MODELLING OF CREEP CRACK GROWTH UNDER CONSTANT AND CYCLIC LOADING

V.I. ASTAFJEV and V.A. PASTUKHOV
Samara State University, 443011, Russia

ABSTRACT

The method for modelling of subcritical creep crack growth, based on stress near tip field analysis is proposed. The critical damage accumulation near a crack tip is assumed as crack growth criterion and leads to crack initiation time and growth rate estimations. For the uncoupled statement of creep/damage problem the main features of creep crack growth under constant and cyclic loading are investigated. It has been shown that for Rabotnov's coupled statement of the problem the new stress asymptotic field near the tip of a growing crack is obtained. The possible development of presented approach is discussed.

KEY WORDS

Fracture, crack growth, creep, continuum damage, scalar damage parameter, creep/damage interaction, stress fields, fatigue.

INTRODUCTION

The evaluation of subcritical crack growth rate is ,important problem for design of various modern structures. Failure of high strength structural members under high temperature often occurs due to slow crack growth from microdefects. Intensive experimental investigations have shown that this process even for normal tension under constant loading is very complicated. Different stages of crack growth may be controlled by different parameters of load such as net stress, elastic stress intensity factor invariant energetic integrals and others (see for example reviews by Van Leeuwen (1979), Sadananda and Shahinian (1983)).

The main causes of such situation are the complexity of mechanical behaviour of materials under high temperature and the variety of local fracture micromechanisms. The biggest part of structural materials deforms accordingly to the elastic-creeping law. Theoretical investigations of near crack tip stress fields for this constitutive equation were analyzed by Cocks and De Voy (1991). They presented a map for choose a governing parameter and equation of crack growth under different conditions.

Models of creep crack growth are based on the stress field analysis and local fracture criteria. Among all types of used criteria, continuum damage approach seems most universal and perspective.

The first aim of present work is to present the main results in modelling of subcritical creep crack growth under constant loading reached by authors in uncoupled statement of creep-damage problem. The second one is to develop this approach on coupled statement and on cyclic loading.

UNCOUPLED STATEMENT OF THE PROBLEM

Constitutive Equations Let's consider the material which deforms accordingly to the next law:

$$\dot{\varepsilon}_{ij} = (1+\nu)(\dot{\sigma}_{ij}-\nu\dot{\sigma}_{kk}\delta_{ij}/(1+\nu))/E + 3B\sigma_e^{n-1}S_{ij}/2 \qquad (1)$$

where E is Young's modulus, ν is Poisson's ratio, n is creep exponent, B is creep constant, S_{ij} is stress deviation tensor ($= \sigma_{ij} - \sigma_{kk}\delta_{ij}/3$), σ_e is effective stress,

$$\sigma_e^2 = 3\ S_{ij}S_{ij}/2 \qquad (2)$$

The process of damage accumulation is described by Rabotnov-Kachanov's kinetic equation

$$\dot{\omega} = A(\sigma*/(1-\omega))^m , \qquad \omega(0) = 0 \qquad (3)$$

Here ω is scalar damage parameter, $\sigma* = \alpha\sigma_1 + (1-\alpha)\sigma_e$, σ_1 is maximal principal stress, A, m, α are material's constants.

The criterion of crack growth is assumed as

$$\omega(a(t)+d,t) = 1 \qquad (4)$$

when d is structural parameter connected with average grain size, a(t) is current crack length.

Governing Equation for Crack Initiation and Growth The integration of equation (3) with taking into account criterion (4) from the time of loading t=0 to current time t gives the governing equation for the process of crack initiation (when t is less than crack start time t_s) and for the process of crack growth (when $t > t_s$):

$$1 = A(m+1)\int_0^t \sigma*^m(a(t)+d,p)dp \qquad (5)$$

Stress Field Approximation Let's assume that stress components in the small near the crack tip creep zone are determined from asymptotic solution for power-law creeping cracked body. This

solution is obtained from HRR field for power-law hardening material (Hutchinson, 1968; Rice and Rosengren, 1968) by substitution of displacements rates instead of displacements:

$$\sigma_{ij}(r \to 0,\varphi) = (C(t)/(BI_n r))^{1/(n+1)}\ \bar{\sigma}_{ij}^{(c)}(n,\varphi) \qquad (6)$$

where r, φ are the polar co-ordinates at the crack tip, I_n is constant connected with n, $\bar{\sigma}_{ij}^{(c)}(n,\varphi)$ are dimensionless functions given in by Hutchinson (1968), C(t) is generalization of C^*-integral of steady state creep which describes stress redistribution from initial elastic state and is given by Riedel and Rice (1980).

Outside the creep zone the elastic solution is valid (see for example Parton and Morozov (1989) or another book on linear fracture mechanics)

$$\sigma_{ij}(r,\varphi) = K_I/(2\pi r)^{1/2}\ \bar{\sigma}_{ij}^{(e)}(\varphi) \qquad (7)$$

Here $\bar{\sigma}_{ij}^{(e)}(\varphi)$ are known dimensionless functions, K_I is elastic stress intensity factor.

Constant Loading The solution for constant loading when only once stress redistribution occurs is given by Astafjev and Pastukhov (1991). Let's assume for simplicity α=1 and consider the main results.

Time of crack start after loading t_s is defined by substitution of $t=t_s$ into the governing equation (5). For ungrowing crack stress redistribution means the stress relaxation under constant loading accordingly (6), where C(t) is monotonic decreasing time-function. The creep zone is defined as zone, where stress relaxation accordingly (6) reduces initial elastic value given by (7). It's size is equal to 0 at the moment of loading and increasing during transition time of steady state creep t_T Riedel and Rice (1980) . The estimation of creep zone size may be obtained by comparison of (6) and (7). If the critical point remains in elastic zone till the initiation of the crack, the result the crack start time has the form:

$$t_s = \frac{(2\pi d)^{m/2}}{A(m+1)K_I^m(a_o)} \qquad (8)$$

where a_o is initial length of crack or defect.

If the creep zone reaches the critical point before the crack start, then t_s is more complicated function of $K_I(a_o)$, $C^*(n,a_o)$, d and material constants.

Results for Crack Growth Rate The change of integration by time on integration by current dimensionless crack length for $t > t_s$ in equation (5) with taking into account value of t_s

gives the governing equation for crack growth rate. It's analytical solution is obtained by Astafjev and Pastukhov (1991) using Laplas transform and asymptotic analysis for $a(t)-a_o >> d$. The result is sensitive to the grade of creep localization:

$$\dot{a} = PK_I^m d^{1-m/2} \quad , R(a,t)<d \qquad (9a)$$

$$\dot{a} = QK_I^{2(n+1-m)/(n-1)} C^{(m-2)/(n-1)} - qd^{(n+1-m)/(n+1)} C^{m/(n+1)} \quad , d<R(a,t)<w-a \qquad (9b)$$

$$\dot{a} = H(a-a_o)^{(n+1-m)/(n+1)} C^{m/(n+1)} \quad , R>w-a \qquad (9c)$$

where w is width of cracked member, R - size of creep zone, P, Q, q, H are defined by material's constants.

These results reflect the main features of creep crack growth observed experimentally. Most interesting are the possibility of change of government loading parameter during the process of crack growth and the description of transition mode of crack growth, when alone parameter don't give a sufficient correlation with crack growth rate. It should be mentioned also the presence of parameter d in the equation (9a) and corrective term of equation (9b) and it's absence in the main term of (9b) and in (9c). It corresponds to the experimentally observed structural sensitivity of K_I-controlled crack growth and the absence of such sensitivity for C^*-controlled.

Cyclic Loading. Numerical simulation of damage accumulation at the critical point under piece-constant cyclic loading was fulfilled on the bases of equations (3), (6), (7) and Riedel-Rice's approximation for C(t). Five different ways of stress changing at critical point were observed. In all cases after the several initial cycles of loading was reached the steady state mode of stress changing accordingly to any cycle. Thus, the damage accumulation and time of crack start are described by the number of cycles. The character of damage accumulation strongly depends upon the ratio between period of loading cycle and Riedel-Rice's transitions time of steady state creep. For it's big values the periodical decreasing of load P to P/2 (for example) leads to increasing of crack start time with respect to the time corresponding to the constant load P. For the small values of ratio crack start time essentially decreasing due to the increasing role of elastic stress field with comparatively high stress level.

The simulation of crack growth by steps on process zone length d leads to generalization of issues for crack initiation rate on the crack growth rate. Thus, the empirical estimations of creep/fatigue crack growth may be formulated in terms of K_I, C^*, number of cycles, period of loading and Riedel-Rice's transitions time of steady state creep.

COUPLED STATEMENT

Constitutive Equations In the coupled statement of the problem the creep/damage interaction is taken into account by

Rabotnov-Leckie-Hayhurst constitutive equations (3) and

$$\varepsilon_{ij} = (3/2)B(\sigma*/(1-\omega))^{n-1} S_{ij}/(1-\omega) \qquad (10)$$

(Rabotnov (1970)).

Analysis of Stress Field The asymptotical analysis of boundary problem for the member with growing crack was fulfilled for $r \to 0$. The eigenvalue problem for the system of ordinary differential equations was obtained. It's solution for plane strain, plane stress and anti-plane shear conditions is given by Astafjev et al. (1991). A new type of stress and damage fields in process zone near a tip of a growing crack is obtained:

$$\psi = \varkappa^m \left[\frac{C^*}{B I_n} \right]^{m/(n+1-m)} \left[\frac{A}{\dot{a}} \right]^{(n+1)/(n+1-m)} r g^{(1)}(\varphi) \qquad (11)$$

$$\sigma_{ij}/\psi = \varkappa \left[\frac{A C^*}{\dot{a} B I_n} \right]^{1/(n+1-m)} f_{ij}^{(1)}(\varphi) \qquad (12)$$

where $f_{ij}^{(1)}(\varphi)$ and $g^{(1)}(\varphi)$ are known dimensionless functions given by Astafjev et al. (1991), $\psi = 1-\omega$ is continuity parameter and $\varkappa$ is undefined parameter.

It has been shown that for small r continuity $\psi=0$ in range $[\pi/2,\pi]$ of polar angle. It means that fully damaged zone (failed zone) adjoins to traction-free crack surfaces.

The introduction of damage state variable into power-law constitutive equations of creeping material permits to describe the stress redistribution and appearance near the crack tip a process zone in which the net stress is bounded and stress and continuity disappear. This is a result of damage influence on creep process taking into account in coupled creep-damage theory. The arising of such zone during creep crack propagation determines the creep opening displacement at the crack tip even if initially crack had infinite small width.

Evaluation of Crack Growth Rate The asymptotic fields are completely specified by current crack growth rate, path-independent integral of steady state creep theory, creep and damage material constants. It should be noted that asymptotic solution is independent of time t explicitly and their implicit time dependence associates only with crack growth rate $\dot{a}(t)$. Besides, the dimensionless parameter $\varkappa$ and crack growth rate $\dot{a}(t)$ in asymptotic solution are undefined and can be sought numerically by solving the main governing equations in whole range of variables $0<r<\infty$, $0<t<\infty$ and $0<\varphi<\pi$.

The evaluation of crack growth rate is fulfilled by gluing the solutions for stress field obtained for near and far from crack tip zones. Accordingly to the results of stress field analysis, these estimations give the upper limit of crack growth rate due to usage of C^* values obtained without crack opening displacement. The finite width and smooth tip of the crack decreasing the stress concentration in the case of transgranular creep crack growth. This fact is described by results obtained in coupled statement.

DISCUSSION

The results for creep growth rate and stress state near a crack tip obtained on the bases of continuum damage concept give the qualitative description of creep crack growth phenomenon. They reflect many important features noted above. Quantitative results aren't sufficiently precise because of essential idealization used in the theoretical study and of complicated character of estimations obtained in terms of loading parameters and material's constants. The later are determined experimentally with strong disperse. But the comparison with available experimental data shows that it is possible to use this results as upper estimation of crack growth rate.

The limitations of used approach connected with limitations of kinetic equation (3) for scalar damage parameter. Even it's application to cyclic loading in the case of uncoupled statement needs in additional grounding because it was established only for the constant or increasing loading. The description of crack growth for combined geometric modes of fracture under complicated ways of loading demands the generalization of presented approach. The tool for this generalization exists, it is the more complicated and universal continuum damage theories based on the vector or tensor damage measures. Evidently, the results for crack growth rate in such complicated models may be reached only numerically and it is more difficult to use them for qualitative analysis. It shows the importance of presented simplified approach and results.

The used methodology of modelling of subcritical crack growth may be applied not only under the creep conditions. It is concluded in the application of damage kinetic equation and results of stress field in cracked body analysis. The first is established on the bases of experimental data for time to failure of uncracked members under constant and increasing load. The second one is obtained by the solution of boundary problem which contains the materials constitutive law also determined experimentally. It seems perspective the application of this approach to ceramic materials.

CONCLUSIONS

The continuum damage concept allows to obtain the important results in the theoretical study of subcritical creep crack growth. The presented model based on the uncoupled statement of creep/damage problem reflects the main features of the process observed experimentally. The coupled statement leads to the description of process zone and creep opening displacement at the crack tip. This approach may be generalized on materials with other constitutive equations and on the more complicated conditions of loading.

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