NON-LINEAR LINE SPRING MODEL AND COMPLEMENTARY ENERGY METHOD OF SURFACE CRACK

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ABSTRACT

A non–linear line spring model for surface flaws in plates or shells is discussed. Based on a complementary energy method, the non–linear constitutive relations of line springs, J–integral and crack opening displacements (CMOD, COD and CTOD) of surface flaws are obtained. A simple method for determining the factors in complementary energy expressions is proposed. The presented non–linear line spring model combined with finite element method is successfully applied to analyze the surface cracked plates under tension. The numerical results show that the calculated values of elastic–plastic CTOD coincide well with those by experiments when $a/t > 0.25$.

KEYWORDS

Non–linear line spring model, complementary energy method, crack opening displacement, surface cracked plate in tension.

INTRODUCTION

The line spring model (LSM) originally proposed by Rice and Levy (1972) has been verified to be a reasonably accurate and cost–effective tool for analyzing the part–through surface cracks in plates or shells (Delale and Erdogan, 1981; German et al.,1983). The LSM has been further developed to estimate the J–integral and crack propagation in the elastic–plastic range (Parks, 1981; Kumar et al.,1983; Miyoshi et al., 1986). Obviously, the LSM combined with finite element method can be successfully used to describe the fracture behavior of complex surface cracked problems. The engineering approach of elastic–plastic fracture mechanics (Kumar et al.,1981) has been applied to the LSM finite element method to simplify the process of analysis (Kumar et al.,1983). Therefore, the attention may be concentrated on the calculations of fully plastic fracture parameters of interest.

The core of LSM is the introductions of "line spring", which is equivalent to a plane strain single edge cracked plate (SECP) under tension and bending. To establish the fully plastic constitutive relations of line springs, the fully plastic solutions of SECP are needed, which are usually calculated by finite element analysis. Some typical fully plastic solutions of SECP have been carried out (Kumar et al.,1981,1983; Shih and needlman, 1984). Miyosh et al.(1986) proposed a complementary energy method to evaluate the stiffness and J-integral of SECP. In their studies the undetermined factors in complementary energy expression are determined by the finite element results of complementary energy of SECP. Only a few values of these factors are given in their paper.

In this paper, a simple method is proposed to define the undetermined factors of complementary energy of SECP, and the complementary energy method is further developed to calculate the crack opening displacement of SECP. The obtained non–linear LSM is incorporated into a plate / shell finite element code, and then, the surface cracked plate in tension are analyzed.

THE COMPLEMENTARY ENERGY METHOD FOR SECP

The material is assumed to follow the Ramberg–Osgood power hardening law. The fully plastic strain–stress relation in uniaxial tension is as follows

$$\varepsilon / \varepsilon_0 = \alpha(\sigma / \sigma_0)^n \tag{1}$$

where ε_0 and σ_0 are reference strain and stress, respectively. α is the material constant. n is the hardening exponent. Generalizing equation (1) to three–dimension by the deformation theory of plasticity leads to

$$\varepsilon_{ij} / \varepsilon_0 = 3/2\alpha(\sigma_e / \sigma_0)^{n-1} S_{ij} / \sigma_0 \tag{2}$$

where, S_{ij} and ε_{ij} are deviatoric stress and fully plastic strain deviator, σ_e is the effective stress, defined by

$$\sigma_e = 3/2 S_{ij} S_{ij} \tag{3}$$

Fig.1 shows a SECP under the state of plane strain subjected to axial force N and bending moment M. The fully plastic complementary energy of the SECP due to the existence of crack given by Miyoshi et al.(1986) is as follows.

$$\Omega_c = [\alpha\sigma_0\varepsilon_0 t^2 / (n+1)] f^{(n+1)/2} \tag{4}$$

$$f = (AN^2 + 2BNM / t + CM^2 / t^2) / [\sigma_0^2 (t-a)^2] \tag{5}$$

where A, B and C are undetermined factors. The fully plastic constitutive relations of SECP are then obtained as

$$\triangle_c = \partial\Omega_c / \partial N = \alpha\varepsilon_0 t^2 f^{(n-1)/2} (AN + BM / t) / [\sigma_0 (t-a)^2] \tag{6}$$

$$\theta_c = \partial\Omega_c / \partial M = \alpha\varepsilon_0 t f^{(n-1)/2} (BN + CM / t) / [\sigma_0 (t-a)^2] \tag{7}$$

where, $\triangle_c$ and θ_c are the fully plastic displacement and rotation of SECP, respectively.

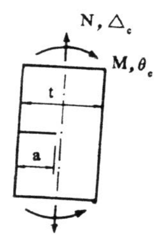

Fig.1 SECP under tension and bending

THE DETERMINATIONS OF FACTORS A, B AND C

The major problem to be solved in the complementary energy method is how to determine these factors in equation (5). A simple method suggested by the authors is given below.

By defining $\lambda = M / Nt$, and substituting it into equations (6) and (7) leads to

$$\triangle_c = \alpha\varepsilon_0 t^2 \Phi^{(n-1)/2} N^n (A + B\lambda) / [\sigma_0^n (t-a)^{n+1}] \tag{8}$$

$$\theta_c = \alpha\varepsilon_0 t \Phi^{(n-1)/2} N^n (B + C\lambda) / [\sigma_0^n (t-a)^{n+1}] \tag{9}$$

$$\Phi = A + 2B\lambda + C\lambda^2 \tag{10}$$

On the other hand, the expressions given by Kumar et al.(1986) are as follows

$$\triangle_c = \alpha\varepsilon_0 a h_3 (a / t, \lambda, n)(N / N_0)^n \tag{11}$$

$$\theta_c = \alpha\varepsilon_0 h_5 (a / t, \lambda, n)(N / N_0)^n \tag{12}$$

where h_3 and h_5 are dimensionless functions of a / t, λ and n, $N_0 = t\sigma_0 f_0$, f_0 is defined as

$$f_0 = \begin{cases} 1.455[-a / t + \sqrt{(1-a / t)^2 + (a / t)^2}] & \lambda = 0 \\ 1.155[-|2\lambda + a / t| + \sqrt{(1-a / t)^2 + (2\lambda + a / t)^2}] & \lambda \neq 0 \end{cases} \tag{13}$$

By comparing equations (8) and (9) with equations (11) and (12), respectively, we can recognize that equations (8) and (9) are explicit forms of equations (11) and (12) approximately. According to the comparisons above, the values of A, B and C can be determined by means of available finite element solutions of h_3 and h_5.

Considering SECP in the cases of pure tension (M = 0) and pure bending(N = 0), which are the two extreme states in combinations of tension and bending, the A, B and C can be obtained as

$$A = (1-a / t)^2 (h_3^t)^{2/(n+1)} \{1.455[-1 + \sqrt{(1-t / a)^2 + 1}]\}^{-2/(n+1)} \tag{14}$$

$$B = [(1-a / t)^2 / (a / t)](h_3^t)^{(1-n)/(1+n)} h_5^t \{1.455[-1 + \sqrt{(1-t / a)^2 + 1}]\}^{-2/(n+1)} \tag{15}$$

$$C = (1-a / t)^{-2(n-1)/(n+1)} \left(h_5^b / 0.364^n \right)^{2/(n+1)} \tag{16}$$

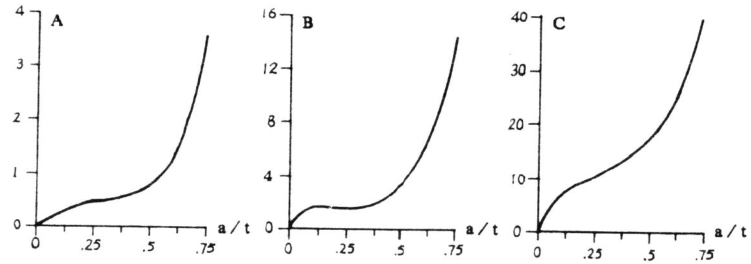

Fig.2 A, B and C verse a / t in the case of n = 5.

where, superscripts t and b on the h represent the pure tension and pure bending, respectively. The h_3 and h_5 in equations (14)–(16) are functions of a / t and n. Thus, the A, B and C given by equations (14)–(16) are virtually functions of a / t and n. The values of h are taken from Shih and Needlman(1984), which have been examined through consistecy checks. The A, B and C varies with a / t in the case of n = 5 are shown in Fig.2. Obviously, the A, B and C can be easily expressed as polynomial forms.

THE FULLY PLASTIC J–INTEGRAL AND CTOD OF SECP

The fully plastic J–integral of SECP can be calculated from the complementary energy as (Miyoshi et al.,1986)

$$J = \partial\Omega_c / \partial a \quad (17)$$

where, a is the crack length of SECP. The J–integral can also be evaluated directly through the finite element analysis by the original definition of path integral. The comparisons made by Miyoshi et al. show that the values of J–integral obtained by the two different methods are in good agreement.

In this section the complementary energy method is developed to estimate the fully plastic crack opening displacement of SECP. The CMOD is defined as the crack mouth opening displacement, COD is defined as the crack opening displacement at the original crack tip, and CTOD is the crack tip opening displacement defined by Shih (1981).

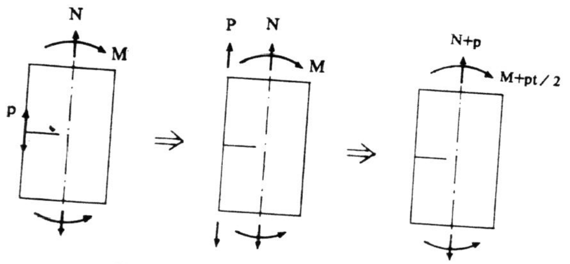

Fig.3 Equivalent process of loading

By acting a pair of forces, P, on the crack mouth, and considering the equivalent process shown in Fig.3, we can obtain the fully plastic complementary energy of SECP as

$$\Omega_c^* = \alpha\varepsilon_0\sigma_0 t^2 f^{*(n+1)/2} / (n+1) \quad (18)$$

$$f^* = [A(N+p)^2 + 2B(N+p)(M+pt/2)/t + C(M+pt/2)^2/t^2] / [\sigma_0(t-a)]^2 \quad (19)$$

Using the Crotti–Engesser theorem for the problem, we have

$$\delta_m = \partial\Omega_c^* / \partial p|_{p=0} = \alpha\varepsilon_0 t\Phi^{(n-1)/2}[A + B\lambda + (B + C\lambda)/2][N/(t\sigma_0)]^n / (1 - a/t)^{n+1} \quad (20)$$

where, δ_m is the value of fully plastic CMOD. The another expression of CMOD is as follows(Kumar et al.,1983)

$$\delta_m = \alpha\varepsilon_0 t h_2(a/t, \lambda, n)(N/N_0)^n \quad (21)$$

The h_2 function represented by factors A, B and C can be obtained by comparing equations (20) with (21) as

$$h_2 = \Phi^{(n-1)/2}[A + B\lambda + (B + C\lambda)/2] f_0^n / [a/t(1 - a/t)^{(n+1)}] \quad (22)$$

Equation (22) can be used to examine the accuracy of equation (20) by available finite element solutions. In Fig.4 some examples about the h_2 versus a / t when $n = 5$, $\lambda = 0, -1/16$ are illustrated. The solid lines are the values of h_2 computed by equation (22), and the solid triangles and circles are finite element solutions (Kumar et al.,1981,1983; Shih and Needlman, 1984). It can be seen that the h_2 given by equation (22) are in good agreement with the finite element solutions provided a / t is less than about 0.25.

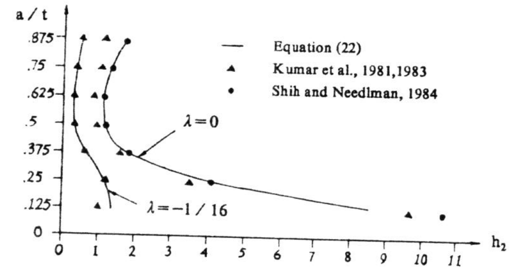

Fig.4 h_2 versus a / t in the case of $n = 5$

In the similar way, by acting a pair of forces, P, on the original crack tip leads to

$$\delta = \alpha\varepsilon_0 t\Phi^{(n-1)/2}[A + B\lambda + (1/2 - a/t)(B + C\lambda)][N/(t\sigma_0)]^n / (1 - a/t)^{n+1} \quad (23)$$

where, δ is the value of fully plastic COD.

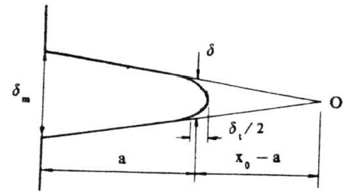

Fig.5 Profile of single edge crack

The profile of single edge crack(Fig.5) suggested by Kumar et al.(1983) is applied to calculate CTOD of the SECP. The relations of CTOD and COD is given as

$$\delta_t = \delta^{n/(n+1)}\{n/[2(x_0 - a)]\}^{1/n}[(n+1)/n]^{(n+1)/n} \quad (24)$$

According to the geometry in Fig.5, we have

$$\delta / \delta_m = (x_0 - a) / x_0 \quad (25)$$

Substituting equations (20) and (23) into (25), after collecting terms, we have

$$x_0 - a = t[(A + B\lambda)/(B + C\lambda) + (1/2 - a/t)] \quad (26)$$

The fully plastic CTOD can be finaly calculated by equations (23)–(26).

NUMERICAL RESULTS

The constitutive relations of the non–linear line spring element are given by equations (6) and (7). The presented line spring elements are incorporated into a plate / shell finite element code to analyze the surface cracked plates in tension. The effect of shear deformation has been considered in the formulation of the isoparametric plate / shell element. The line spring elements as additional elements are connected to nodes of plate / shell elements along the length of crack. The details about the implementation of the LSM in the plate / shell finite elements may refer to references (German et al., 1983; Huang and Gao, 1990). The four noded plate / shell elements are employed. The finite element mesh of the quarter plate contain 81 plate / shell elements, and 16 line–spring elements(Huang and Gao, 1990). The analysis of the fully plastic problem with the deformation theory of plasticity is equal to a non–linear elastic problem. The Modified Newton–Raphson iteration strategy is adopted. The initial elastic stiffness matrix can be extracted from equations (2) for plate and (6), (7) for line springs. The computations are executed on VAX–11 / 780 computer.

For comparison with available experimental results, three crack geometries are calculated:

$a/t=0.497,\quad a/c=0.343;$

$a/t=0.342,\quad a/c=0.323;$

$a/t=0.205,\quad a/c=0.297.$

The material constant $\alpha=1.83$, and hardening exponent $n=13$. The elastic–plastic CTOD are obtained by the engineering approach (Kumar et al. 1981), which are the summations of modified elastic CTOD and fully plastic CTOD. Fig.6 shows the elastic–plastic CTOD in the deepest points of surface cracks versus applying stress, in which the experimental results (Garwood, 1988) are also given for comparison. It can be seen that the calculated CTOD results coincide well with the experimental results for the first and second crack geometries. For the third one, the values of calculated CTOD are rather greater than the experimental results when the applying stress, σ^{∞}/σ_0, is greater than 1.0.

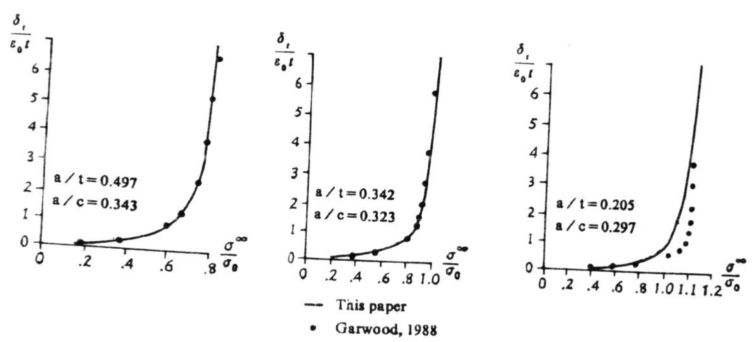

Fig.6 Elastic–plastic CTOD versus applying stress

CONCLUDING REMARKS

A non–linear line spring model for surface cracked components is discussed in the paper. The complementary energy method and engineering approach of elastic–plastic fracture mechanics are employed. A simple method is suggested

to determine the factors in the complementary energy expression of SECP by means of available finite element results. The surface cracked plates in tension are examined by the model with finite element method. The calculated results of elastic–plastic CTOD are in good agreement with those of experiments when a / t is not less than 0.25.

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