distribution on crack front



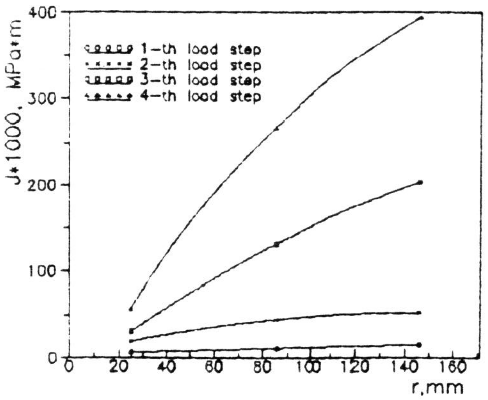


Fig.6. The J-integral exchange on upper through-wall crack for 4 load steps

Fig.5. Discrete model for steam-generator collector perforate zone

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