MODELS OF ELASTIC-PLASTIC STATE AND CRACK GROWTH FOR MIXED MODES UNDER BIAXIAL LOAD

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ABSTRACT

The new model concepts of an elastic-plastic state of material in region around the angled crack tip were formulated. The new boundary conditions were proposed and the new gradient characteristic having a physical sense of material structural parameter was introduced. Stress-strain state (SSS) main parameters were shown as functions of the structural parameter in complete range of mixed models. On the base of above results the elastic-plastic models were elaborated for both the crack growth rate and the fatigue life prediction. The models comprise complex of the main mechanical material characteristics as well as the singular SSS parameters for mixed modes of fracture. Comparison with experimental data for material having the different properties confirms a correctness of the approaches elaborated.

KEYWORDS

Mixed mode fracture; elastic-plastic stress-strain state; crack growth rate; fatigue life prediction.

INTRODUCTION

Solution of problems of fatigue life prediction for the structure elements on the stage of crack growth foresees their realization stage-by-stage. The main stages are as follows: analysis of the stress-strain state (SSS) and calculation of the crack growth duration from initial sizes up to critical ones. Moreover, one must take into account more completely both the complex of the material physical and mechanical properties obtained during the imitational experiments and the structure exploitation conditions as well as a character of its virtual damages. As a rule, the articles of new engineering are in conditions of the complex stress state, while the virtual damages orientation is arbitrary about the system of an operative loads.

Previous investigations have showed that the stress state type influence on characteristics of crack resistance under static and cyclic loading is displayed through the plastic deformation zone in region around the angled crack tip (for general case). Interpretation of this region state is not unique, and solutions are not adequate. Formulation and solution of the similar problems belong to the fundamental problems of mechanics of the mixed modes fracture.

In the first part of present paper the angled cracks elastic-plastic state models under biaxial load of an arbitrary direction are considered while in the second one - their growth conditions.

MODELS OF AN ELASTIC-PLASTIC STATE

Let the through rectilinear crack be in the nominal stresses biaxial field of an arbitrary direction at the plane stress state. Consider both quantitative (elastic-plastic singularity amplitude K_σ) and qualitative (dimensionless fields of the stress-strain state parameters $\tilde{\sigma}_{ij}, \tilde{\varepsilon}_{ij}$) aspects of the problem as independent ones. Because of the physical fields continuity in continuum, a distribution of SSS dimensionless parameters in the crack tip will be given with corresponding mathematical model of material deformation displaying the qualitative aspect of problem under consideration. To describe the quantitative aspect of problem we use the approximate analytical method by Neuber.

It is known that the classical Hutchinson-Rosengren-Rice's (HRR) solution for dimensionless stress fields of symmetric tension gives the good results. Therefore to obtain the qualitative aspects of problem it is worth to use the elastic-plastic deformation model taking into account only the nonlinear components in the corresponding terms. Proceeding from above model (Hutchinson, 1968) the homogeneous differential equation is obtained governing with the θ-distribution ($\theta\in(-180;180)$) of the stress tensor dimensionless components in the crack tip in terms of stress functions: Ary's

$$\left[n(S-2)-\frac{\partial^2}{\partial\theta^2}\right]\left[\tilde{\sigma}_e^{n-1}\{S(S-3)\Phi-2\Phi''\}\right]+\{n(S-2)+1\}\{n(S-2)\}\tilde{\sigma}_e^{n-1}\times$$
$$\times\{S(2S-3)\Phi-\Phi''\}+6\{n(S-2)+1\}(S-1)(\tilde{\sigma}_e^{n-1}\Phi')'=0 \qquad (1)$$

in which n - is the strain hardening exponent in the Ramberg-Osgud model, σ_e - is the stress intensity, Φ - is the Ary's stress function, $S=(2n+1)/(n+1)$. Thus, equation (1) describes the boundary values problem having the corresponding boundary condition. To determine these boundary conditions for the nonlinear mechanics of mixed modes fracture is the one of probblems. For the particular case of symmetric tension there was (Hutchinson, 1968) used condition of the stress field symmetry

$$\frac{\partial}{\partial\theta}\sigma_\theta(0^\circ)=\frac{\partial}{\partial\theta}\sigma_r(0^\circ)=0$$

as well as a requirement of unloading the crack faces $\sigma_\theta(\pm\pi)=$ $=\sigma_r(\pm\pi)=0$. HRR solution for symmetric tension has the following peculiarity. In the known point of θ-distribution because of the higher order derivative discontinuity, the situation of the physical fields infinite gradients takes a place. The similar situation leads to a need of an artificial and ungrounded transition through the region adjoining with above point. Since the real materials have the finite gradient properties, when solving (1) we have introduced (Dolgorukov, Makhutov, Shlyannikov, 1990) criterial gradient characteristic being determined in the following way:

$$G=\max\left(\frac{\partial}{\partial r}\sigma_{ij}\right)\Big|_{r=\delta^*} \quad \text{or} \quad G_r=\frac{1}{(n+1)r}\Big|_{r=\delta^*} \qquad (2)$$

Thus, the value of criterial gradient characteristic is determined by an elastic-plastic singularity form $(r^{-1/(n+1)})$ and material properties -n, δ^* is the structurally sensitive parameter. Consequently, the condition which has to be performed when solving (1) in symmetrical formulation as well as in unsymmetrical one will be written as follows:

$$\frac{\partial}{\partial\theta}\sigma_{ij}\leq G \qquad (3)$$

Solution (1) taking into account the condition (3) is shown in Fig. 1 illustrating the kinetics of isolines of elastic-plastic stresses intensity as well as the plasticity zones form depending on criterial value (2). These data remove the contradiction existing up to the present moment between the plasticity zone conception for an ideally plastic material in both the Hutchinson's solution and the kinematic models by Dugdale, 1960. To solve the mixed problem of an elastic-plastic crack meckanics we introduce the new model boundary condition conception. Suppose a simple load to take place for general case of asymmetrical problem in the crack front near the angle of a fracture initiation θ^*. Then direction of a fracture initiation θ^* may be determined by means of an elastic solution, hence the elastic criterion of fracture direction, for example, $\sigma_{r\theta}=0$ ($\sigma_\theta=\max$) is applicable to an elastic-plastic situation. As a result, the boundary conditions for equation (1) on the crack faces may be completed with the inner boundary condition $\sigma_{r\theta}(\theta^*)=0$. Moreover, solution of asymmetrical problem has been carried out with a taking into account results of investigations Budiansky and Rice, 1973 according to which $W(\pi)=W(-\pi)$ or $\sigma_r(\pi)=\pm\sigma_r(-\pi)$. Based on above formulated set of the boundary conditions for deformation compatibility equation (1), Dolgorukov, 1991 has obtained solution for the fields of dimensionless displacements, deformations and stresses in all the range of mixed modes from the symmetrical tension up to the pure shear taking into account the structural parameter δ^* influence. As

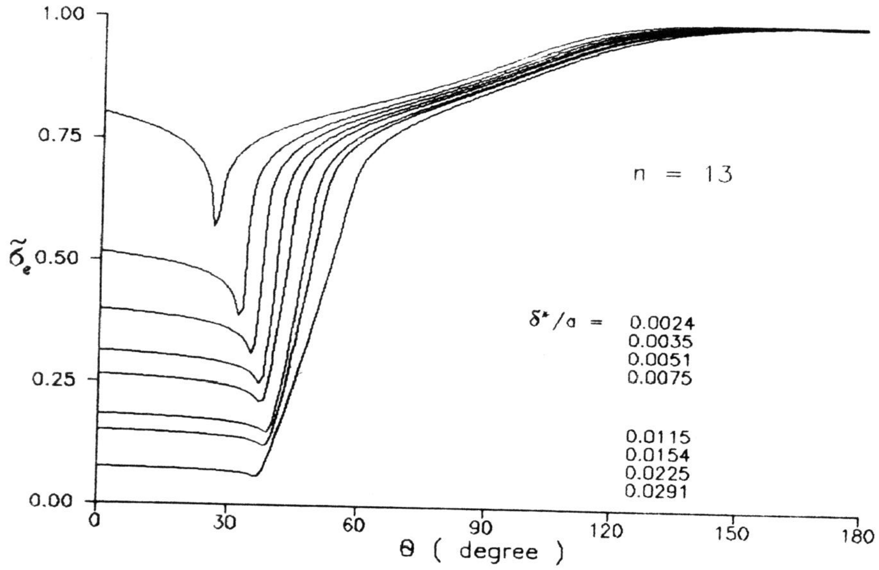

Fig. 1 Kinetics of a plastic stress intensity $\tilde{\sigma}_e$ change against the δ^* value

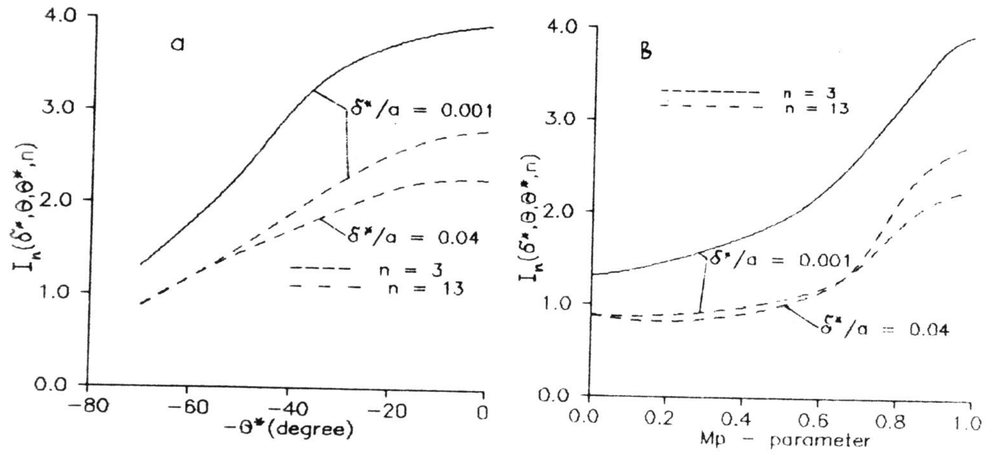

Fig. 2. Values of I_n vs θ^* (a) and M_p (b)

it also follows from Fig. 1, we have the known results by Hutchinson, 1968 for the plane stress state at $\delta^*\leq 0.0024$. Increasing of $\delta^*>0.0024$ leads to change of the stress intensity distribution. Thus, we achieve the situation when the material structural parameter δ^* affects upon dimensionless θ-distributions of elastic-plastic stress components without a direct entrance into the deformation compatibility equation (1). Hence,

$$\tilde{\sigma}_{ij}=\tilde{\sigma}_{ij}[\delta^*,\theta,\theta^*(\alpha,\eta),n] \qquad (4)$$

Moreover, an influence of specific type of the fracture mixed modes is taken into account via the angle of fracture initiation θ^* dependence on the angle of crack initial orientation α and on relation of biaxial nominal stresses η. Quantitative characteristic of the mixed modes influence with a taking into account δ^* under the plane stress state may be obtained by the Rice's J-integral dimensionless part which has the following form (Hutchinson, 1968):

$$I_n=\int_{-\pi}^{\pi}\left\{\frac{n}{(n+1)}\tilde{\sigma}_e^{n+1}\cos\theta-\left[\sin\theta\left(\tilde{\sigma}_r\left(\tilde{u}_\theta-\frac{\partial\tilde{u}_r}{\partial\theta}\right)-\tilde{\sigma}_{r\theta}\left(\tilde{u}_r+\frac{\partial\tilde{u}_\theta}{\partial\theta}\right)\right)+\frac{1}{(n+1)}(\tilde{\sigma}_r\tilde{u}_r+\tilde{\sigma}_{r\theta}\tilde{u}_\theta)\cos\theta\right]\right\}d\theta \qquad (5)$$

Proceeding from) we obtain the following structure for I_n:

$$I_n=I_n[\delta^*,\theta,\theta^*(\alpha,\eta),n]=\int_{-\pi}^{\pi}f(n,\tilde{\sigma}_e,\tilde{\sigma}_{ij},\tilde{u}_i,\theta)d\theta \qquad (6)$$

In Fig. 2 there are shown the results obtained from solution (1) with the above formulated boundary conditions for both dependences I_n on M_p-parameter by Shih, 1974 and on the angle of crack growth orientation θ^* in all the range of fracture mixed modes under the plane stress state. As it is seen, when the strain hardening exponent increases from n=3 up to n=13, then the structural parameter δ^* influence increases as well. Approach described is just for the plane stress as well as for the plane strain.

MODELS OF CRACK GROWTH

On the previous stage of the problem formulated solution the method is stated to obtain the dimensionless elastic-plastic stress fields in the angled crack tip. The final generalizing stage of the fatigue life calculation on the stage of crack growth comprises numerical information on the stress-strain state and experimental investigation results. The above obtained dimensionless elastic-plastic stress fields invariant to the object analysed and K-tarings of specimens or structure elements taking into account the specific article geometry belong to numerical results. As a rule, they are determined numerically by the finite elements method. Resistance to fatigue in the crack front depends on the local stresses and strains. One considers that inside the plastic zone conditions for the crack propagation by the process zone value (material structural parameter δ^*) in the fracture elemental statement submit to the low-cycle fatigue regularities. This fact assumes a use of the material deformation characteristics such as the limiting either static or cyclic deformation ε_f and the plastic zone size r_p. In given work the connection between the limiting stresses σ_f and strains ε_f with the fatigue life N_f in the form of low-cycle fatigue Manson-Coffin's and Ellin's equations has been used. Moreover the possibility of biaxial loading influence is taken into account on the SSS limiting characteristics change via the corresponding functions and parameters of an uniaxial tension σ'_{f1} and ε'_{f1}.

Proceeding from the above approaches and the elastic-plastic solutions obtained we have proposed the two models for prediction of crack growth rate and fatigue life for the cyclic fracture mixed modes. Intention of the first model was to express the crack growth characteristics under an arbitrary biaxial loading through the corresponding parameters of an uniaxial symmetric tension. The similar parameters were the Paris's equation ones C_o and m_o. As a result, Shlyannikov and Braude, 1992 have obtained the equation for prediction of the crack growth by the first model as follows

$$\left(\frac{da}{dN}\right)_\eta = C_o \left\{ \frac{\sqrt{1-\lambda_o+\lambda_o^2}\,[A(\lambda)+\gamma B(\lambda)]}{\sqrt{1-\lambda+\lambda^2}\,[A(\lambda_o)+\gamma B(\lambda_o)]} \right\}^{-\frac{n}{\beta}} (S_{max})^{m_o} \qquad (6)$$

in which β - is the Manson-Coffin's constant, S - is the parameter of strain energy density by Sih, 1974; λ - is the relation of main stresses in crack growth direction. The region of above equation application is limited by the linear part of the fatigue fracture diagram.

When elaborating the second model based on the HRR-solution one has set a target to describe all the fatigue fracture diagram beginning with the fatigue thresholds ΔK_{th} taking into account the material structural parameter δ^* To this end one has obtained the equation which allows to calculate the value equivalent to ΔK^{eq}_{th} over the material deformation characteristics and the value of ΔK_{th1} for any type of mixed fracture under an uniaxial symmetric tension . Parameters of the singular elastic-plastic SSS as well as material properties with a taking into account the type of biaxial loading has entered into the following equation for prediction of crack growth by the second model:

$$\frac{da}{dN} = 2\delta^* \left\{ \frac{\sigma^2 \pi a F_{r1} \mu_i (1+\xi\tilde{\sigma}^2) - (\Delta K^{eq}_{th})^2}{16\sigma'_f \varepsilon'_f E \delta^* I_n[\delta^*, \theta, \theta^*(\alpha,\eta), n)} \tilde{\sigma}_\theta (\tilde{\sigma}_\theta - \tfrac{1}{2}\tilde{\sigma}_r) \tilde{\sigma}_e^{n-1} \right\}^{\frac{1}{(b+c)}} \qquad (7)$$

Both above models have the common stages of their realization foreseeing the calculation of an elastic-plastic boundary contour, definition of crack growth direction and trajectory, its rate calculation and resifual fatigue life under the fracture mixed modes. In Fig.3 a comparison is displayed of the computational (according to the equations 6,7) and experimental data on prediction of crack growth rate (Fig. 3,a) and residual fatigue life (Fig. 3,b) under the fracture mixed modes. Experiments have been carried out for the eight aluminium alloys. Their properties are reduced in the table under an

	E GPa	σ_b MPa	σ_{ys} MPa	n	Mark
Al.alloy 1	71	320	160	3.72	■
Al.alloy 2	75	390	225	4.53	o
Al.alloy 3	72	439	285	4.94	★
Al.alloy 4	72	445	310	9.68	Δ
Al.alloy 5	71	420	320	3.71	+
Al.alloy 6	72	478	369	5.14	◇
Al.alloy 7	70	345	300	10.0	x
Al.alloy 8	72	563	506	10.05	✱

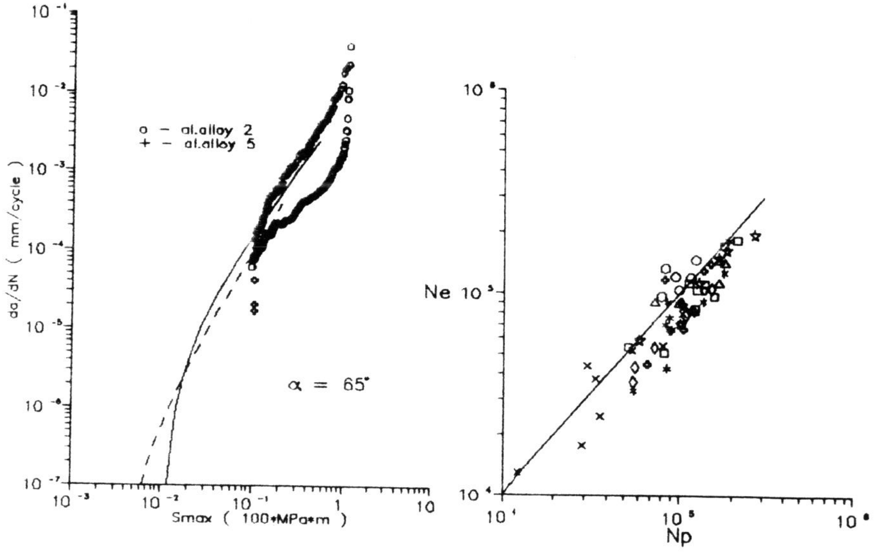

Fig.3. Comparison between the theoretical predictions and experimental data for aluminium alloys

uniaxial (η=0) and biaxial (η=0.5;1) tension with the angles of crack initial orientation α=0°,25°,45°,65°,90°. Comparison of computational results with experimental ones shows their good agreement with each other. Thus, to describe correctly the crack growth processes under the mixed modes one must perform the careful analysis of the singular elastic-plastic state in region around the crack tip.

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