INTERNAL PLASTIC INCOMPATIBILITIES ACCUMULATION AS FACTOR OF HARDENING AND DAMAGE OF DEFORMED HETEROGENEOUS MATERIALS

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ABSTRACT

This paper presents author's investigations on accumulation of plastic incompatibilities and related internal stress in a material being plastically deformed. Macroscopic effects of the factors pointed have been analized such as resistance to plastic deforming, predisposition to fracture of heterogeneous material deformed, internal stress energy decomposing over multiple structural levels and influence of elastic unisotropy of structural elements on elastic deformation of heterogeneous body with plastic incompatibilities.

KEYWORDS

Plastic incompatibility, superfluous deformation, internal stress, heterogeneous material, structural level.

INTRODUCTION

Internal plastic incompatibilities accumulated in a media been deformed and related internal microstress are the main factors "situated" within the gap between mechanics and physics of plasticity. There is a global reason for an accumulation process mentioned in heterogeneous materials. This is a difference of structural elements in their compliances in a course of deformation. A clear idea pointed above is a good base for a quite rigorous sructural-micromechanical theory to be worked out that would descript macroscopical effects of internal micro-scale incompatibilities of plastic deformation. The most simple variant of a similar approach was realized by authors (1985,1986,1990) for heterogeneous materials where a piece-uniform approximation is a quite good one.

Let's consider the aggregate of continuously jointed structural elements (s = 1,2,...) which are denoted by the following terms:
modulus of elasticity- $\hat{c}_s$, elastic- $\hat{\varepsilon}_s^{el}$ and plastic- $\hat{\varepsilon}_s$ deformation. superfluous elastic- $\hat{\xi}_s^{el}$ and plastic- $\hat{\xi}_s$ deformation:

$$a)\ \hat{\xi}_s^{el} = \hat{\varepsilon}_s^{el} - \langle \hat{\varepsilon}_s^{el} \rangle_V \qquad b)\ \hat{\xi}_s = \hat{\varepsilon}_s - \langle \hat{\varepsilon} \rangle_V , \qquad (1)$$

where $\langle\ldots\rangle$ simbol means the average value , V is macrovolume of the aggregate. In the framework of piece-uniform approximation the fragment's proper stress, internal stress and macrostress have, accordingly, forms

$$a)\ \hat{\sigma}_s = \langle \hat{\sigma} \rangle_{Vs} = \hat{c}_s\ \hat{\varepsilon}_s^{el} \quad b)\ \hat{\sigma}_s^{int} = \hat{\sigma}_s - \hat{\sigma}^{ext} \qquad (2)$$

$$c)\ \hat{\sigma}^{ext} = \langle \hat{\sigma}_s \rangle_V ,$$

where Vs is a volume of the fragment "s". The media continuity condition for the terms defined have a simplest form

$$\hat{\xi}_s^{el} + \hat{\xi}_s = 0 \qquad (3)$$

in accordance with Zisman and Rybin (1986). Also the following equations are just:

$$a)\ \langle \hat{\xi}_s^{el} \rangle_V = -\langle \hat{\xi}_s \rangle_V = 0 \quad b)\ \langle \hat{\sigma}_s^{int} \rangle_V = 0 . \qquad (4)$$

One may show from (1)-(4) that

$$\hat{\sigma}^{ext} = \hat{c}_o \cdot\cdot\ \hat{\varepsilon}^{el} - \langle \Delta\hat{c}_s \cdot\cdot\ \hat{\xi}_s \rangle_V , \qquad (5)$$

where

$$\hat{\varepsilon}^{el} = \langle \hat{\varepsilon}_s^{el} \rangle_V , \quad \Delta\hat{c}_s = \hat{c}_s - \hat{c}_o , \quad \hat{c}_o = \langle \hat{c}_s \rangle_V . \qquad (6)$$

i.e. violation of the mixture rule for elasticity modulus take place if plastic incompatibilities accumulate.

Internal stress as defined are "superfluous" over macroscopic one, therefore former values may be easily expressed through superfluous deformations. It followes from (2) and (5) that

$$\hat{\sigma}_s^{int} = - \hat{c}_s \cdot\cdot\ \hat{\xi}_s + \Delta\ \hat{c}_s \cdot\cdot\ \hat{\varepsilon}^{el} + \langle \Delta\ \hat{c}_s \cdot\cdot\ \hat{\xi}_s \rangle_V \ . \qquad (7)$$

First term in (7) descripts the size mismatch effect, the second one- modulus effect and third - superposition of both. The simplification of (7) for elastically uniform material is obvious:

$$\hat{\sigma}_s^{int} = - \hat{c}_s \cdot\cdot\ \hat{\xi}_s = \hat{c}_s \cdot\cdot\ \hat{\xi}_s^{el} , \quad \hat{c}_s = \hat{c}_o . \qquad (8)$$

Thus we have obtained the principal terms and relationships of structural micromechanics. For they to be applied it is necessary only to find the superfluous plastic deformations with related internal stress of elements and then to analyze

the structure dependent macroscopic effects. At first we shall consider the latter problem. Decision of the former one is strongly dependent of concrete materials nature and will be discussed in conclusion of a present paper.

RESISTANCE TO DEFORMING

A specific work on plastic deformation increment $\delta\hat{\varepsilon}$ as it was shown by Rybin and Zisman (1990) is equel to

$$\delta A = \hat{\sigma}^{ext} \cdot\cdot\ \delta\ \hat{\varepsilon} = \delta Q + \delta\ \Gamma + \delta\ W^{int} , \qquad (9)$$

$$\delta W^{int} = \langle \hat{\sigma}_s^{int} \cdot\cdot\ \delta \hat{\xi}_s^{el} \rangle_V = -\langle \hat{\sigma}_s^{int} \cdot\cdot\ \delta \hat{\xi}_s \rangle_V .$$

where δQ, $\delta\ \Gamma$ and δW^{int} are specific increments of energies dissipated, related to lattice defects (dislocation nuclei, interfaces ect.) and internal stress correspondingly. It followes from (9) that a deforming macrostress is

$$\hat{\sigma}^{ext} = \hat{\sigma}^{eff} + \hat{\sigma}^{*} , \qquad \hat{\sigma}^{eff} = \frac{\partial\ Q}{\partial\ \hat{\varepsilon}} , \qquad (10)$$

$$\hat{\sigma}^{*} = \frac{\partial\ \Gamma}{\partial\hat{\varepsilon}} + \frac{\partial\ W^{int}}{\partial\hat{\varepsilon}} = \frac{\partial\ \Gamma}{\partial\ \hat{\varepsilon}} - \langle\ \hat{\sigma}^{int} \cdot\cdot\ \frac{\partial\ \hat{\xi}_s}{\partial\ \hat{\varepsilon}} \rangle_V ,$$

where $\hat{\sigma}^{eff}$ is the conventional characteristic of plasticity mechanics and $\hat{\sigma}^{*}$ is an additional resistance related with transfer of the bringed mechanical energy into a latent form. In other words the latter is the athermal component of stress.

One may see that $\hat{\sigma}^{*}$ tensor with neglecting of $\partial\ \Gamma/\partial\hat{\varepsilon}$ is defined by plastic incompatibilities accumulation rate and accordingly by the rate of internal stress increasing. So, for the tensor considered to be evaluated one need to know $\hat{\xi}_s$ dependence on macroscopic plastic deformation only. Naturally there are other kinds sources of long-range stress in a body deformed. They give the conventional athermal resistance to deforming - "σ^{o}" that is related with lattice dislocations long-range elastic inter-action . Unlike one the value considered is due directly to the heterogeneous (polycrystal, multy-phase ect.) structure.

SPECIFIC ENERGY OF INTERNAL STRESS AS DAMAGE FACTOR

Let's compare the plastic deformation work δA of continuously jointed structural elements' aggregate with the same deformation work δA_o of the same elements which are in disconnected state. The latter is equel to

$$\delta A_0 = \langle \hat{\sigma}_s \cdot\cdot \delta \hat{\varepsilon}_s \rangle_V . \qquad (11)$$

In accordance with (3) and (9) we'll obtain

$$\delta A_0 = \delta Q + \delta \Gamma \qquad (12)$$

and, comparing with (9), find

$$\delta A - \delta A_0 = \delta W^{int}. \qquad (13)$$

In other words W^{int} value characterizes an expenditure of energy bringed to the sample for the material being deformed to retain continuity. Interpretation pointed permits to consider W^{int} value as a suitable characteristic of state for general analysis of heterogeneous material damage.

Really the material will not be able to digest the bringed energy and, at the same time, retain continuity if the value

$$W^{int} = \int_0^{\hat{\varepsilon}^T} \frac{dW^{int}}{d\hat{\varepsilon}^T} \cdot\cdot d\,\hat{\varepsilon}^T, \qquad (14)$$

where $\hat{\varepsilon}^T$ is the total deformation, reaches a certain critical value W_{cr}. The last is determined by strengh of interfaces and their specific area. A coarse estimmate of W_{cr} is

$$W_{cr} \simeq (\gamma' + \gamma'' - \gamma_b) / D , \qquad (15)$$

where γ' and γ'' are specific energies of free surfaces of contacting elements, γ_b is one of interface and D is the typical size of structural element.

Naturally formulated discontinuity criterion is too hard since it corresponds with a material "crumbling" to particles and leaves out of an account the crack propagation as affected of external stress. However the W^{int}/W_{cr} ratio is the good measure of a deformed material predisposition to fracture.

STRUCTURAL LEVELS

The heterogeneous materials often possess a number of structural levels and elements within every level pointed usually distinguish in their compliances. Let's enumerate levels in accordance with Figure 1, begining from most large-scalic one. For generalization of principal formulas of structural micromechanics one have to define self-contained (proper) deformations on an arbitrary level q and corresponding superfluous deformations:

$$\text{a)}\ \hat{\varepsilon}_s^{el(q)} = \langle \hat{\varepsilon}^{el} \rangle_{V_s^{(q)}} \qquad \text{b)}\ \hat{\xi}_s^{el(q)} = \hat{\varepsilon}_s^{el(q)} - \hat{\varepsilon}_s^{el(q-1)} \qquad (16)$$

$$\text{a)}\ \hat{\varepsilon}_s^{(q)} = \langle \hat{\varepsilon} \rangle_{V_s^{(q)}} \qquad \text{b)}\ \hat{\xi}_s^{(q)} = \hat{\varepsilon}_s^{(q)} - \hat{\varepsilon}_s^{(q-1)} . \qquad (17)$$

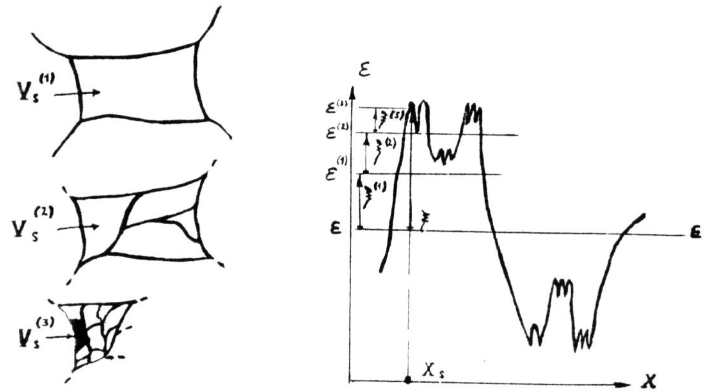

Fig.1. On definition of structural levels and related parameters.

The sense of definitions made are illustrated on the Fig.1. Beside that we'll define proper stress of corresponding elements and relative internal stress (generalization of above defined internal one):

$$\text{a)}\ \hat{\sigma}_s^{(q)} = \langle \hat{\sigma} \rangle_{V_s^{(q)}} \qquad \text{b)}\ \Delta\hat{\sigma}_s^{(q)} = \hat{\sigma}_s^{(q)} - \hat{\sigma}_s^{(q-1)} . \qquad (18)$$

Correlation between such terms and above mentioned ones is trivial:

$$\text{a)}\ \hat{\sigma}_s^{int} = \sum_{q=1}^{Q} \Delta \hat{\sigma}_s^{(q)} \qquad \text{b)}\ \hat{\xi}_s^{(q)} = \sum_{q=1}^{Q} \hat{\xi}_s^{(q)} \qquad (19)$$

and allowes to decompose δW^{int} value over structural levels:

$$\text{a)}\quad \delta W^{int} = \sum_{q=1}^{Q} \delta W^{int(q)} \qquad \text{b)}\quad \delta W^{int(q)} = -\langle \Delta\hat{\sigma}_s^{(q)} \cdot\cdot\, \hat{\xi}_s^{(q)} \rangle_V \,. \quad (20)$$

We used (9) for (20) to be obtained and took in account the original "ortogonality condition" that follows from (16) and (17) :

$$\langle \Delta\hat{\sigma}_s^{(q')} \cdot\cdot\, \hat{\xi}_s^{(q)} \rangle_V = 0 \quad \text{if} \quad q \neq q' \,. \quad (21)$$

Decomposing obtained permits definition of proper values $W_{cr}^{(q)}$ on every structural level and provides the terms for independent analysis of a fracture probability on different levels.

EFFECTS OF ELASTIC UNISOTROPY OF STRUCTURAL ELEMENTS IN MATERIAL PLASTICALLY DEFORMED

Let's equate an external load to nought and find, having in a view (7), the residual stress within structural elements:

$$\hat{\sigma}_s^r = \hat{c}_s \cdot\cdot \left(\hat{c}_o^{-1} \cdot\cdot \langle \Delta\hat{c}_s \cdot\cdot \hat{\xi}_s \rangle_V - \hat{\xi}_s \right) \,. \quad (22)$$

As one may see internal stress of elastically non-uniform material will redistribute when external stress disappear after plastic deformation finishing.

There is another interesting result followed from (5):

$$\hat{\varepsilon}_R^{el} = \langle \hat{\varepsilon}_s^{el} \rangle_V = \hat{c}_o^{-1} \cdot\cdot \langle \Delta\hat{c}_s \cdot\cdot \hat{\xi}_s \rangle_V \,. \quad (23)$$

It means that residual elastic deformation of an elastically non-uniform body is not zero on the average. In this one differs from the residual internal stress, that are, naturally, self-balanced :

$$\langle \hat{\sigma}_s^r \rangle_V = 0 \,. \quad (24)$$

Essentially the effect is that for a stress field to be self-balanced after external unloading the high-modulus particles have to be subjected to less value of elastic deformations then the low modulus ones must (equel volume parts of both phases are suggested for such qualitative explaning).

It seems that results (22) and (23) are quite useful for two

important applications at least. The first is related with thermal expansion of heterogeneous bodies when $\hat{\sigma}^{ext}=0$ and "stress-free" (pseudo-plastic) deformations $\hat{\varepsilon}_s$ and $\hat{\xi}_s$ are thermal ones:

$$\text{a)}\quad \hat{\xi}_s^{th} = \hat{\varepsilon}_s^{th} - \langle \hat{\varepsilon}_s^{th} \rangle_V \,, \quad (25)$$

$$\text{b)}\quad \hat{\varepsilon}_s^{th} = \alpha_s \hat{I} \,(T - T_o),$$

where $\hat{I}$ is the double rank unit tensor, α_s - thermal expansion coefficient of s-th element and T_o - initial (stress-free) temperature. Using equations (25) and above results one may evaluate the residual microstress appeared during heating or cooling of the heterogeneous body.

The second application is related with the X-ray tensometry of polycrystals, where elastic unisotropy of crystallites misoriented are usually neglected in a practice. The matter of a problem is that residual stress are measured in fact through the ellastic deformation and considered addition $\hat{\varepsilon}_R^{el}$. when not eccounted, may result in a serious mistake.

CONCLUSION

For the results obtained to be applied one have to know parameters $\hat{\xi}_s$ and their evolution in media being deformed.

A similar problem was analized for polycrystals, Rybin and Zisman (1985). Let's consider one for two-phase composite with reinforcing degree - p. It is clear that

$$p\,\hat{c}_h \cdot\cdot \hat{\varepsilon}_h^{el} + (1-p)\,\hat{c}_m \cdot\cdot \hat{\varepsilon}_m^{el} = \hat{\sigma}^{ext} \,, \quad (26)$$

where m - index is connected with matrix and h-one with reinforcing phase. If a hard phase is subjected to elastic deforming only one may find with (3) and (26) that

$$\text{a)}\quad \hat{\varepsilon}_m = \hat{\varepsilon}^T - \frac{\hat{c}_m^{-1}}{1-p} \cdot\cdot \left(\hat{\sigma}^{ext} - p\,\hat{c}_h \cdot\cdot \hat{\varepsilon}^T \right) \quad (27)$$

$$\text{b)}\quad \hat{\varepsilon} = (1-p)\,\hat{\varepsilon}^T - \hat{c}_m^{-1} \cdot\cdot \left(\hat{\sigma}^{ext} - p\,\hat{c}_h \cdot\cdot \hat{\varepsilon}^T \right)$$

and then obtain

$$\text{a)}\quad \hat{\xi}_m = p\,\hat{\varepsilon}^T - \frac{p\hat{c}_m^{-1}}{1-p} \cdot\cdot \left(\hat{\sigma}^{ext} - p\,\hat{c}_h \cdot\cdot \hat{\varepsilon}^T \right) \tag{28}$$

$$\text{b)}\quad \hat{\xi}_h = (p-1)\,\hat{\varepsilon}^T + \hat{c}_m^{-1} \cdot\cdot \left(\hat{\sigma}^{ext} - p\,\hat{c}_h \cdot\cdot \hat{\varepsilon}^T \right),$$

where the sum $\varepsilon^T = \hat{\varepsilon}^{el}_s + \hat{\varepsilon}_s = \hat{\varepsilon}^{el} + \hat{\varepsilon}$ is a total macrodeformation of the body.

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