LIMITING STATE OF CONSTRUCTION ELEMENTS WITH DIFFERENT KIND OF DAMAGES

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ABSTRACT

This paper presents author's investigation on the determination of the limiting state of construction elements with different kind of defects (notches, punctures, cracks, etc.). In case of availability of notches or punctures, the method of defining the limiting state is based on the joint use of criteria of strength of continuum mechanics and mechanics of fracture. For check of the above suggested method, the following two examples are considered : axial tension of orthotropic strip with elliptical hole and tension of isotropic plate with punched hole. In case of availability of defects similar to that of cracks, the famous criteria of mechanics of fracture are used for the definition of limiting state, but fundamental attention is paid to find the most unfavourable combination and arrangement of loads. For illustration, the following problem about axial tension of orthotropic plate with crack under the action of additional system of local loads is considered.

KEYWORDS

Damage, hole, puncture, crack, stress intensity factor, criteria of strength, method of section, anisotropy, local loads

LIMITING STATE OF CONSTRUCTION ELEMENTS WEAKENED OF DAMAGES UNSIMILAR TO THAT OF CRACKS

The problem of definition of limiting state of construction elements with defects like that of notches and punctures is considered.

Presently, there exists two basis methods (Panasjuk et al,1988) to define the limiting state of loaded bodies.From the position of classical and new series of criteria of mechanics of continuous media the strength of the body is defined by the state of stress of the material at that point. In agreement with these criteria, the limiting load for the body will correspond to the case, in which the limiting state of the material will be achieved at all the points of the dangerous section. From the point of view of mechanics of fracture, the limiting state of the body with crack is attained, when critical stresses exist even for small volume of the body.

The other methods (Tsay and Han,1978), describing the limiting state of plate made of laminated composites with holes, proposes that fracture occurs when tensional stress at point lying at certain distance from the boundary the hole attains strength at tension of material without concetrator. The indicated methods (Tsay and Han,1978) require the experimental determination of critical measures of zone of limiting state of material around the hole. Then for the evaluation of limiting load, linear mechanics of fracture is applied.

Suggested below is a new method for the determination of critical state of plate with damage. In this method critical measures of zone of the limiting state of material near the hole (or damage) and critical load are determined theoretically.

The essence of the suggested method lies in the following. From the experimental data, it is known that at the moment of fracture of plate with defect, the limiting stress state is not attained along the whole weakened section. The zone of limiting state of material d arises at the dangerous section under certain load σ . In this zone, structure of the material is observed to be changed forming a crack. With the further increase in load, the zone of limiting state increases till the crack attains critical length. Afterwords practically instantaneous fracture of the plate is observed.

Criteria of the continuum mechanics help to define measures of the zone in which limiting state of the materiad at given load is attained. But with the help of only these criteria, critical measures of the zone of limiting state of the material as well as the critical load at which the whole constructional element is separated in to parts should not be defined.

Criteria of the mechanics of fracture, on the other hand, permit to define the critical dimension of the defect simular to crack at the given load. But they don't permit to define size of the zone in which material is situated in limiting state.

Consequently in joint application of criteria of the methods of mechanics of continuous media and the mechanics of fracture leads to define the solution of the given problem. From the continuum mechanics it is necessary to take the method defining the length of zone of limiting state of the material near the defect from the given load, and from the mechanics of fracture - definition of the critical length of the crack near the defect from the given load.

To find the size of the zone of limiting state of the material, it is necessary to have formulas for determination of the principal stresses along the dangerous section. Then using these formulas and criterion of the strength of material S

of the continuum mechanics, can be found size of zone d of limiting state of material. We get following relation

$$d = f(\sigma, S). \qquad (1)$$

Using methods of the mechanics of fracture, the dependence of the stress intencity factor on the applied load can be found. Then applying criterion g of the mechanics of fracture, the critical length of defect d is found. Mathematical dependence of d on σ and g can be presented analogous to formula (1):

$$d = f_1(\sigma, g). \qquad (2)$$

Critical load σ_c is defined with the joint solutions of equation (1) and (2). When $d = d_c$

$$f(\sigma, S) - f_1(\sigma, g) = 0. \qquad (3)$$

Due to mathematical complications, arising in obtaing the equation (3) in it's obvious form, the solution can preferably be obtained using graphical methods, in majority of problems (Fig. 1). Here curve 1 is relation between dimension of the zone of limiting state of the material around the defect and the external load σ ; and the curve 2 is relation between critical length of the radial crack d and σ . Abscissa of the point of intersection of the two curves gives critical dimension of the zone of limiting state of material, and ordinate is the value of load σ , under which occurs complete rupture of element in parts.

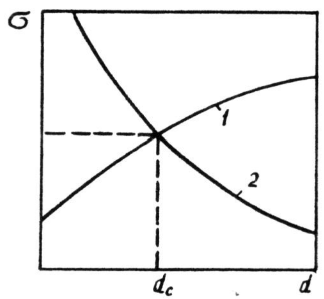


Fig. 1. Graphical determination of critical stress $\sigma = \sigma_c$

Example 1. Consider the application on the suggested method for the calculation of the limiting state of the orthotropic plate of finite width with the elliptical hole. X and y co-ordinate axis normal to the plane of elastic symmetry. On sufficiently large distance from concentrator, at the cross section of the plate act distributed uniform tensile stresses of intensity σ_0 =const.

For the criterion of the strength of continuous material criterion of statical fracture of materials is adopted with the addition on the condition of separation

$$\sigma_{eq} \geq \sigma_u \vee \sigma_1 \geq S_c , \qquad (4)$$

where σ_{eq} - equivalent (desing) stresses; σ_u - ultimate strength; $\vee$ - sign of logical summation; σ_1 - principal stress; S_c - resistance to direct pull.

Irvin's condition is used as the condition of fracture

$$K_1 = K_c , \qquad (5)$$

where K_1 and K_c – stress intensity factor (SIF) of Mode 1 and it's critical value in case of plane state of stress.

Further, in plotting curve 1 (Fig. 1), it's necessary to have formulas for defining principal stresses σ_1 and σ_2 acting at the dangerous section. Approximation of the formulas for σ_1 and σ_2 can be obtained from the known solution (Savin, 1968) about axial tension of infinitely isotropic plate with elliptical hole with the help of method of sections. Then using the formulas obtained for σ_1 and σ_2 and criterion of strength (4) can be obtained dependences of size of the zone of limiting state of the material near the hole on the external load σ_0 (curve 1 on Fig.1).

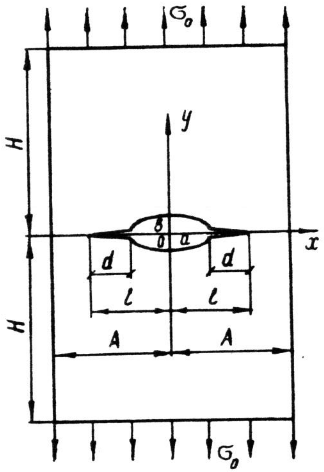
For plotting curve 2 (Fig.2), which is dependence of SIF on the length of radial crack situated in the plate with hole (Fig.2). Expression for SIF can be found on the basis of solution of (Borodachev and Kulii, 1983) problem about the tension of strip with central crack.Then, using expression obtained for SIF and condition of strength (5), dependence 2 (Fig.1) of critical length of radial crack on σ_0 is established.

For verification the correctness of the calculation in the method suggested, the experimental data of the trials of fracture due to tension of plane graphite fiber-epoxy composite specimens (thickness =2.7 mm, width =35 mm, diameter of a hole $2a$ = = 5-14 mm) (Fig.3, dash line) were used. There are experimental values (shaded region) on Fig. 3 too.

Theoretical curves $f(S_c)$ and $f(\sigma_u)$ (Fig.3) are obtained by using the criterion of the mechanics of fracture ($K_1 = K_c$) and two different criteria of continuum mechanics ($\sigma_1 \geq S_c$ and $\sigma_{eq} \geq \sigma_u$ correspondingly). There theoretical curves gave double-sided estimates of the parameters of the limiting state. Maximum difference between theoretical and experimental data did not exeed 12% in using the criterion of strength $\sigma_1 \geq S_c$. In applying the criterion $\sigma_{eq} \geq \sigma_u$ error is smaller. Yet if arithmetical mean value of stress is obtained from the criteria $\sigma_{eq} \geq \sigma_u$ and $\sigma_1 \geq S_c$, then the error of the method do not exeed 5%.

Example 2. Let's see plate from D16AT (2A = 140 mm,2H = 400 mm, t = 1.83 mm, σ_u = 385 MPa, $\sigma_{0.2}$ = 270 MPa, K_c = 30.6 MPa·m$^{1/2}$, μ = 0.3, E = 0.71·10^5 MPa) with through rupture (Fig.4).Required to find limiting external load $\sigma_0 = \sigma_{0c}$,at which fracture of plate will be happened.

The through hole,arising from the impact of drift on the plate, is nearly circular in form. Surface of the contour of the hole has a complicated microrelief. For simplication of the solution of the problem, the damage zone is substituted by an elastic ring, elastic characteristics of which correspond to the average characteristics of the material in the deformed zone. To verify the correctness of the suggested method to define the limiting state,the plate from D16AT were tested.Using plates were



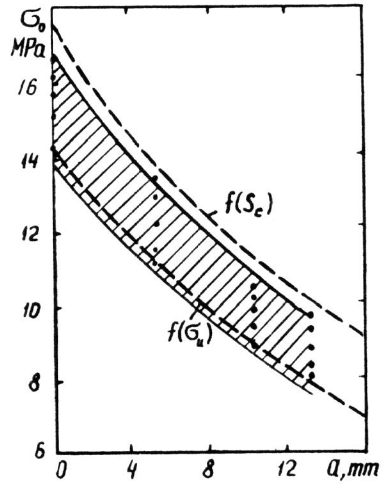


Fig. 2. Plate with radial cracks, appearing on the contour of notch

Fig. 3. Experimental (shaded region) and theoretical (shaded lines) date of the critical stresses σ_0 for the plates with notches of different radius R = a ; Curve $f(S_c)$ plotted by using criteria of strength S_c and K_c; curve $f(\sigma_u)$ – from the use of σ_u and K_c

punched by drift of mass m = 0.4-1.2 g and with the speed V = 1.0 and 1.5 km/s.

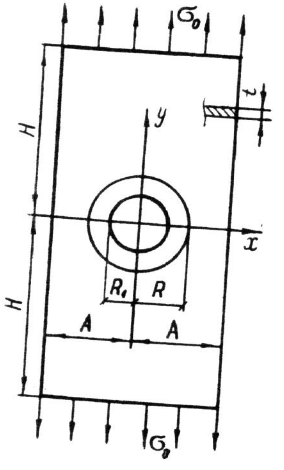
In the Fig.5, for the damaged plates,theoretical (shaded lines) and experimental (continuous lines) are given. Comparison of experimental and calculated data in the considered examples indicates the effectiveness of the proposed method which defines the limiting state of the constructional elements from the different materials with defects.

INFLUENCE OF LOCAL LOADS ON LIMITING STATE OF ELEMENTS WITH CRACKS

To evoluate strength construction elements with cracks it is necessary to know the place application of loads the most unfavourable from the point of these elements strength.

Essential influence on distribution of stress renders local applied loads, especially it is not far from cracks or another stress concentrator, moreover anisotropy of material influences on stress state too.

To estimate the dependence of local loads application and ani-



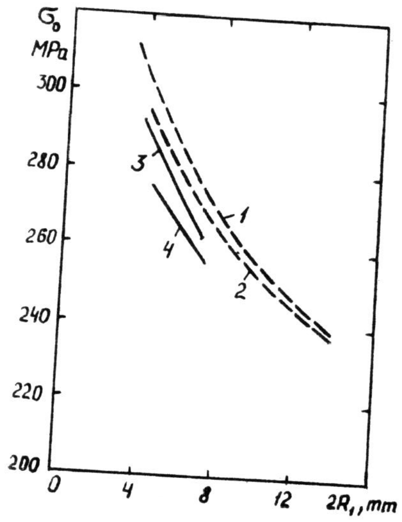


Fig. 4. Plate with damaged zone of material of length δ = R-R.

Fig. 5. Dependence of critical stress $\sigma_0 = \sigma_{0c}$ on the diameter of the hole in the plate; 1 and 2 are theoretical curves for the thickness of rings (δ = 0.65 and 1.3 mm) correspondingly, μ_1 = 0.26, E_1=0.47·10 MPa); 3 and 4 are experimental curves (velocity of drift V = 1.5 and 1.0 km/s correspondingly).

sotropy of material we will consider on the example of the orthotropic plate with crack which occupies segment $[-l,l]$ on the x axis. The elliptical hole, which was consided before degenerates to given crack when b = 0. Suppose that the plate is in a state of plane stress under the action of arbitrary system body forces X(x,y) and Y(x,y), which are symmetrical to the surface of crack.

Especial attention in mechanics of fracture problems is payed to finding out of stress intensity factor for elastic orthotropic plates as well as for isotropic. The stress and displacement state near the crack tip and limiting state can be determined by means this factor. So this problem can be limited by definition of SIF.

According to results of (Borodachev and Shevshenko, 1987) while the arbittrary system crack surface (x axis) the stress intensity factors can be determined for crack tip with $\pm l$ co-ordinates using the expression :

$$K_1^{\pm} = \frac{2}{\sqrt{\pi l}} \int_{-l}^{l} \left(\frac{l \pm x}{l \mp x}\right)^{1/2} dx \int_{-\infty}^{\infty} d\xi \int_{0}^{\infty} [P_{21}(x-\xi,\eta)X(\xi,\eta) + P_{22}(x-\xi,\eta)Y(\xi,\eta)]\, d\eta \,, \quad (6)$$

$$P_{21}(x,y) = \frac{1}{C}\sum_{i=1}^{2}(-1)^{i}\frac{A(q_i)x}{x^2+q_i^2y^2}, \quad P_{22}(x,y) = \frac{1}{C}\sum_{i=1}^{2}(-1)^{i+1}\frac{[A(q_i)-a_{66}]q_i y}{x^2+q_i^2y^2},$$

$$A(q) = a_{11}q^2 - a_{22}, \qquad C = 2\pi a_{11}(q_2^2 - q_1^2).$$

where and further $a_{ij}(i,j=1,2,6)$ – elastic constante, and q_1, q_2 – positive roots of the equation $a_{11}q^4 - (2a_{12}+a_{66})q^2 + a_{22} = 0$.

The arbitrary body forces X,Y can be changed to the system of concentrated forces $T_i(i=1,2,\ldots,n)$-parallel and $N_j(j=1,2,\ldots,m)$-normal to the x axis with the sufficiant degree of accuracy. In this case expression (6) for determination of stress intensity factors may be written as :

$$K_1^{\pm} = \sum_{i=1}^{n} T_i\, \bar{K}_{Ti}^{\pm}(x_i,y_i) + \sum_{j=1}^{m} N_j\, \bar{K}_{Nj}^{\pm}(x_j,y_j), \quad (7)$$

where T_i and N_j is intensity of parallel and normal x axis of concentrated forces,respectively. Apart from that, $\bar{K}_{Ti}^{\pm}(x_i,y_i)$ and $\bar{K}_{Nj}^{\pm}(x_j,y_j)$ – stress intensity factors for corresponding tips of crack from the action of unit forces at point, with co-ordinates indicated in corresponding (normal or parallel) diractions.

Expression for factors $\bar{K}_{Nj}^{\pm}(x_j,y_j)$ has next final form

$$\bar{K}_{Nj}^{\pm}(x_j,y_j) = [(1+D)/Q_j^{\pm}(q_1) + (1-D)/Q_j^{\pm}(q_2)]/\sqrt{2\pi l},$$

$$Q_j^{\pm}(q) = \{[1+(1\mp B_j)^2/q^2H^2][B_j^2+q^2H^2-1+\sqrt{(B_j^2+q^2H_j^2+1)^2-4B_j^2}]\}^{1/2},$$

where $D = a_{66}/a_{11}(q_1^2 - q_2^2)$, $B_j = x_j/l$, $H_i = y_j/l$.

Factors $\bar{K}_{Ti}^{+}$ are determined numerically by the use of quadrature formula of Gauss (Abramovitz and Stegun, 1979)

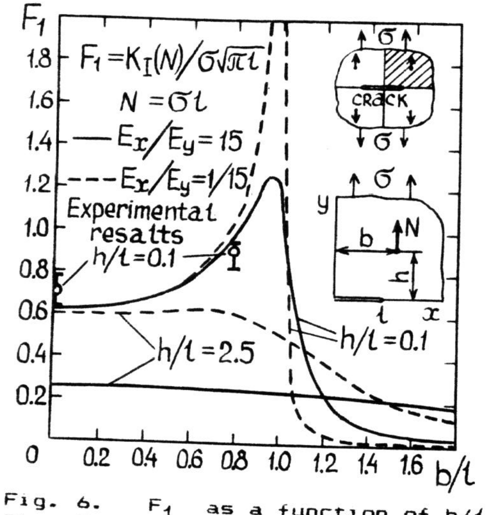
$$\bar{K}_{Ti}^{+}(x_i,y_i) = (4l/\sqrt{\pi l})\sum_{j=1}^{k} W_j P_{21}(Z_j),$$

where $W_j = 2\pi\zeta_j/(2k+1)$, $Z_j = l(1+2\zeta_j)$,

$$\zeta_j = \cos^2[(\pi/2)(2j-1)/(2\pi-1)].$$

On example of essential anisotropic material graphite fiber – epoxy composite consider using formula (7) for the calculation of value SIF.

Results of calculation present in form of diagrams of dependence relative value of stress intensity factors on co-ordinats (b,h)-points of aplication of concentrated forces for case when crack is parallel(normal) to diraction maximum rigidity of composite (Fig.6,Fig.7). Maximum values on diagrams show points of application concentrated loads, which are the most unfavourable from the point view of strength of these elements with cracks.



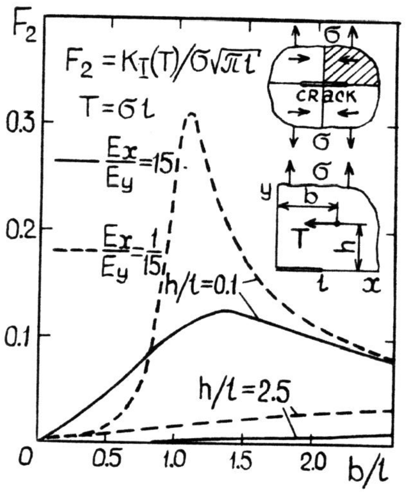


Fig. 6. F_1 as a function of b/l , h/l and E_x/E_y .
Fig. 7. F_2 as a function of b/l , h/l and E_x/E_y .

Experimental research on fracture due to tension have been developed on graphite fiber-epoxy composite specimens (thickness = 2.7 mm, width = 35 mm), with crack ($2l$ = 14 mm) directed along fibers E_x/E_y = 15. Experimental results in Fig. 6 for two case of loading of tension concentrated forces give satisfactory correlation with the numerical data.

REFERENCES

Abramowitz M. and Stegun I.A. (1979). Handbook of Mathematical Funktions with Formulas, Graphs and Mathematical Tables, (in Russian), Nauka, Moskow.

Borodachev N.M. and Kulii M.P. (1983). Generalization of Method of Plane Sections for Determining Stress Intensity Factor, (in Russian), Problemy Prochnosti, N 2, P. 23-27.

Borodachev N.M. and Shevchenko O.A. (1987). Determination of Stress Intensity Factors for Crack in Orthotropic Body in which Body Forces are Acting, Prikladnaia Mechanika, 23, N 9, P.78-84.

Panasjuk W.W., Andreikiv A.E. and Parton W.Z. (1988). Fundamentals of Mechanics of Fracture, (in Russian), Naukowa Dumka, Kiev.

Savin G.N. (1968). Distribution of Stress Near the Holes, (in Russian), Naukowa Dumka, Kiev.

Tsay S. and Han H. (1978). Fracture Analysis of Composites, In: Nonelastic Properties of Composites, (in Russian). (Gerakovich K.), 16, P. 104-139, Mir, Moskow.