MODIFIED CRACK CLOSURE INTEGRAL (MCCI) USING QUARTER-POINT THREE DIMENSIONAL ELEMENTS

K. BADARI NARAYANA

Structures Group, ISRO Satellite Centre
Bangalore 560 017, India

ABSTRACT

In the present work, modified crack closure integral (MCCI) method is used to develop expressions for strain energy release rates (SERR) of 20-noded isoparametric singular elements in 3-D problems. Numerical results are presented for a penny shaped crack in cylindrical body, center crack tension (CCT) and edge crack shear (ECS) specimens. Estimates of SERR obtained for 20-noded singular elements are compared with the available results in literature.

KEY WORDS

Stress intensity factors (SIF), Strain energy release rates (SERR), Singular elements, Modified crack closure integral (MCCI), 20-noded isoparametric brick elements.

INTRODUCTION

The most practical crack configurations encountered in real life industrial problems are three dimensional in nature. These configurations are either through cracks, part-through cracks or embedded cracks. (Through cracks in solids and part-through or embedded cracks in any material requires 3-D analyses.) Finite element method (FEM) has become the most popular numerical tool for the estimation of stress intensity factors (SIF), strain energy release rate (SERR) G in fracture problems. Stress analysis by FE methods basically provides displacement and stress distributions in a structure. From these distributions the fracture parameters are post-processed. In literature, one finds that various methods have been used to estimate SIF and/or G in different modes and their distributions along crack fronts. The most commonly used techniques are (i) crack opening displacement method (Chan et al., 1970) (ii) virtual crack extension method (Parks 1974, Hellen 1975) (iii) nodal force method using singular crack tip elements (Raju and Newman, 1977) and (iv) direct estimation of SIF as separate DOF using singular hybrid elements (Atluri and Kathiresan, 1975). A recent development of VCE method is referred to as the equivalent domain integral (EDI) method for J-integral calculations in 3D problems (Nikishov and Atluri, 1987).

Irwin's crack closure integral (CCI) is one of the significant concepts for estimation of strain energy release rate (SERR) G in individual as well as mixed-mode situations. SERR can be estimated from the CCI concept by considering an incremental crack extension and evaluating the work done to close the crack to the original configuration.

Using the CCI concept, Rybicki and Kanninen (1977) have proposed FE calculation of SERR from modified crack closure integral (MCCI) through a single analysis using nodal forces and displacements in the elements forming the crack tip in 2-D problems. Later, Buchhalz (1984) developed MCCI expressions valid for LST and 8-noded quadrilateral elements. MCCI expressions are basically element dependant and their derivation needs a systematic approach, particularly, in the case of crack tip/front singular elements. For this purpose, a general procedure was proposed earlier for 2-D problems (Ramamurthy et al., 1986; Badari Narayana, 1991 and Raju, 1986). Using this procedure, the element dependant MCCI expressions are derived (using shape functions, displacement and stress distributions in the elements forming the crack tip). MCCI expressions valid for 8-noded and 20-noded regular brick elements in 3-D problems were recently proposed (Badari Narayana et al., 1991, 1992).

In the present work, the above mentioned general procedure is used to derive the MCCI expressions in 3-D problems (with cracks) modelled using 20-noded isoparametric singular elements. Application of the procedure is illustrated through several examples such as a penny shaped crack in cylindrical bar, CCT and ECS specimens.

MODIFIED CRACK CLOSURE INTEGRAL (MCCI)

Figure 1 shows a quarter elliptical corner crack in a 3-D solid with the plane of the crack front parallel to the xz-plane. Local coordinate system at any point along the crack front is denoted by PYQ-system with P-axis normal to the crack front. As the crack front undergoes an extension by an infinitesimally small area ΔA_q, using Irwin's CCI concept, the crack opening mode SERR, G_I, is obtained at any point q along the crack front as

$$G_I(q) = \underset{\Delta A_q \to 0}{Lt} \frac{1}{\Delta A_q} \int_{\Delta A_q} \sigma y\,(P,O,Q)\; U_y\,(P-\Delta P,\pi,Q)\; dpdq \quad (1)$$

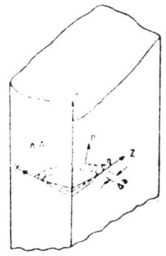

FIG. 1. QUARTER ELLIPTICAL CORNER CRACK IN A 3D SOLID : NOTATION AND CO-ORDINATE SYSTEM.

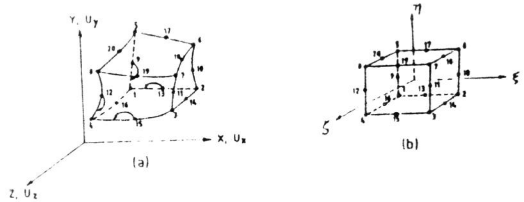

FIG. 2. 20 NODED ISOPARAMETRIC SINGULAR ELEMENT
a). SINGULAR ELEMENT IN CARTERSIAN CO-ORDINATE SYSTEM
b). PARENT ELEMNET IN LOCAL CO-OREINATE SYSTEM

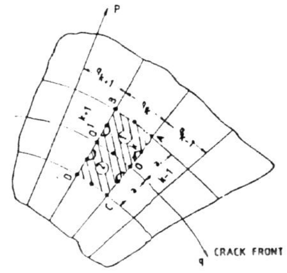

FIG. 3. TYPICAL FE MESH IN THE CRACK PLANE AT THE CRACK FRONT

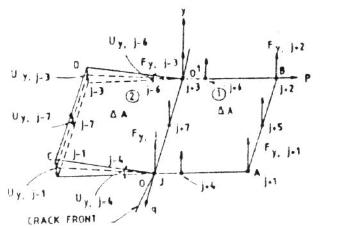

FIG. 4. NODAL FORCES AND DISPLACEMENTS : 20 NODED SINGULAR ELEMENT

in which ΔA_q is sub-area of ΔA around the point q, σ_y is normal stress with reference to the original crack configuration and U_y is the crack opening displacement with reference to the extended crack configuration. (Note that r = p in θ = 0 plane and r = - p in θ = π plane and ΔA_q = $\partial p.\partial q$). In numerical calculations, the limiting process is replaced by evaluating average G with reference to the virtual crack incremental area ΔA_q. Average G is given by the integrand. Similar expressions can be written for mode-II and mode-III crack deformations.

FE ESTIMATION OF MCCI

In a finite element analysis, the crack front is divided into a number of segments by different elements (Fig.3). The details of the elements ahead and behind the crack front are indicated for a general kth segment. For sufficiently small virtual crack extension ΔP, the incremental area ΔA_q in Eqn.(1) may be identified at this stage with the full area of the kth element. Average G at the mid-point of the segment OO' is evaluated by considering the stress and displacement components in the integrand of CCI in Eqn. (1) with those in the elements OABO' and COO'D respectively.

MCCI Expressions for 20-Noded Singular Elements : Let OABO' form one of the faces of an 20-noded isoparametric singular element with the corresponding element with face COO'D behind the crack front. Let the face OABO' be represented in the natural $\xi\zeta$-coordinate system ($-1 \le \xi, \zeta \le 1$). Referring to Fig. 2, the shape functions for the element in this plane are given (Zienckiwicz, 1971) as

$$N_i = 1/4\ (1+\xi\xi_i)\ (1+\zeta\zeta_i)\ (\xi\xi_i+\zeta\zeta_i-1)\xi_i^2\zeta_i^2$$
$$+\ 1/2\ (1-\xi^2)\ (1+\zeta\zeta_i)\ (1-\xi_i^2)$$
$$+\ 1/2\ (1-\zeta^2)\ (1+\xi\xi_i)\ (1-\zeta_i^2) \qquad (2)$$

Figure 3 shows a typical FE mesh in the crack plane at the crack front. The nodal forces and displacements with reference to Mode-I are shown in Fig. 4. Referring to Fig. 2., by shifting the mid side nodes, 13, 15 (on the sides AO, BO', CO and DO' (Fig.3)) to quarter point position, a strain (or stress) singularity in the elements located on either side of the crack front is obtained. Consistent with the above element shape function, in the isoparametric formulation the stress distribution σ_y in the singular element in terms of ξ and ζ is expressed as

$$\sigma_y(\xi,\zeta) = \frac{b_0}{(1+\xi)} + b_1 + \frac{b_2\zeta}{(1+\xi)} + b_3\zeta + b_4(1+\xi) + \frac{b_5\zeta^2}{(1+\xi)}$$
$$+\ b_6\ (1+\xi)\zeta + b_7\zeta^2 \qquad (3)$$

From the equivalent nodal forces for this stress distribution, the coefficients b_k (K = 0,1,..7) are related to the nodal forces $F_{y,j+i-1}$ by the area integral as

$$F_{y,j+i-1} = \int_{\Delta A_q} [N_i(\xi,\zeta)]^T\ \sigma_y(\xi,\zeta)\ dpdq \qquad (4)$$

where dpdq is an infinitesimal area of the element in the real plane. In the natural domain, the above integral (Eqn.4) takes the form

$$F_{y,j+i-1} = \int_{-1}^{+1}\int_{-1}^{+1} [N_i(\xi,\zeta)]^T\ \sigma_y(\xi,\zeta)\ |J|\ d\xi\, d\zeta \qquad (5)$$

where $|J|$ is Jacobian transformation matrix. For 20-noded singular element, the det$|J|$ is conveniently expressed as

$$|J| = J_0 + J_1\xi + J_2\zeta \qquad (6)$$

where J_0, J_1 and J_2 are constants depending on real coordinates of the nodes. The coefficients J_1 and J_2 account for the curvature effects of the crack front OO'. For a straight crack front OO' the coefficient J_2 is zero. Now, using Eqn. (6) in Eqn. (5), carrying out necessary integration and simplfiying, the coefficients b_k (k=0,1,..,7) are obtained. Nodal forces in Eqn.(5) are extracted through the standard procedure of multiplying the stiffness matrix of the element with the corresponding nodal displacements.

With reference to the shape functions given in Eqn.(1), the displacement distribution U_y in the element COO'D is taken as

$$U_y = a_0 + a_1(1+\xi') + a_2\zeta' + a_3(1+\xi')(\zeta') + a_4(1+\xi')^2 + a_5(\zeta')^2$$
$$+\ a_6\ (1+\xi')^2\ \zeta' + a_7\ (1+\xi')\ (\zeta')^2 \qquad (7)$$

The constants a_i (i = 0,1,...,7) are evaluated in terms of crack opening displacements on the surface COO'D (nodes 1 to 4 and 13 to 16). Now, using the U_y distribution given by Eqn.(7) in the element behind the crack front and the σ_y distribution given by Eqn:(3) and the associated geometric transformation of the element ahead of the crack front, MCCI expression for mode-I SERR, G_I in Eqn. (1) for the kth element will take the form

$$G_I(q_k) = \frac{1}{2\Delta A_K}\int_{-1}^{1}\int_{-1}^{1}\sigma_y(\xi,\zeta)\ U_y(\xi',\zeta')\ |J|\ d\xi\, d\zeta \qquad (8)$$

where A_K is the area of the kth element ahead of crack fron in the real domain. The transformation for the 20-noded singular element between (ξ,ζ) and (ξ',ζ') systems can be obtained as

$$(1+\xi')^2 + (1+\xi)^2 = 4 \quad \text{and} \quad \zeta' = -\zeta \qquad (9)$$

Carrying out the necessary integration, the expression for G_I given by Eqn.(8) can be further simplified in terms of a's and b's (constants of displacement and stress functions) as

$$\begin{aligned} G_I(q_k) = 1/(2\,\Delta A_k)\ [\,&(b_0 + b_1 + 4/3\ b_4 + 1/3\ b_5 + 1/3\ b_7)\ a_0 \\ &+ (\ \pi/2\ b_0 + 4/3\ b_1 + \ \pi/2\ b_4 + \ \pi/6\ b_5 + 4/9\ b_7\ )\ a_1 \\ &+ (\ 1/3\ b_2 + 1/3\ b_3 + 4/9\ b_6\ )\ a_2 \\ &+ (\ \pi/6\ b_2 + 4/9\ b_3 + \ \pi/6\ b_6\ )\ a_3 \\ &+ (\ 8/3\ b_0 + 2b_1 + 32/15\ b_4 + 8/9\ b_5 + 2/3\ b_7)\ a_4 \\ &+ (\ 1/3\ b_0 + 1/3\ b_1 + 4/9\ b_4 + 1/5\ b_5 + 1/5\ b_7\ )\ a_5 \\ &+ (\ 8/9\ b_2 + 2/5\ b_3 + 32/45\ b_6\ )\ a_6 \\ &+ (\ \pi/6\ b_0 + 4/9\ b_1 + \pi/6\ b_4 + \pi/10\ b_5 + 4/15\ b_7\ )a_7] \end{aligned} \tag{10}$$

Where $a_0, a_1, \ldots, a_7$ and $b_0, b_1, \ldots, b_7$ are the constants of displacement and stress functions. The above expression can be further expressed in terms of nodal forces and displacements. Since the resulting expression is quite lengthy it is not attempted to give it in this form.

Using a similar procedure, one can derive the corresponding expressions for Mode-II SERR, G_{II} and Mode-III SERR, G_{III}.

NUMERICAL EXAMPLES

A penny shaped crack in a cylindrical bar, a centre crack tension (CCT) specimen and an edge crack under shear (ECS) specimen are analysed to demonstrate the capabilities of MCCI technique to generate accurate numerical results comparable with the available results reported in the literature.

Circular Cracks in Cylindrical Body : The diameter and the length of the cylindrical body (Fig.5) considered are 10a, where a is the crack size. A uniform tension of $\sigma_y = 1.0$ is applied at the edges of the cylinder. The mesh contains 96 elements and 684 nodes with about 2052 degrees of freedom. The size of the smallest element used along the crack front is a/20.

The penny shaped crack considered in the present analysis was analysed earlier by Sneddon (1946) who considered this crack in an infinite body. Tracey (1973) conducted FE analysis for circular cracks in finite bodies and obtained SIF using a displacement approach. The SIF values of the present analysis are normalized by Sneddon's circular crack result. The variation of SIF along the crack front for the penny shaped crack is shown in Fig. 5. The maximum deviation of the present result is 2.1 % as compared to Sneddon's exact

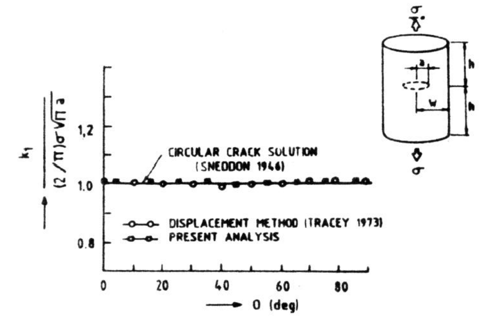

FIG. 5. VARIATION OF SIF ALONG THE CRACK FRONT : SOLID CYLINDRICAL BAR WITH PENNY SHAPED CRACK ($\frac{h}{w}$ = 5.0 & $\frac{a}{w}$ = 5.0)

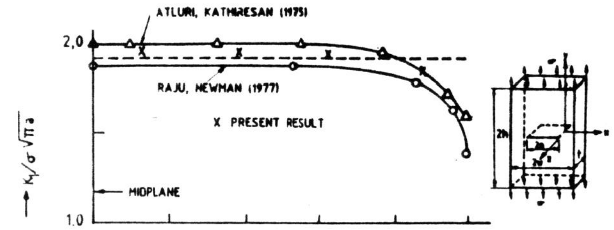

FIG. 6. VARIATION OF SIF ACROSS THE SPECIMEN THICKNESS : CCT SPECIMEN ($\frac{h}{w}$ = 0.5, $\frac{a}{w}$ = 0.5 & $\frac{t}{a}$ = 0.5)

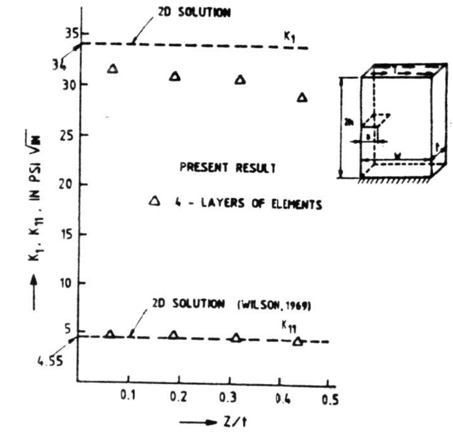

FIG. 7. VARIATION OF SIF ACROSS THE SPECIMEN THICKNESS : ECS SPECIMEN ($\frac{h}{w}$ = 1.142, $\frac{a}{w}$ = 0.5 & $\frac{t}{a}$ = 1.0)

solution and about 1.4 % from Tracey's finite element result.

Centre Crack Tension (CCT) Specimen : The centre crack tension (CCT) specimen configuration is shown in Fig.6. The FE idealization using 20-noded singular elements consists of 4 layers of elements along the crack front and has 548 nodes, 84 elements and 1644 DOF. The size of the smallest element used throughout the analysis is a/10. Due to double symmetry, only 1/8th of the domain is modelled for the FE analysis.

This specimen was analysed earlier by Raju and Newman (1977) and Atluri and Kathiresan (1975). The present analysis is carried out on a specimen with h/w = 0.5, t/a = 0.5 and a/w = 0.5 for which results are available from these two references. The variation of SIF across the thickness is shown in Fig. 6. For this specimen the SIF value at the centre is estimated within 1% by the present solution.

Edge crack shear (ECS) specimen : An edge crack in a rectangular solid subjected to uniform shear is shown in Fig. 7. This is analysed as an example of mixed-mode case. The specimen dimensions are h/w = 1.142, a/w = 0.5 and t/a = 1.0.

The FE model used for this problem is of the same type as that of CCT specimen analysed in the previous example. The results are compared (in Fig. 7) with those of Wilson who used the boundary collocation method. The results compare very well.

CONCLUSIONS

Modified crack closure integral (MCCI) expressions are obtained for 20-noded isoparametric singular elements. MCCI expressions are used for evaluation of strain energy release rates (SERR) G using full elemental area within each element. The accuracy and usefulness of the proposed procedure is illustrated through numerical evaluation of a penny shaped crack in a circular bar, centre crack tension (CCT) specimen and an edge crack shear (ECS) specimen in thick slabs.

REFERENCES

Atluri, S.N. and Kathiresan, K. (1975). Proc. 12th Ann. Meeting of the Soc. of Engng. Sci., 391-399.

Badari Narayana, K. (1991). Ph.D. Thesis, Indian Inst. of Science, Banalore, India.

Badari Narayana, K., Dattaguru, B., Ramamurthy, T. S. and Vijayakumar, K. (1991).Proc. Int. Conf. on Comput. Engng. & Sci., (ICES), Melbone, Australia.

Badari Narayana, K., Dattaguru, B., Ramamurthy, T. S. and Vijayakumar, K. (1992). (Communicated).

Buchholz, F. G. (1984). Proc. 4th World Congr. and Exhib. in FEM, Interlaken, 650-659.

Chan, S.K., Tuba, I.S. and Wilson, W. K. (1970). Engng. Fract. Mech., 2, 1-17.

Hellen, T.K. (1975).Int. J. Numer. Meth. in Engng.,9,187-207.

Nikishov, G. P. and Atluri, S. N. (1987). Int. J. Numer. Meth. Engng., 24, 1801-1821.

Parks, D. M. (1974). Int. J. of Fract., 10, 487-502.

Raju, I. S. (1986). NASA-CR-178186.

Raju, I. S. and Newman Jr., J. C. (1977). NASA-TN-8414.

Ramamurthy, T. S., Krishnamurthy, T., Badari Narayana, K., Dattaguru, B. and Vijayakumar, K. (1986). Mech. Res. Commun., 13(4), 179-186.

Rybicki, E. F. and Kanninen, N. F. (1977). Engng. Fract. Mech., 9, 931-938.

Sneddon, I. N. (1946). Proc. Royal. Soci. of London, 187, 229-260.

Wilson, W. K. (1969). Ph.D. Thesis, University of Pittsburgh, PA, USA.

Tracey, D. M. (1973). Int. J. of Fract., 9, 340-343.

Zienckiwicz, O.C. (1971). The finite element method in Engineering Science, 2nd ed., McGraw Hill, N.Y.